FIITJEE **JEE (Advanced)-2018**

ALL INDIA TEST SERIES

FULL TEST – VIII

Paper 1

Time Allotted: 3 Hours **Maximum Marks: 183**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
4. Each part is further divided into Two sections: **Section-A & Section-C**
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with black pen for each character of your Enrolment No. and write your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

1. ***Section-A (01– 07, 19 – 25, 37 - 43)*** *contains 21 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer*
   Partial Marks **+1** for each correct option provided no incorrect options is selected.

   Section-A (08 – 13, 26 – 31, 44 - 49) *contains 18 questions. Each of 2 Tables with 3 Columns and 4 Rows has three questions. Column 1 will be with 4 rows designated (I), (II), (III) and (IV). Column 2 will be with 4 rows designated (i), (ii), (iii) and (iv). Column 3 will be with 4 rows designated (P), (Q), (R) and (S).*
   Each question has ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

2. ***Section-C (14 – 18, 32 – 36, 50 - 54)*** *contains 15 Numerical based questions with answer as numerical value from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

Name of the Candidate	
Enrolment No.	

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's constant	h	=	6.625×10^{-34} J·s
		=	6.625×10^{-27} erg·s
1 Faraday		=	96500 coulomb
1 calorie		=	4.2 joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

Physics

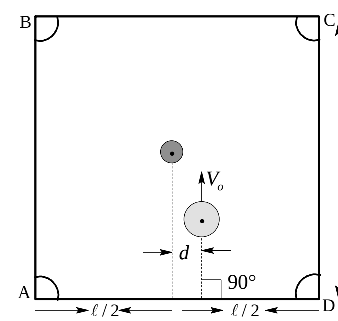
PART – I

SECTION – A
(One or More than one correct type)

This section contains **SEVEN** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

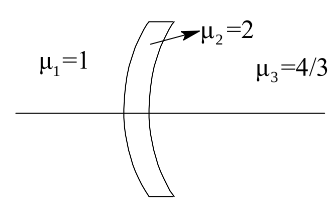
1. Which of the following statements is/are correct for mechanical standing wave on a stretched wire?
(A) Elastic potential energy of a small element at antinode is constant and minimum.
(B) Elastic potential energy of a small element at node is constant and maximum.
(C) Total energy of an element is constant.
(D) Total kinetic energy between two consecutive nodes become maximum twice in one time period.

2. The queen is put at the center of a perfectly smooth carom board (square with side ℓ). The striker strikes the queen with a speed V_o as shown in the figure. Radius of the queen is $\sqrt{10}cm$ and that of the striker is $2\sqrt{10}cm$. Coefficient of restitution for the collision between the queen and the striker is $1/2$ and that for the collision between the queen and the walls of the board is 1.
(Assume $\ell >>>$ radius of queen)



(A) The value of 'd' for which the queen gets in the hole A is 3 cm.
(B) The value of 'd' for which the queen gets in the hole A is 2 cm.
(C) The value of 'd' depends on the coefficient of restitution between the queen and the striker.
(D) The value of 'd' is independent of the coefficient of restitution between the queen and the striker.

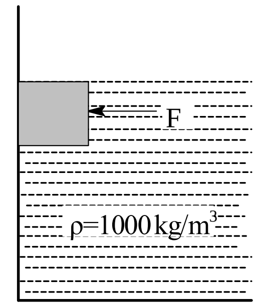
3. A thin lens of same radius of curvature 20cm is having two different medium on its two sides extending upto infinity as shown in the figure. Then



(A) It may behave as a converging lens of focal length 60cm.
(B) It may behave as a diverging lens of focal length 60cm.
(C) It may behave as a converging lens of focal length 80cm.
(D) It may behave as a diverging lens of focal length 80cm.

Space for Rough work

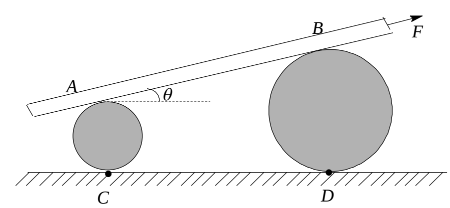
4. A cubical block of mass 5kg and side 10cm is pressed against a rough wall $(\mu=0.9)$ with a force F passing through the centre of cube inside a swimming pool as shown in the figure. Then:



(A) The cube will remain in equilibrium if the force $F \geq 355/9$ Newton.
(B) The cube will remain in equilibrium if the force is $F<355/9$ Newton.
(C) The friction force acting on the cube is 40N if $F=110/3\,\text{N}$.
(D) The friction force acting on the cube is 40N if $F=50\text{N}$.

5. A typical fission reaction is
$${}_{92}U^{235} + {}_{0}n^{1} \rightarrow \left[{}_{92}U^{236}\right] \rightarrow {}_{z_1}X^{A_1} + {}_{z_2}Y^{A_2} + \in {}_{0}n^{1}$$
Which of the following statement(s) is/are correct for above reaction?
(A) $z_1 + z_2 = 92; A_1 + A_2 + \in = 236$
(B) The ratio of masses of $X \,\&\, Y$ is found experimentally to be roughly $3/2$
(C) The number of neutrons $(\in)$ released in the fission of a particular element will depend upon the final fragments that are produced.
(D) The two decay fragments usually have a neutron proton ratio approximately equal to that of the original nucleus.

6. A thin plank of mass m is kept on two rollers such that the centre of mass of the plank is midway between the points of contact with the rollers. Friction is sufficient everywhere to prevent slipping. A force 'F' whose magnitude can be varied is applied parallel to the plank as shown in figure.



(A) System cannot remain in equilibrium if F is greater than mg $\sin\theta$
(B) Friction on the plank on both contact points is always directed towards F, if the system is in equilibrium.
(C) Direction of friction on roller at points C and D is towards right if the system is in equilibrium.
(D) If the rollers are clamped to the surfaces below it so that they cannot move, the system cannot remain in equilibrium for $F \geq mg\sin\theta$

Space for Rough work

7. A hydrogen like atom is observed to emit six wavelengths originating from all possible transitions between a group of levels. These levels have energies between -0.85eV and $-0.544\,\text{eV}$ (including both these levels). Then
(A) The atomic number of the atom is 2
(B) The atomic number of the atom is 3
(C) The smallest wavelength emitted in these transitions is 4052nm.
(D) The difference of principal quantum number of the two levels is 3

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 8, 9 and 10 by appropriately matching the information given in the three columns of the following table.

The column – 1 below represent some wave phenomenon, column – 2 shows the information about frequency related to phenomenon and column – 3 gives the quantities on which phenomenon depends.

	Column 1		**Column 2**		**Column 3**
(I)	Interference	(i)	Different frequencies	(P)	Time
(II)	Beats	(ii)	Apparent frequency changes	(Q)	Relative motion
(III)	Doppler effects	(iii)	Constant frequency	(R)	Length of air column
(IV)	Resonance	(iv)	Same frequencies	(S)	position

8. Which combination is used for calculating speed of sound?
(A) $(\text{I})\ (\text{iv})\ (\text{P})$
(B) $(\text{IV})\ (\text{iv})\ (\text{R})$
(C) $(\text{IV})\ (\text{iv})\ (\text{Q})$
(D) $(\text{II})\ (\text{i})\ (\text{P})$

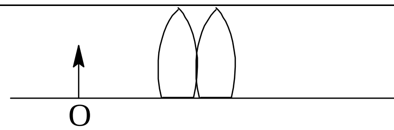
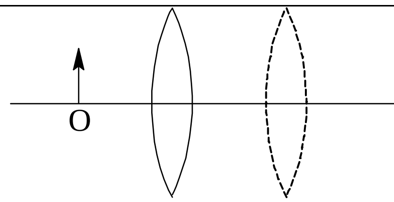
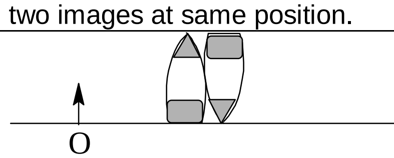
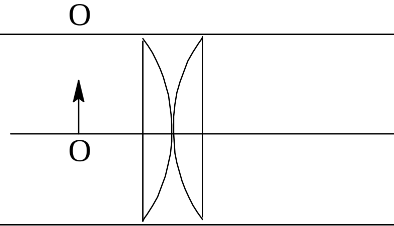
9. For which combination, phenomenon is not detected if the difference in frequencies is very large:
(A) $(\text{II})\ (\text{iv})\ (\text{Q})$
(B) $(\text{II})\ (\text{i})\ (\text{S})$
(C) $(\text{II})\ (\text{iii})\ (\text{P})$
(D) $(\text{II})\ (\text{i})\ (\text{P})$

10. For which combination, phenomenon is not observed if the observer remains stationary:
(A) $(\text{III})\ (\text{ii})\ (\text{R})$
(B) $(\text{III})\ (\text{iv})\ (\text{P})$
(C) $(\text{I})\ (\text{iv})\ (\text{S})$
(D) $(\text{I})\ (\text{ii})\ (\text{Q})$

Space for Rough work

Answer questions 11, 12 and 13 by appropriately matching the information given in the three columns of the following table.

A thin biconvex lens of small a aperture and of focal length f forms image of an object having certain intensity. If this lens is cut into two equal parts in two ways and are used to form image of the same object placed at same distance. In the table below column – 1 represents certain ways in which lens or combination of lenses is placed, column – 2 represent the focal length of the lens or combination of lenses and column – 3 represent the intensity of image in comparison to formed by complete lens:

	Column 1		Column 2		Column 3
(I)		(i)	f	(P)	Decreases
(II)	 Same lens is shifted rightward to get two images at same position.	(ii)	$f/2$	(Q)	Increases
(III)		(iii)	$2f$	(R)	Remain same
(IV)		(iv)	Infinity	(S)	Indeterminate

11. Which combination is correct?

(A) (I) (iii) (Q) (B) (II) (ii) (Q)

(C) (I) (ii) (P) (D) (III) (iv) (R)

Space for Rough work

12. Which combination is used for calculating focal length of a convex lens?

(A) $(\text{IV})\ (\text{ii})\ (\text{Q})$
(B) $(\text{II})\ (\text{i})\ (\text{P})$
(C) $(\text{II})\ (\text{i})\ (\text{R})$
(D) $(\text{III})\ (\text{iii})\ (\text{R})$

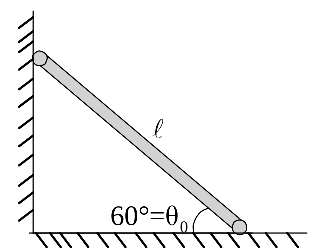
13. Which combination is correct?

(A) $(\text{III})\ (\text{iv})\ (\text{S})$
(B) $(\text{III})\ (\text{ii})\ (\text{R})$
(C) $(\text{IV})\ (\text{iv})\ (\text{P})$
(D) $(\text{IV})\ (\text{i})\ (\text{Q})$

SECTION – C
(Single digit integer type)

This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

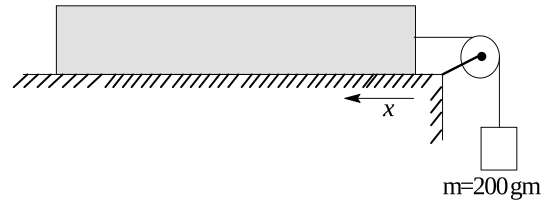
14. A ladder of mass m and length ℓ stands against a frictionless wall with its feet on a frictionless floor. If it is let go at an initial angle $\theta_0 = 60°$ then the angle 'θ' at which the ladder loses contact with the wall is given as $\sin^{-1}\left(1/\sqrt{N}\right)$, find 'N'



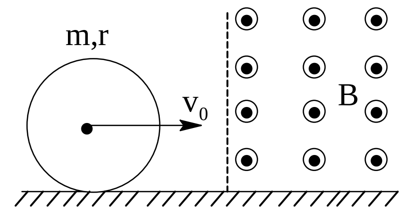
15. An electric charge distribution produces an electric field $\vec{E} = C\left(1 - e^{-\alpha r}\right)\frac{\hat{r}}{r^2}$ where $C = \frac{1}{4\pi\varepsilon_0}$ & α are constant. If the net charge within the radius $r = \frac{1}{\alpha}$ is $\left(1 - e^{-N}\right)$, then find the value of 'N' ?

Space for Rough work

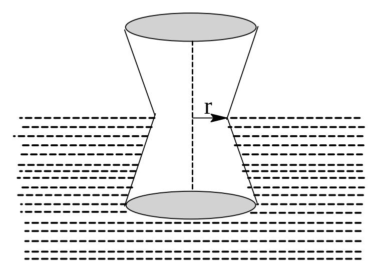
16. Uniform rope of mass = 5kg and length 1 meter is lying on a rough horizontal surface. Coefficient of static friction varies from right end of the rope as $\mu = \mu_0 x$ where $\mu_0 = 0.5$ per meter. A block of mass 200 gm is hanging from an ideal string which passes over an ideal pulley as shown in the figure. The minimum value of x (in cm) for which tension at some cross-section of rope becomes zero is $10\times\text{N}$. Find N.



17. A ring of mass m and radius r is made of an insulating material carries uniformly distributed charge. Initially it rests on a frictionless horizontal tabletop with its plane vertical. The charge on the ring, such that it starts rolling on entering completely into the region of the magnetic field, is $\frac{\sqrt{\text{N}}mv_0}{rB}$,then find the value of 'N'



18. A solid object of mass $\frac{22}{7}$kg is in the shape of pellet drum is half submerged in water of density 1000 kg/m^3 with dimensions as shown in the figure. Find the time period (in seconds) of small vertical oscillations of the drum. [Take $r=\frac{22}{7}$cm]



Space for Rough work

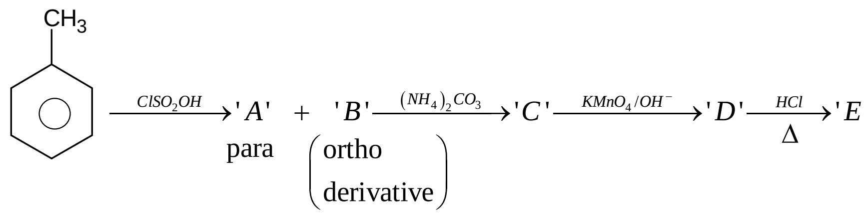
Chemistry

PART – II

SECTION – A
(One or More than one correct type)

This section contains **SEVEN** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

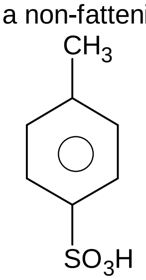
19.



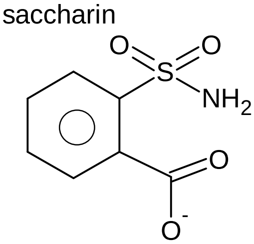
(A) 'E' is a non-fattening sweetener

(B) 'E' is saccharin

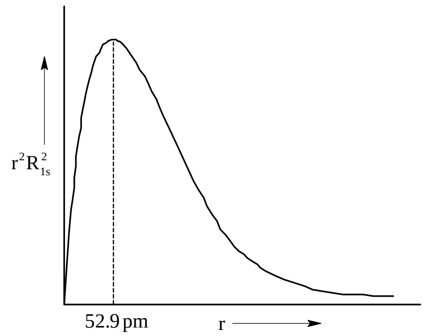
(C) 'A' is



(D) 'D' is



Space for Rough work

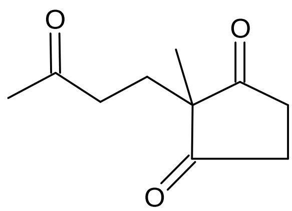
20. Which of the following option is/are correct?
(A) Atomic orbitals are completely described as the regions where the probability of finding the electron is maximum.
(B)


The weighted average of large number of observations for measuring the radius of 1s orbital is greater than $52.9\,pm$ (r^2R^2 dr represents the total probability of finding the electron between r and r+dr).
(C) The energy of 4s is always lower than 3d for multi electronic atom/ ion.
(D) Energy needed to excite an electron from $n=2$ to $n=4$ state is $\frac{25}{28}$ times the energy needed to excite an electron from n=2 to n=5 for a single electron atom / ion.

21. The correct statement(s) about the surface properties is (are):
(A) Soap lather is colloidal solution in which gas is dispersed in liquid.
(B) The surface coverage increases on increasing the pressure for chemisorption and the surface coverage is higher for undissociative process than the dissociative process (e.g. H_2 to 2H) under identical conditions.
(C) On increasing the concentration of cationic surfactant, surface tension decreases before CMC
(D) CMC for non-ionic surfactant is higher than anionic surfactant.

22. $NiO(Green)$ is doped with colorless Li_2O, to give black solid $Li_xNi_{1-x}O$ which acts as semiconductor:
(A) $Li_xNi_{1-x}O$ exhibit both cationic and anionic vacancies
(B) $Li_xNi_{1-x}O$ exhibit Schottky defect
(C) Doping of NiO with Li_2O induces mixed valency of Ni
(D) NiO becomes p-type semiconductor

Space for Rough work

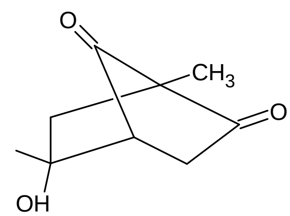
23. 

$\xrightarrow[\Delta]{H^{\oplus}} 'A' \xrightarrow[80^\circ C]{NaCN,HCN} 'B' \xrightarrow[Cold]{HCl} 'C' \xrightarrow[Br_2]{NaOH} 'D'$

(A) Compound 'D' gives positive iodoform test

(B) Compound 'D' gives positive carbylamines test

(C) Compound 'C' gives positive 2, 4-DNP test

(D) Compound 'A' is 

24. The correct option(s) is /are:

(A) F-F < Cl-Cl < Br-Br < I-I (Bond length)

(B) Bond angle of $F_{eq.}$ - S - $F_{eq.}$ bond is less in CH_2SF_4 than SOF_4

(C) H_2S < O_3 < SO_2 < NO_2 (Bond angle)

(D) AsH_3 < SbH_3 < NH_3 < H_2O (boiling point)

25. $Cl_2(g) \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2Cl(g)$

$Cl(g) + CHCl_3(g) \xrightarrow{k_2} HCl(g) + CCl_3(g)$

$CCl_3(g) + Cl(g) \xrightarrow{k_3} CCl_4(g)$

$k_1 = 4.8\times10^3 \quad k_{-1} = 1.2\times10^3 \quad k_2 = 1.3\times10^{-2} \quad k_3 = 2.1\times10^2$

(A) Order of reaction is 3/2

(B) Magnitude of overall rate constant is 2.6×10^{-2}

(C) If conc. of $CHCl_3$ is increased four times rate of reaction increase by a factor of two

(D) If conc. of Cl_2 is increased four time rate of reaction increase by a factor of two

Space for Rough work

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 26, 27 and 28 by appropriately matching the information given in the three columns of the following table.

Consider all the gases as ideal and irreversible process is carried out at constant P_{Final}.

	Column 1		Column 2		Column 3
(I)	$A(g) \rightarrow A(g)$ (10atm, 1mol, 300K,V, ideal gas) (1atm, 1mol 10V)	(i)	$q=0$	(P)	$\Delta U=0$
(II)	$B(s) \xrightarrow{\text{At } 0^0\text{C, 1atm.}} B(liq)$ (melting)	(ii)	$W=0$	(Q)	$\Delta H=0$
(III)	$A(g) \rightarrow A(g)$ (10atm, 1mol, 300K, $C_v=1.5R$) (1atm, 1mol)	(iii)	$\Delta S_{system}>0$	(R)	$Vol._{Final}<Vol._{Inital}$
(IV)	Mixing of ideal gases at constant T and P in an isolated container	(iv)	$\Delta G=0$	(S)	$T_{\text{Final for irr. process}} > T_{\text{Final for rev. process}}$

26. Which of the following combination represents isothermal reversible process?

(A) (II) (ii) (Q)
(B) (I) (iii) (S)
(C) (I) (iii) (Q)
(D) (IV) (iv) (P)

27. Which of the following is correct combination when "B" as H_2O and others are gases as specified?

(A) (II) (ii) (Q)
(B) (II) (iv) (R)
(C) (IV) (iii) (S)
(D) (II) (iii) (S)

Space for Rough work

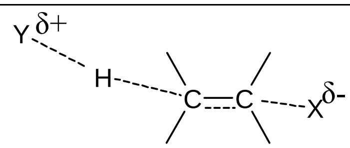
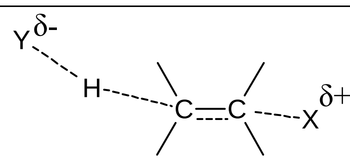
28. Which of the following combination represents the adiabatic process?

(A) (III) (i) (P) (B) (III) (i) (Q)

(C) (III) (i) (S) (D) (IV) (iii) (S)

Answer questions 29, 30 and 31 by appropriately matching the information given in the three columns of the following table.

Consider X as leaving group and Y as a nucleophile or base:

	Column 1 (Activated complex of initial substrate)		Column 2 (Mechanism)		Column 3 (Effect)
(I)	$\overset{\delta+}{Y}$-----R------$X^{\delta+}$	(i)	SN^2	(P)	Large decrease
(II)	$\overset{\delta-}{Y}$-----R------$X^{\delta+}$	(ii)	E_2	(Q)	Large increase
(III)		(iii)	SN^1	(R)	Small decrease
(IV)		(iv)	E_1	(S)	Small increase

29. A neutral nucleophile attacks on a substrate containing neutral leaving group. Which of the following represent the correct combination of effect of increased solvent polarity on reaction rate?

(A) (I) (i) (R) (B) (II) (i) (P)

(C) (I) (iii) (Q) (D) (II) (iii) (S)

Space for Rough work

30. A negatively charged nucleophile attacks on a substrate containing neutral leaving group. Which of the following represent the correct combination of effect of decreased solvent polarity on reaction rate?

(A) (I) (iii) (R)
(B) (II) (i) (P)
(C) (II) (i) (Q)
(D) (I) (iii) (S)

31. A neutral base attacks on a substrate containing an anion as leaving group. Which of the following represent the correct combination of effect of increased solvent polarity on reaction rate?

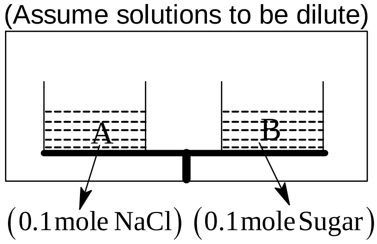
(A) (IV) (iv) (P)
(B) (III) (iv) (Q)
(C) (III) (ii) (S)
(D) (III) (ii) (R)

SECTION – C
(Single digit integer type)

This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

32. On combustion, 1g of 'A' yields 2.9g CO_2. 'A' on easy dehydration with conc. H_2SO_4, gives a hydrocarbon 'B'. 'A' reacts with Na to liberate 0.00275 mole of $H_2(g)$. The empirical formula of 'A' is $C_xH_yO_z$. Then the value of $\frac{x+y}{17}$ will be:

33. Two beakers $A(0.1\,\text{mole NaCl in } 1\,\text{kg } H_2O)$ & $B(0.1\,\text{mole sugar in } 1\,\text{kg } H_2O)$ is placed in a small sized closed container. The molality of solution of Beaker A changes to $\frac{'x'}{40}$. The value of 'x' will be
(Assume solutions to be dilute)



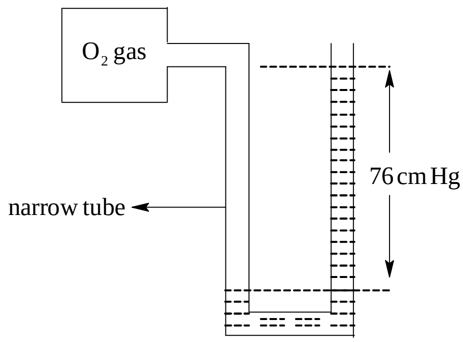
Space for Rough work

34. How many compounds show the cis-trans isomerism?

$\left[Co(NH_3)_4 Cl_2\right]^+$, $\left[Pt(Cl)(Br)(H_2O)(NH_3)\right]$, $\left[Cr(en)_2 ClBr\right]^+$

$\left[Co(NH_3)_3 Cl_3\right]$, $\left[Co(en)_2 Cl_2\right]^+$, $\left[Co(NH_3)_4 (H_2O)(Br)\right]^+$

35.



In above figure, the gas filled in this bulb is subjected for the combustion of 10 moles of CH_4. Maximum number of moles of CO_2 formed in this process is 2. The volume of the bulb is "11.2 x" litre at 280K. Find the approximate integer value of x.

[R=0.08 litre atm $mol^{-1}K^{-1}$, Atmospheric pressure=1atm.]

36. Upon treatment with ammonical H_2S, the metal ion that precipitates as a sulphide is /are,

$Fe^{3+}, Fe^{2+}, Zn^{+2}, Mg^{2+}, Ni^{2+}, Al^{3+}, Cr^{3+}, Cu^{2+}, Ca^{2+}$

Space for Rough work

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

This section contains **SEVEN** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. $I_n = \int_{\frac{n}{2}}^{\left(\frac{n+1}{2}\right)} \frac{\sin\left(\pi \sin^2 \pi x\right)}{\left(\sqrt{2}\right)^x} dx, \quad n \in I$

(A) $\frac{I_n}{I_{n+4}} = 2$

(B) $\frac{I_n}{I_{n+4}} = \frac{1}{\sqrt{2}}$

(C) $\frac{\sum_{n=0}^{\infty} I_{8n}}{I_0} = \frac{4}{3}$

(D) $\frac{\sum_{n=0}^{\infty} I_n}{I_0} = 2$

38. A parabola $S = 0$ has its vertex at $(-9, 3)$ and it touches the x-axis at the origin then equation of axis of symmetry of the aforesaid parabola can be.

(A) $x - y + 12 = 0$

(B) $x - 2y + 15 = 0$

(C) $2x - y + 21 = 0$

(D) $x + y + 6 = 0$

39. The first term of an infinite geometric series is 21. The second term and the sum of the series are both positive integers. All possible values of the second term can be

(A) 12

(B) 14

(C) 18

(D) 20

Space for Rough work

40. Let $f:[0,1]\to[0,1]$ be a continuous function such that $f\left(f\left(x\right)\right)=1$ for all $x\in[0,1]$ then:

(A) $f\left(x\right)$ is many one function

(B) $y=f\left(x\right)$ intersects the line $y=x$ for some $x\in[0,1)$

(C) $\int_0^1 f\left(x\right)\,dx$ has maximum value 1

(D) $\int_0^1 f\left(x\right)dx$ can be less than $\frac{3}{4}$.

41. A parallelopiped is formed, using three non-zero non-coplanar vectors $\vec{a},\vec{b}\ \&\ \vec{c}$ with fixed magnitudes. Angles between any of the vector with normal of the plane determined by the other two is α and the volume of parallelopiped is T and its surface area is Y. If

$\left(\frac{\mathrm{Y}}{\mathrm{T}}\right)=4\left(\frac{1}{|\vec{a}|}+\frac{1}{|\vec{b}|}+\frac{1}{|\vec{c}|}\right)$ then:

(A) $\cos^2\alpha+\cos\alpha=\frac{3}{4}$

(B) $\sin^2\alpha+\sin^4\alpha=\frac{21}{16}$

(C) $\cos^2\alpha+\cos\alpha=\frac{3+2\sqrt{3}}{4}$

(D) $\sin^2\alpha+\sin^4\alpha=\frac{5}{16}$

42. $f:R\to R,\ f\left(x\right)=\begin{cases}(-1)^n & x=\frac{1}{2^n},n\in I-\{0\}\\ 0 & \text{otherwise}\end{cases}$

Which of the statements are **incorrect**?

(A) $y=f\left(x\right)f\left(2x\right)$ is continuous at $x=0$

(B) $y=f\left(x\right)+f\left(2x\right)$ is continuous at $x=0$

(C) $y=f\left(x\right)$ is continuous at $x=2$

(D) $y=f\left(x\right)$ is continuous at $x=3$

43. z_1,z_2,z_3 are three non zero distinct points satisfying $|z-1|=1\ \&\ z_2^2=z_1z_3$ then

(A) $\frac{z_3-z_2}{z_2+z_3-2}$ is purely imaginary

(B) $\mathrm{Arg}\left(\frac{z_2-1}{z_1-1}\right)=2\mathrm{Arg}\left(\frac{z_3}{z_2}\right)$

(C) $\mathrm{Arg}\left(\frac{z_2-1}{z_1-1}\right)=2\mathrm{Arg}\left(\frac{z_3}{z_1}\right)$

(D) $\left|\frac{1}{z_2}-\frac{1}{z_3}\right|+\left|\frac{1}{z_1}-\frac{1}{z_2}\right|=\left|\frac{1}{z_1}-\frac{1}{z_3}\right|$

Space for Rough work

(Matching type - Single Correct Option)

This section contains **SIX** questions of matching type. The section contains **TWO** tables (each having 3 columns and 4 rows). Based on each table, there are **THREE** questions. Each question has **FOUR** options (A), (B), (C), and (D). **ONLY ONE** of these four options is correct.

Answer questions 44, 45 and 46 by appropriately matching the information given in the three columns of the following table.

Trips are taken from warehouse P to cities A, B, C to deliver goods. f_a , f_b , f_c are frequencies of trips to cities A, B, C from P, where overall large number of trips are made, say n in total. E_a , E_b , E_c are expenditure incurred per trip per km from warehouse to cities A, B, C respectively. d_{AB} stands for distance between cities A & B. **(Column 1 and Column – 2 are two given situations for which we have to choose optimum placing of warehouse P, provided in Column - 3)**

	Column 1 (Frequencies of trip to various cities)		**Column 2 (Value of expenditure as per route)** (E_a, E_b, E_c are expenditure in rupees per km.)		**Column 3 (Optimum placing of ware house to minimize overall costing)**
(I)	$f_a = f_b = f_c = \frac{n}{3}$	(i)	$E_a = E_b = E_c = \mu$ (Where μ is a constant)	(P)	P has infinitely many positions
(II)	$f_a = f_b = \frac{n}{2}, \quad f_c = 0$	(ii)	$E_a = d_{PA}, E_b = d_{PB}, E_c = d_{PC}$	(Q)	P must be the centroid of triangle of triangle formed by A, B, C (cities being vertices)
(III)	$f_a = f_c = \frac{n}{2}, \quad f_b = 0$	(iii)	$E_a = \frac{1}{d_{PA}}, E_b = \frac{1}{d_{PB}}, E_c = \frac{1}{d_{PC}}$	(R)	P is such that AB, BC, CA subtends $120°$ at P
(IV)	$f_a = \lambda d_{BC}, f_b = \lambda d_{AC}, f_c = \lambda d_{AB}$ $\left(\lambda = \frac{n}{d_{AB} + d_{BC} + d_{AC}}\right)$	(iv)	$E_a = E_b = \frac{E_c}{2} = \mu$ (Where μ is a constant)	(S)	P must be incentre of triangle ABC

44. Choose the **correct** option

(A) $(\mathrm{I})\ (\mathrm{iii})\ (\mathrm{Q})$
(B) $(\mathrm{III})\ (\mathrm{i})\ (\mathrm{P})$
(C) $(\mathrm{II})\ (\mathrm{ii})\ (\mathrm{R})$
(D) $(\mathrm{IV})\ (\mathrm{i})\ (\mathrm{S})$

45. Choose the **correct** option

(A) $(\mathrm{III})\ (\mathrm{ii})\ (\mathrm{Q})$
(B) $(\mathrm{IV})\ (\mathrm{ii})\ (\mathrm{R})$
(C) $(\mathrm{I})\ (\mathrm{i})\ (\mathrm{R})$
(D) $(\mathrm{II})\ (\mathrm{iii})\ (\mathrm{S})$

Space for Rough work

46. Choose the **incorrect** option

(A) $(\mathrm{II})\ (\mathrm{iv})\ (\mathrm{P})$
(B) $(\mathrm{I})\ (\mathrm{ii})\ (\mathrm{Q})$
(C) $(\mathrm{IV})\ (\mathrm{ii})\ (\mathrm{S})$
(D) $(\mathrm{I})\ (\mathrm{iv})\ (\mathrm{S})$

Answer questions 47, 48 and 49 by appropriately matching the information given in the three columns of the following table.

Match the following **Column(s)**

	Column 1		Column 2		Column 3
(I)	If A & B are two different matrices such that $A^3=B^3 \,\&\, A^2B=B^2A$ and B is non singular.	(i)	$\det(A)=\pm 1$	(P)	$\det A \neq \det B$
(II)	$A=\left[a_{ij}\right]_{4\times 4}$ such that $a_{ij}=\begin{cases}2 & \text{when } i=j\\ 0 & \text{when } i\neq j\end{cases}$	(ii)	A is non-Singular matrix	(Q)	If $\det(A)>0$ $\det(2A)-\det(\text{adj}A)=7$
(III)	Let B be skew symmetric Matrix of order 3×3 with real entries given $I-B$ and $I+B$ are non singular matrices if $A=(I+B)(I-B)^{-1}$ here I represent Identity matrix	(iii)	A is orthogonal matrix	(R)	$\left\{\dfrac{\det(\text{adj}(\text{adj}A))}{7}\right\}=\dfrac{1}{7}$ where $\{.\}$ represent fractional part function
(IV)	Consider $I_{n,m}=\int_0^1 \dfrac{x^n}{x^m-1}dx$ and $J_{n,m}=\int_0^1 \dfrac{x^n}{x^m+1}dx\ \forall\ n>m,\ \ n,m\in N$ And consider the matrices $A=(a_{ij})_{3\times 3}$ where $a_{ij}=\begin{cases}I_{6+i,3}-I_{i+3,3} & i=j\\ 0, & i\neq j\end{cases}$ $B=\begin{bmatrix}J_{6,5} & 72 & J_{11,5}\\ J_{7,5} & 63 & J_{12,5}\\ J_{8,5} & 56 & J_{13,5}\end{bmatrix}$	(iv)	A is symmetric Matrix	(S)	$\det\left(A^2+B^2\right)=0$

47. Which of the following is the only **correct** combination?

(A) $(\mathrm{I})\ (\mathrm{iv})\ (\mathrm{P})$
(B) $(\mathrm{II})\ (\mathrm{iii})\ (\mathrm{Q})$
(C) $(\mathrm{III})\ (\mathrm{iii})\ (\mathrm{Q})$
(D) $(\mathrm{IV})\ (\mathrm{ii})\ (\mathrm{R})$

Space for Rough work

48. Which of the following is the only **incorrect** combination?

(A) (III) (i) (Q) (B) (II) (iv) (R)

(C) (I) (iv) (P) (D) (IV) (ii) (P)

49. Which of the following is the only **correct** combination?

(A) (I) (ii) (S) (B) (II) (iv) (Q)

(C) (III) (i) (P) (D) (IV) (i) (P)

SECTION – C
(Single digit integer type)

This section contains **FIVE** questions. The answer to each question is a single Digit integer ranging from 0 to 9, both inclusive.

50. A line through any point on the curve $x^2 - y^2 = 1, z = 0$ intersects two lines $y = x, z = 1$ and $y = -x, z = -1$. If the locus of this line is $\alpha x^2 + \beta y^2 + \gamma z^2 + \delta = 0$ there the value of $\alpha + \beta + \gamma + \delta$ is

51. Let three lines L_1, L_2 & L_3 belonging to the family $x - 2y + 6 + \lambda(x - y + 2) = 0$, where λ is a parameter, be interior angle bisectors of ΔABC. If the equation $x + 3y - 4 = 0$ represents side AB of the triangle, then find the value of

$$\left[\frac{\Delta}{\sum\left(r\cot\frac{A}{2} + a\right)}\right]$$

(Note: Symbols used have usual meaning in ΔABC *and* $[.]$ denotes G.I.F)

52. Let PT be a tangent from the point $P\left(5, 3 + \sqrt{3}\right)$ to the circle $x^2 + y^2 + 4x - 6y - 3 = 0$, with centre C, at T and AB is secant which passes through P such that BT is the normal at T. If $\mathrm{Ar}(\Delta CAB) + \mathrm{Ar}(\Delta CAT) = \frac{\lambda}{25}$, then find the value of $\left(\left[\sqrt{\lambda}\right] - 15\right)$ ($[.]$ denotes G.I.F)

Space for Rough work

53. If $y = \lambda_1 e^{ax} + \lambda_2 x e^{bx}$, where λ_1, λ_2 are arbitrary constants; is general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ then the value of $\frac{a}{b}$ is

54. Every ray of light, emerging from $(1,2)$, after striking at an elliptical curve, whose eccentricity is $\frac{2\sqrt{5}}{2+\sqrt{5}+\sqrt{45}}$, always passes though (3,6)after reflection . If $\mathrm{P}\ (\alpha,\beta)$ is a point on this curve such that it is at unit distance from origin then $|2\alpha-\beta|$ is

Space for Rough work

FIITJEE **JEE(Advanced)-2018**

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FULL TEST – VIII

PAPER-1

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	**AD**	19.	**ABD**	37.	**AC**
2.	**AD**	20.	**BD**	38.	**AB**
3.	**AC**	21.	**ABCD**	39.	**ABCD**
4.	**AD**	22.	**CD**	40.	**AC**
5.	**ABCD**	23.	**BC**	41.	**AB**
6.	**AB**	24.	**ABC**	42.	**AC**
7.	**BCD**	25.	**ABD**	43.	**ABD**
8.	**B**	26.	**C**	44.	**B**
9.	**D**	27.	**B**	45.	**C**
10.	**C**	28.	**C**	46.	**D**
11.	**C**	29.	**A**	47.	**C**
12.	**B**	30.	**C**	48.	**C**
13.	**A**	31.	**C**	49.	**A**
14.	**3**	32.	**2**	50.	**0**
15.	**1**	33.	**3**	51.	**1**
16.	**4**	34.	**6**	52.	**4**
17.	**2**	35.	**4**	53.	**1**
18.	**2**	36.	**4**	54.	**0**

Physics

PART – I

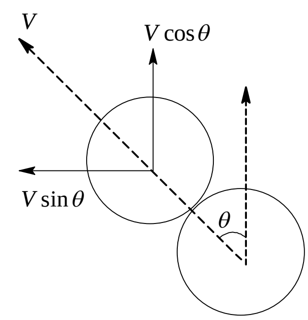
SECTION – A

1. AD

Elastic potential energy $\propto \left(\frac{\partial y}{\partial x}\right)^2$,kinetic energy $\propto \left(\frac{\partial y}{\partial t}\right)^2$. So for antinode elastic potential energy is constant & minimum. $\frac{\partial y}{\partial x}$ always changes for all other points.

2. AD

Let's assume, time taken by the queen to get into the hole is t



$t = \frac{\ell}{2V\sin\theta}$ & also $t = \frac{\ell}{2V\cos\theta} + \frac{\ell}{eV\cos\theta}$

Therefore

$$\frac{\ell}{2V\sin\theta} = \frac{\ell}{2V\cos\theta} + \frac{\ell}{eV\cos\theta}$$

$$\therefore \tan\theta = \frac{1}{3}$$

$$\therefore \frac{d}{\sqrt{(r_1+r_2)^2 - d^2}} = \frac{1}{3}$$

$$d^2 = 9 \Rightarrow d = 3cm$$

3. AC

When parallel light falls from left side

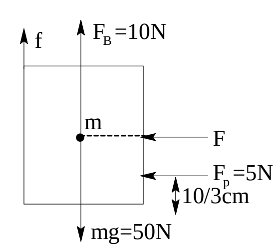
$$\frac{\mu_3}{f} = \frac{(\mu_3 - \mu_2)}{R} + \frac{(\mu_2 - \mu_1)}{R}$$

$$\frac{4/3}{f_1} = \frac{(4/3-2)}{R} + \frac{(2-1)}{R} \Rightarrow f_1 = 80cm$$

When parallel light falls from right side

$$\frac{1}{f_2} = \frac{(1-2)}{-R} + \frac{(2-4/3)}{-R} \Rightarrow f_2 = 60cm$$

4. AD



For translatory equilibrium $f + F_B = mg \Rightarrow f = 40N$

For rotational equilibrium $40\times5 + 5\times\frac{5}{3} = N(5)$

N'=125/3 newton as $F + F_p = N' \Rightarrow F = 110/3\,Newton$

But for $F = 110/3\,Newton$ object will not be in translational equilibrium so for translational equilibrium

$$\frac{9}{10}(F+5) = 40 \Rightarrow F = \frac{355}{9}\text{newton}$$

5. ABCD

6. AB
Normal Reaction on any of the roller always has a horizontal compound directed towards right. Therefore, friction acting on the rollers at every point at contact should have component towards left.
Hence option (C) is incorrect.
Since friction on roller is in opposite direction of F, friction on the plank is in the direction of F. Therefore, for the system to be in equilibrium, F should be less than $mg\sin\theta$ so option A and B are correct.
If rollers are clamped so they cannot move, friction on rollers can be towards left as well as right. Therefore friction on the plank at point A and B can be opposite to F, hence F can be greater then $mg\sin\theta$, so option D is incorrect.

7. BCD
$$0.85=\frac{13.6Z^2}{n_1^2}\ and\ 0.544=\frac{13.6Z^2}{n_2^2}$$
As $n_2-n_1=3\Rightarrow n_1=12, n_2=15$
$$\frac{hc}{\lambda}=RhcZ^2\left(\frac{1}{12^2}-\frac{1}{15^2}\right)\Rightarrow\lambda=4052nm$$

8. B

9. D

10. C

11. C

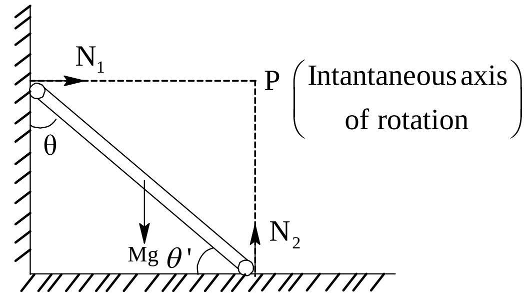
12. B

13. A
Sol. **[Q. 11 – 13]**
Use $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

SECTION – C

14. 3
x coordinate of centre of mass of the rod
$$x=\frac{\ell}{2}\sin\theta$$
$$v_x=\frac{\ell}{2}\cos\theta\omega\ and\ a_x=\frac{\ell}{2}\left(\cos\theta\alpha-\sin\theta\omega^2\right)$$
When $N_1=0\Rightarrow\cos\theta\alpha=\sin\theta\omega^2---(1)$
Using COME:
$$mg\frac{\ell}{2}(\cos\beta-\cos\theta)=\frac{1}{2}m\frac{\ell^2}{4}\omega^2+\frac{1}{2}m\frac{\ell^2}{12}\omega^2$$

$\left[v = \frac{\ell}{2}\omega \text{ from IAOR}\right]$



$$\omega^2 = \frac{3g}{\ell}(\cos\beta - \cos\theta) - - - (2)$$

$$\alpha = \frac{3g}{2\ell}\sin\theta - - - - (3)$$

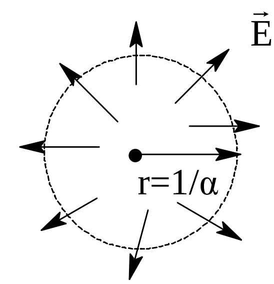
Substituting the value of ω^2 and α in equation (1) gives

$$\Rightarrow \cos\theta = \frac{2}{3}\cos\beta \Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \sin\theta' = \frac{1}{\sqrt{3}} \text{ so N=3}$$

15. 1

As the electric field is radial, by applying gauss law, we can write

$$\int \vec{E}.d\vec{s} = \frac{Q}{\varepsilon_0}$$



For $r = \frac{1}{\alpha}$, $\vec{E} = C\left(1 - e^{-\alpha\times 1/\alpha}\right)\frac{\hat{r}}{(1/\alpha)^2}$

$$\therefore \oint \vec{E}.d\vec{s} = C\left(1 - e^{-1}\right)\alpha^2 \times 4\pi (1/\alpha)^2$$

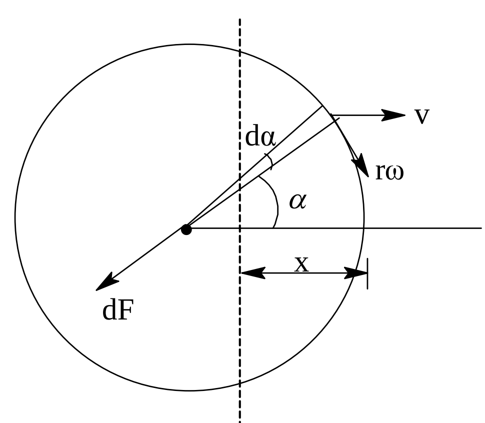
$$\Rightarrow \frac{Q}{\varepsilon_0} = 4\pi C\left(1 - e^{-1}\right) \Rightarrow Q = \left(1 - e^{-1}\right) \Rightarrow \therefore N = 1$$

16. 4

$$\int_0^{x_{\min}} 0.5x\frac{mg}{\ell}dx = 2 \Rightarrow x_{\min} = 40\text{cm}$$

17. 2

$$\text{mr}^2\beta = \int_0^{\theta}\frac{2q}{2\pi}d\alpha\, vBr\cos\alpha$$



$$\beta = \frac{dw}{dt} = \frac{qvB\sin\theta}{\pi\ mr} - - - - (1)$$

$$-ma = \int_0^{\theta} 2\frac{q}{2\pi}d\alpha\, rwB\cos\alpha$$

$$a = \frac{dv}{dt} = \frac{qrwB\sin\theta}{\pi\ m} - - - - (2)$$

$$= \frac{dw}{dv} = \frac{v}{r^2 w} \Rightarrow r^2\int_0^{w} wdw = -\int_{v_0}^{v} v\,dv$$

x=r-rcosθ

dx=rsinθdθ

$$\Rightarrow v = \frac{v_0}{\sqrt{2}}$$

$$\frac{vdv}{dx} = \frac{qrwB\sin\theta}{\pi m}$$

$$\int_{v_0}^{v_0/\sqrt{2}} \frac{vdv}{\sqrt{v_0^2 - v^2}} = \frac{qrB}{m\pi}\int_0^{\pi} \sin^2\theta d\theta$$

$$\Rightarrow q = \frac{\sqrt{2}mv_0}{Br}$$

18. 2

For slight displacement $\left(\pi r^2 x\right)\rho g = ma \Rightarrow T = 2\pi\sqrt{\frac{m}{\pi r^2 \rho g}} \Rightarrow T = 2\sec$

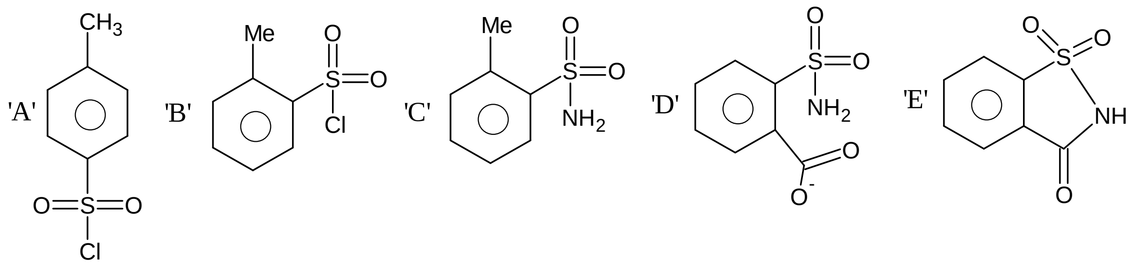
Chemistry

PART – II

SECTION – A

19. ABD

Sol.



20. BD

A.O. is a single e^- wave function

Area of the plot for r<52.9 pm is smaller than the area of the plot of r>52.9pm.

At higher atomic number every of $3d<4s$

$$\Delta E_{2\to 5} = RhcZ^2\left[\frac{1}{2^2}-\frac{1}{5^2}\right] = RhcZ^2\left[\frac{21}{100}\right]$$

$$\Delta E_{2\to 4} = RhcZ^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right] = RhcZ^2\times\frac{3}{16}$$

$$\Delta E_{2\to 5} = \frac{28}{25}\Delta E_{2\to 4}$$

21. ABCD

For dissociative process (e.g.H_2 to 2H), more pressure is required to get the same extent of chemisorption as two species have to be chemisorbed.

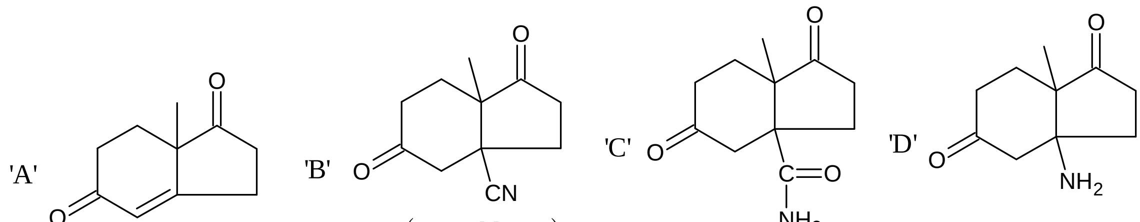
In anionic surfactant negatively charged $-COO^-$ group will repeal each other so at lower concentration micelles will form.

22. CD

Ni^{2+}	O^{2-}	Ni^{2+}	O^{2-}
O^{2-}	Ni^{2+}	O^{2-}	Li^{+}
Ni^{2+}	O^{2-}	Ni^{3+}	O^{2-}

Li^+ ion occupy Ni^{2+} sites to form substitutional defects. In order to maintain the charge neutrality, every Li^+ ion is balanced by Ni^{3+} ion and it becomes a p-type semiconductor.

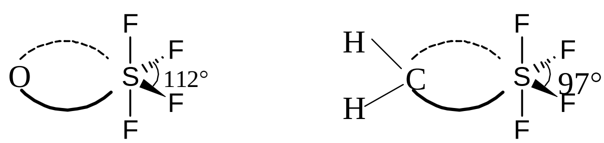
23. BC



$(1,4\ addition)$

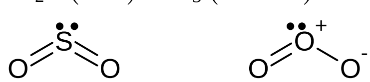
24. ABC

F-F < Cl-Cl < Br-Br < I-I (Bond length)



Oxygen being more EN, the electron density at S in OSF_4 will decreases so it occupies less space so bond angle $F_{eq.}$ - S - $F_{eq.}$ will increased.

$H_2S(92°) < O_3(116.8°) \quad < SO_2(119.5°) \ < NO_2(134°)$



$AsH_3 < NH_3 < SbH_3 < H_2O(BP)$

25. ABD

Rate of reaction = $k_2[Cl][CHCl_3]$ and $\frac{k_1}{k_{-1}} = \frac{[Cl]^2}{[Cl_2]}$

$$[Cl] = \sqrt{\frac{k_1}{k_{-1}}[Cl_2]}$$

Rate of reaction = $k_2 . \sqrt{\frac{k_1}{k_{-1}}}[Cl_2]^{1/2}[CHCl_3]$

$$k = 1.3 \times 10^{-2} \times \sqrt{\frac{4.8 \times 10^3}{1.2 \times 10^3}} = 2.6 \times 10^{-2}$$

26. C
27. B
28. C

Sol. **(Q. 26 – 28)**

(I) Isothermal expansion

(II) Water & Ice will remain in equilibrium to 0°C & 1 atm pressure so $\Delta G = 0$

(III) For adiabatic reversible $T_2P_2^{\frac{1-\gamma}{\gamma}} = T_1P_1^{\frac{1-\gamma}{\gamma}} \Rightarrow T_2 = 300\left(\frac{P_1}{P_2}\right)^{-2/5} = 300\left(\frac{10}{1}\right)^{-2/5} = 120K$

For adiabatic irreversible

$$C_v(T_2\text{-}T_1)\text{=-}P_{ext}(V_2\text{-}V_1)\text{=-}P_2\left[\frac{nRT_2}{P_2}\text{-}\frac{nRT_1}{P_1}\right]$$

$$\frac{C_v}{nR}(T_2\text{-}T_1) = \text{-}P_2\left[\frac{T_2}{P_2}\text{-}\frac{T_1}{P_1}\right]$$

$$\frac{(T_2\text{-}T_1)}{\gamma\text{-}1}\text{=-}T_2+\frac{T_1P_2}{P_1}$$

$$T_2\text{=}T_1\left[\frac{(\gamma\text{-}1)}{\gamma}\frac{P_2}{P_1}+\frac{1}{\gamma}\right]$$

$$T_2\text{=}192K$$

(IV) Mixing of ideal gas at constant T & P in an isolated container

$\Rightarrow q = 0, w = 0, \Delta U = 0, \Delta S_{system} > 0, \Delta G < 0$

29. A
30. C
31. C

Sol. **(Q. 29 to 31)**

Increasing in polarity of the solvent will increase the rate if T.S. has more charge density in comparison to reactant**.**

SECTION – C

32. 2

% of carbon $= \frac{12\times 2.9\times 100}{44} = 79.09$

$$\text{R-OH+Na} \longrightarrow \text{R-ONa+}\frac{1}{2}H_2\left(0.00275\,\text{mole}\right)$$

Moles of oxygen $= 0.00275\times 2$

Mass of oxygen $= 0.00275\times 2\times 16 = 0.088g$

% of oxygen $= 8.8$

So % hydrogen $= 100 - 79.09 - 8.8 = 12.11$

Empirical = $C_{12}H_{22}O$

So $\frac{x+y}{17} = \frac{12+22}{17} = 2$

33. 3

In equilibrium, vapour pressure of solution will remain same

$$\frac{\Delta p}{p*} = \left(\frac{n_{sugar}}{n_{sugar}+n_{H_2O}}\right)_{\text{Beaker B}} = \left[\frac{2\ n_{NaCl}}{2\ n_{NaCl}+n_{H_2O}}\right]_{\text{BeakerA}}$$

$$\frac{0.1}{0.1+\frac{(1-a)\times 1000}{18}}=\frac{2\times 0.1}{2\times 0.1+\frac{(1+a)\times 1000}{18}}$$

$\Rightarrow a = 1/3$

So molarity of NaCl $=\frac{0.1}{\frac{4}{3}}=\frac{0.3}{4}=\frac{3}{40}$

34. 6

$\mathrm{I, II, III, IV, V, VI}$ shows cis & trans isomerism

35. 4

Sol. Pressure of O_2 gas filled inside bulb $=(76+76)\,cm\,Hg = 2\,atm$

O_2 is a limiting reagent.

$$\underset{\text{4 mole}}{\underset{\downarrow}{CH_4}} + \underset{\text{2 mole}}{\underset{\downarrow}{2O_2}} \rightarrow CO_2 + 2H_2O_{(l)}$$

$PV = nRT$

$2\times V = (4)(0.08)(280)$

$V = 44.8L = 11.2\times 4L$

$x = 4$

36. 4

$Fe^{2+}, Zn^{2+}, Ni^{2+}, Cu^{2+}$

Forms precipitate as $\mathrm{FeS, ZnS, NiS, CuS}$ respectively Al^{3+}, Cr^{3+} ions ppt as hydroxides

Fe^{3+} forms precipitate as $\mathrm{FeS}\,(\text{not } Fe_2S_3)$.

Mathematics

PART – III

SECTION – A

37. AC

$$I_{n+k} = \int_{\frac{n+k}{2}}^{\frac{n+k+1}{2}} \frac{\sin\left(\pi \sin^2 \pi x\right)}{\left(\sqrt{2}\right)^x} dx$$

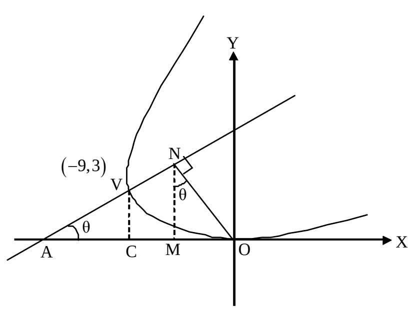
Let $x = \frac{k}{2} + t$

$$I_{n+k} = \int_{\frac{n}{2}}^{\frac{n+1}{2}} \frac{\sin\left(\pi \sin^2 \pi\left(\frac{k}{2}+t\right)\right)}{\left(\sqrt{2}\right)^{\frac{k}{2}+t}} dt$$

$$I_{n+k} = \frac{1}{2^{\frac{k}{4}}} \int_{\frac{n}{2}}^{\frac{n+1}{2}} \frac{\sin\left(\pi \sin^2 \pi t\right)}{\left(\sqrt{2}\right)^t} dt$$

$$I_{n+k} = \frac{I_n}{2^{\frac{k}{4}}}$$

38. AB



$VC = 3 units, CO = 9 units$

As per property: $AV = VN$ and $\frac{AN}{AV} = 2 = \frac{MN}{VC}$

$$\Rightarrow OM = 6\tan\theta, AC = CM = 3\cot\theta \Rightarrow 9 = 3\left(2\tan\theta + \frac{1}{\tan\theta}\right)$$

$$\Rightarrow m = 1 \, or \, m = \frac{1}{2}$$

$\Rightarrow Eq^n$ of Axis can be

$$(y-3) = 1(x+9)$$

$$or$$

$$(y-3) = \frac{1}{2}(x+9)$$

39. ABCD

Let the series be $21, 21r, 21r^2, ...$

$\text{Sum}=\dfrac{21}{1-r}$ is a positive integer

Also 21r is a positive integer

$S=\dfrac{(21)(21)}{21-21r}$ as $21r \in N$ hence $21-21$ r must be an integer

Also $21r<21$

Hence 21-21r may be equal to 1, 3, 7 or 9

i.e must be a divisor of (21) (21)

$\text{hence } 21-21r = 1 \text{ or } 3 \text{ or } 7 \text{ or } 9$

21r=20, 18, 14 or 12

40. AC

$f(x)$ is continuous on a closed interval so it attains a minimum value α.

Since α is in the range of f, $\therefore f(\alpha)=1$. If $\alpha=1, f(x)=1 \forall x$ *and* $\int_0^1 f(x)\,dx = 1$

Now, if $\alpha<1$, by intermediate value theorem, since f is continuous it attains all values between α *and* 1. So for all $x \ge \alpha, f(x)=1$.

There fore

$$\int_0^1 f(x)\,dx = \int_0^\alpha f(x)\,dx + (1-\alpha)$$

Since $f(x) \ge \alpha, \int_0^\alpha f(x)\,dx > \alpha^2$ and the equality is strict because f is continuous and thus cannot be α for all $x<\alpha$ *and* 1 *at* α. So

$$\int_0^1 f(x)\,dx > \alpha^2 + (1-\alpha) = \alpha\left[\alpha - \frac{1}{2}\right]^2 + \frac{3}{4} \ge \frac{3}{4}$$

$$\therefore \frac{3}{4} < \int_0^1 f(x)\,dx \le 1$$

41. AB

$$T=|\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_1|$$

$$=|\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_2|$$

$$=|\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_3|$$

$$Y=2\left(|\vec{a}\times\vec{b}|+|\vec{b}\times\vec{c}|+|\vec{c}\times\vec{a}|\right)$$

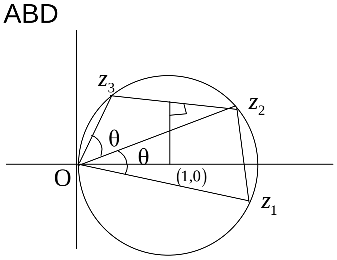
$$=2\left(|\vec{a}||\vec{b}||\sin\theta_1|+|\vec{b}||\vec{c}||\sin\theta_2|+|\vec{c}||\vec{a}||\sin\theta_3|\right)$$

$$\frac{Y}{T}=\frac{2}{|\cos\alpha|}\left(\frac{1}{|\vec{a}|}+\frac{1}{|\vec{b}|}+\frac{1}{|\vec{c}|}\right)=4\left(\frac{1}{|\vec{a}|}+\frac{1}{|\vec{b}|}+\frac{1}{|\vec{c}|}\right)\Rightarrow|\cos\alpha|=\frac{1}{2}$$

42. AC

$\lim\limits_{x\to 0} f(x)f(2x)$ is $-1\, or\, 0$ depending upon $x=\frac{1}{2^n}\, or\, x\neq\frac{1}{2^n}$ but $f(x)+f(2x)$ always tends towards zero $f(2)=f\left(\frac{1}{2^{-1}}\right)=-1$ but $\lim\limits_{x\to 2} f(x)$ is 0, $\lim\limits_{x\to 3} f(x)=f(3)=0$

43. ABD



1st option – chord (Z_3-Z_2) is $\perp$ to line joining $\frac{z_2+z_3}{2}$ and 1.

2nd option – angle by chord (Z_3-Z_2) at $(1,0)$ is double of angle at $(0,0)$

4th option – Ptolemy's theorem

44. B

For option B total cost of all the n trips= $\frac{n\lambda}{2}\left(d_{PA}+d_{PB}\right)$

i.e. we have to minimise $d_{PA}+d_{PB}$.

$d_{PA}+d_{PB}\geq d_{AB}$ where equality holds when p lies on line segment joining A & B

45. C

For option C total cost of all the n trips

$=\frac{n\lambda}{3}\left(d_{PA}+d_{PB}+d_{PC}\right)$

$d_{PA}+d_{PB}+d_{PC}$ is minimum when P is interior point such that AB, BC, CA subtends $120°$ at P.

46. D

For option D total cost of all the n trips

$=\frac{n\lambda}{3}\left(d_{PA}+d_{PB}+2d_{PC}\right)$

As $d_{PA}+d_{PC}\geq d_{AC}$ & $d_{PB}+d_{PC}\geq d_{BC}$

$\Rightarrow d_{PA}+d_{PB}+2d_{PC}\geq d_{AC}+d_{BC}$

Equality holds when P is vertex C

47. C

$A^3=B^3$ & $A^2B=B^2A$ subtracting both we get

$A^2(A-B)=B^2(B-A)$

$\Rightarrow \left(A^2+B^2\right)(A-B)=0$

$\Rightarrow \det.\left(A^2+B^2\right)=0$ (otherwise $A=B$ which is not true)

48. C

49. A

Sol. **(Q. 48 to 49)** By given information

$$a_{ij} = \begin{cases} 2 & \text{when } i=j \\ 0 & \text{when } i \neq j \end{cases}$$

$$A=\left[a_{ij}\right]$$

$$A=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$|A|=2^4$$

$$|\text{adj}\,A| = |A|^{n-1} = (16)^3 = 2^{12}$$

$$\Rightarrow |\text{adj}(\text{adj}\,A)| = |\text{adj}\,A|^3 = 2^{36} = \left(2^3\right)^{12} = (1+7)^{12}$$

$$= {}^{12}C_0 + {}^{12}C_1\, 7^1 + \dots + {}^{12}C_{12}\ \ 7^{12}$$

$$\Rightarrow \left\{\frac{2^{36}}{7}\right\} = \frac{1}{7}$$

Given $A=(I+B)(I-B)^{-1}$

Now, $AA^T = (I+B)(I-B)^{-1}(I+B)^{-1}(I-B)$

$$=(I+B)(I+B)^{-1}(I-B)^{-1}(I-B) = I\times I = I$$

$\therefore |AA^T| = |I| \Rightarrow \ |A|^2 = 1 \Rightarrow |A| = \pm 1$

Given $|A| > 0 \therefore |A| = 1$

$\therefore \det(2A) - \det(\text{adj}\,A)$

$=2^3 \det A - (\det A)^2$

$=8\times 1 - 1^2 = 7$

$$a_{ij} = \begin{cases} I_{6+i,3} - I_{i+3,3}, & i=j \\ 0, & i \neq j \end{cases}$$

$$I_{6+i,3} - I_{3+i,3} = \int_0^1 \left(\frac{x^{i+6}}{x^3-1} - \frac{x^{i+3}}{x^3-1} \right) dx = \int_0^1 x^{i+3}\,dx = \frac{1}{i+4}$$

$$\Rightarrow A=\begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix} \Rightarrow |A| = \frac{1}{5\times 6\times 7} = \frac{1}{210}$$

For $B=\begin{bmatrix} J_{6,5} & 72 & J_{11,5} \\ J_{7,5} & 63 & J_{12,5} \\ J_{8,5} & 56 & J_{13,5} \end{bmatrix}$

Applying $C_1 \to C_1+C_3$

$$|B|=\begin{vmatrix} J_{6,5}+J_{11,5} & 72 & J_{11,5} \\ J_{7,5}+J_{12,5} & 63 & J_{12,5} \\ J_{8,5}+J_{13,5} & 56 & J_{13,5} \end{vmatrix}$$

$$J_{6,5}+J_{11,5}=\int_0^1 \frac{x^6}{x^5+1}dx+\int_0^1 \frac{x^{11}}{x^5+1}dx=\int_0^1\left(x-\frac{x}{x^5+1}\right)dx+\int_0^1\left(x^6-x+\frac{x}{x^5+1}\right)dx=\frac{1}{7}$$

$$\begin{vmatrix} \frac{1}{7} & 72 & J_{11,5} \\ \frac{1}{8} & 63 & J_{12,5} \\ \frac{1}{9} & 56 & J_{13,5} \end{vmatrix}=\frac{1}{7\times8\times9}\begin{vmatrix} 72 & 72 & J_{11,5} \\ 63 & 63 & J_{12,5} \\ 56 & 56 & J_{13,5} \end{vmatrix}=0$$

SECTION – C

50. 0

The required line shall be represented by

$(y-x)+\lambda_1(z-1)=0\ \&\ (y+x)+\lambda_2(z+1)=0$............(1)

Where (x, y, z) is any general point on the line.

At $z=0, x^2-y^2=1$

$(y-x)=\lambda_1$....................(2)

$(y+x)=-\lambda_2$..................(3)

$(2)\times(3)$

$\Rightarrow y^2-x^2=-\lambda_1\lambda_2$

$\Rightarrow -1=-\lambda_1\lambda_2 \Rightarrow \lambda_1\lambda_2=1$

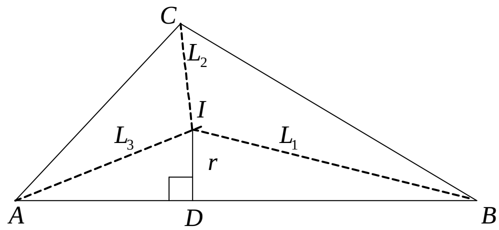
Substituting $\lambda_1\ \&\ \lambda_2$ from (1)

$$\frac{(y-x)}{(z-1)}\times\frac{y+x}{z+1}=1 \Rightarrow y^2-x^2=z^2-1$$

$x^2-y^2+z^2-1=0$

$\alpha+\beta+\gamma+\delta=0$

51. 1



Intersection point of $x-2y+6=0$ *and* $x-y+2=0$ is the Incentre of the ΔABC

$I(2,4)$

$r =$ in radius = Perpendicular distance of I from the line AB

$$=\left|\frac{2+12-4}{\sqrt{10}}\right|$$

$$r=\sqrt{10}$$

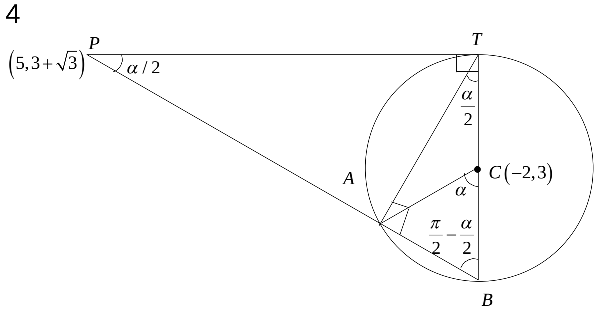
$$\sum\left(r\cot\frac{A}{2}+a\right)=r\cot\frac{A}{2}+a+r\cot\frac{B}{2}+b+r\cot\frac{C}{2}+c$$

$$=s-a+a+s-b+b+s-c+c$$

$$=3s$$

$$\left[\frac{\Delta}{\sum\left(r\cot\frac{A}{2}+a\right)}\right]=\left[\frac{rs}{3s}\right]=\left[\frac{\sqrt{10}}{3}\right]=1$$

52. 4



$$CT=CB=r=4$$

$$PT=\sqrt{5^2+\left(3+\sqrt{3}\right)^2+4\times5-6\left(3+\sqrt{3}\right)-3}$$

$$PT=6$$

Let $\angle ACB=\alpha$

$$Ar(\Delta CAB)=\frac{1}{2}\cdot r\cdot r\cdot\sin\alpha=8\sin\alpha$$

$$Ar(\Delta CAT)=\frac{1}{2}\cdot r\cdot r\cdot\sin(\pi-\alpha)=8\sin\alpha$$

In ΔPBT

$\tan\frac{\alpha}{2} = \frac{BT}{PT} = \frac{8}{6}$

$\Rightarrow \sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{4}{3}}{1+\frac{16}{9}} = \frac{24}{25}$

$Ar(\Delta CAB) + Ar(\Delta CAT) = 16\sin\alpha = 16\times\frac{24}{25} = \frac{384}{25}$

$\Rightarrow \lambda = 384$ hence $\left[\sqrt{384}\right] - 15 = 19 - 15 = 4$

53. 1

Satisfying then given solution in differential equation

get $\left(\lambda_1\left(a^2-2a+1\right)+\lambda_2\left(\left(b^2-2b+1\right)x+2\left(b-1\right)\right)\right)e^{ax}=0$ which must be true for every $\lambda_1 \,\&\, \lambda_2 \in R$ so $a^2-2a+1=0, b^2-2b+1=0 \Rightarrow a=b=1$

54. 0

$(1,2)\ and\ (3,6)$ are foci of ellipse

$2ae = 2\sqrt{5} \Rightarrow 2a = \frac{2\sqrt{5}}{e} = 2+\sqrt{5}+\sqrt{45}$

$\sqrt{(\sin\theta-1)^2+(\cos\theta-2)^2}+\sqrt{(\sin\theta-3)^2+(\cos\theta-6)^2} = \left(1+\sqrt{45}\right)+\left(1+\sqrt{5}\right)$

$\left(1+\sqrt{5}\right) and \left(1+\sqrt{45}\right)$ are maximum distance of $(1,2)$ are $(3,6)$ from circle

$x^2+y^2=1 \Rightarrow (1,2),(3,6),(\alpha,\beta)$ are collinear

$2\alpha-\beta=0$