FIITJEE

JEE (Advanced)-2018

ALL INDIA TEST SERIES

FULL TEST – IX

Paper 2

Time Allotted: 3 Hours **Maximum Marks: 231**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
4. Each part is further divided into three sections: **Section-A, Section-C & Section-D**.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with black pen for each character of your Enrolment No. and write your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

1. ***Section-A (01 – 03, 24 – 26, 47 – 49)*** *contains 9 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
   Section-A (04 – 08, 27 – 31, 50 – 54) *contains 15 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for correct answer and* ***–2 marks*** *for wrong answer.*
   Partial Marks **+1** for each correct option provided no incorrect options is selected.
   Section-A (09 – 10, 32 – 33, 55 – 56) *contains 3 paragraphs. Based upon paragraph, 2 multiple choice questions have to be answered. Each question has* ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer. There is no negative marking.*
2. ***Section-C (11 – 20, 34 – 43, 57 – 66)*** *contains 30 Numerical based questions with answer as numerical value from* ***0 to 9*** *and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*
3. ***Section-D (21 – 23, 44 – 46, 67 – 69)*** *contains 9 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Name of the Candidate	
Enrolment No.	

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$ C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's constant	h	=	6.625×10^{-34} J·s
		=	6.625×10^{-27} erg·s
1 Faraday		=	96500 coulomb
1 calorie		=	4.2 joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

PART – I (Physics), PART – II (Chemistry), PART – III (Mathematics):
(SECTION – D)

For questions **21 to 23, 44 to 46, 67 to 69.**
Numerical answer type questions with answer XXXXX. XX

If answer is 348.4 / 251.37 / 213

Correct Method :

0	0	3	4	8	.	4	0
0	0	2	5	1	.	3	7
0	0	2	1	3	.	0	0

Wrong Method :

	3	4	8		.	4	
3	4	8			.		4
		3	4	8	.		4
	3		4	8	.	4	
	2		5	1	.	3	7
		2	1	3	.		
		2	1	3	.	0	
		2	1	3	.		0
		3	4	8	.	4	0
		2	5	1	.	3	7
		2	1	3	.	0	0

Physics

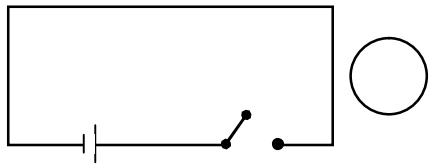
PART – I

SECTION – A

Straight Objective Type

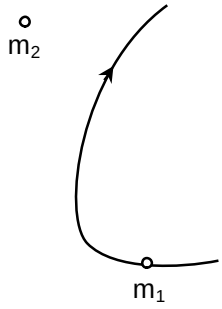
This section contains **3 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which only **ONE** is correct

1. Consider the situation as shown in figure.
If the switch is closed and after some time it is open again, the closed circuit loop will show

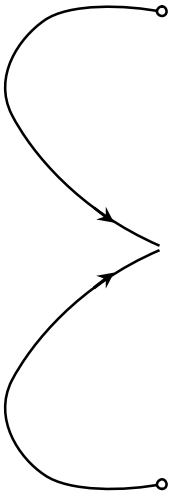


(A) A clockwise current pulse then anticlockwise current pulse.
(B) An anticlockwise current pulse then a clockwise current pulse;
(C) An anticlockwise current pulse then no current then a clockwise current pulse;
(D) A clockwise current pulse then no current then an anticlockwise current pulse;

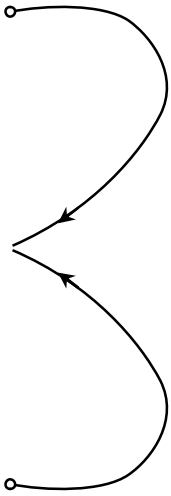
2. Two interacting particles form a closed system whose center of mass is at rest. Figure illustrates the positions of both particles at a certain moment and the trajectory of the particle of mass m_1. Select the trajectory of the particle of mass m_2 if $m_2 = m_1$.



(A)



(B)

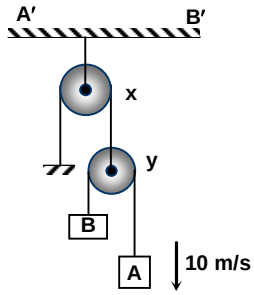


(C)

(D)

Space for rough work

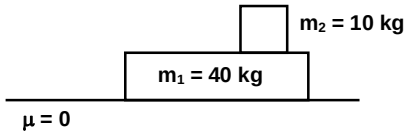
3. In given diagram A′B′ frame is moving with speed 5 m/s in downward direction. Block A is moving with speed 10 m/s in downward direction. Speed of block B will be:
(A) 5 m/s in downward direction
(B) 5 m/s in upward direction
(C) 10 m/s in downward direction
(D) 10 m/s in upward direction



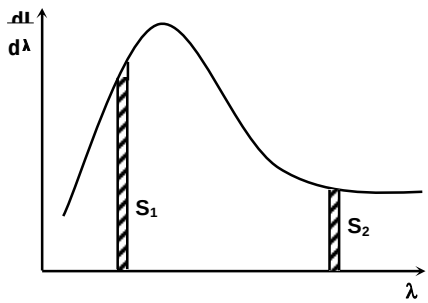
Multiple Correct Choice Type

This section contains **5 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE OR MORE THAN ONE** is/are correct

4. In the figure, a slab of mass m_1 = 40 kg rest on a frictionless floor, and a block of mass m_2 = 10 kg rests on top of the slab. Between block and slab, the coefficient of static friction is 0.60; and the coefficient of kinetic friction is 0.40. Possible value of forces that should be applied to the block m_2 (upper block), so that it causes slipping between the blocks is
(A) 60 N (B) 70 N
(C) 75 N (D) 90 N



5. Two separate segments of equal area are isolated in the energy distribution of black body radiation as shown in figure. Let us assume that the number of photons emitted by the body per unit time in the segments are n_1 and n_2 and the energy of the photons are E_1 and E_2 respectively. Then

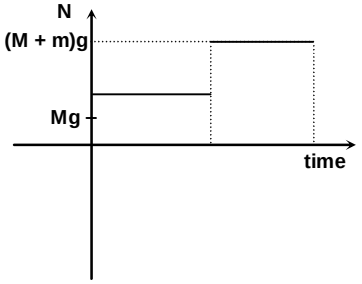


(A) $n_1 > n_2$ (B) $E_1 > E_2$
(C) $n_2 > n_1$ (D) $E_2 > E_1$

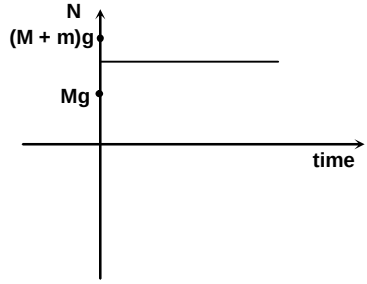
Space for rough work

6. In a long cylindrical container of mass M (including liquid) a solid ball of mass m is released with zero initial speed from the top of container. Viscosity of the liquid is zero and density of the material of ball and that of liquid are ρ_m and ρ_ℓ. N denotes magnitude of normal reaction of the bottom of the cylinder to the liquid. v denotes velocity and upward direction of velocity is drawn along positive direction of y-axis. (given $\rho_m > \rho_l$ and collision of ball with bottom of cylinder is elastic and ignore impulse due to impact with bottom). Choose the correct option(s).

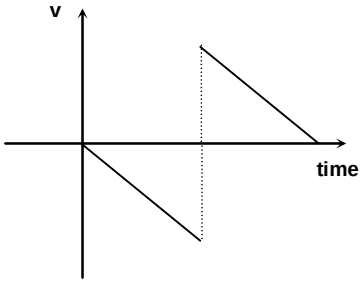
(A)



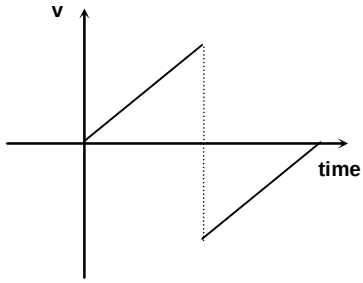
(B)



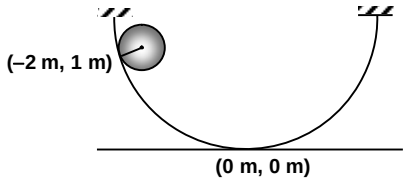
(C)



(D)



7. A disc of mass 3 kg and radius of 7 cm is placed on a rigid parabolic path ($x^2 = 4y$) at position (–2, 1) as shown. It starts from this point at t = 0. If friction is sufficient for pure rolling of the disc. Then (take g = 9.8 m/s^2 and $\sqrt{2} = 1.4$)



(A) Velocity of disc at the position (0, 0) is $9.8\sqrt{\frac{2}{15}}$ m/s

(B) Angular velocity of disc at (0, 0) is $140\sqrt{\frac{2}{15}}$ rad/s.

(C) Normal reaction at bottom point of the path is 49 N approx.

(D) Normal reaction at bottom point of the path is 98 N approx.

8. In a moving coil galvanometer the number of turns N = 30, area of the coil A = 4×10^{-3} m^2 and the magnetic field strength B = 0.3T. To increase its voltage sensitivity by 50% we

(A) increase number of turns to 45

(B) increase area to 9×10^{-3} m^2

(C) increase magnetic field to 0.45 T

(D) Change the material of wire such that its specific resistance would be 2/3rd of the sepecific resistance of the present wire.

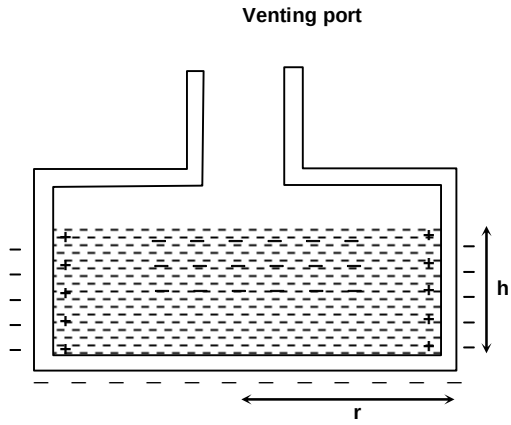
Space for rough work

Comprehension Type

This section contains **1 group of questions.** The group has 2 multiple choice question based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE** is correct

Paragraph for Question Nos. 9 and 10

A safety engineer needs to evaluate the practice of storing inflammable conducting liquids in non-conducting containers. The company supplying the liquid uses a cylindrical plastic container of radius r = 20 cm and fills it to a height of h = 20 cm, which is not equal to its full height. During handling, the exterior surface acquires a charge density of $\sigma = 2\times10^{-6}\,C/m^2$, approximately uniform. This induces a charge separation within the liquid, because it is conducting (see figure).



9. How much charge is induced within the bulk of the liquid? Take $\pi \approx 3$.
(A) 0.36 μC (B) 0.72 μC
(C) 0.18 μC (D) 0.54 μC

10. The minimum potential energy required to cause a spark is 10 mJ. Assume that the capacitance of the central part relative to ground is nearly 36 pF. Calculate the potential energy stored in the capacitor. Take $\pi \approx 3$.
(A) 1.8 mJ (B) 3.6 mJ
(C) 5.4 mJ (D) 7.2 mJ

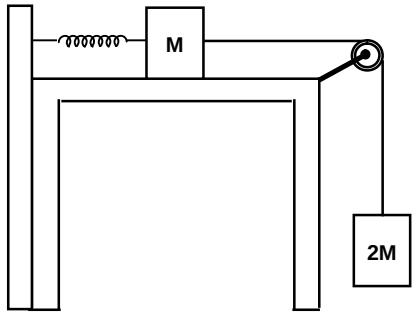
Space for rough work

SECTION – C

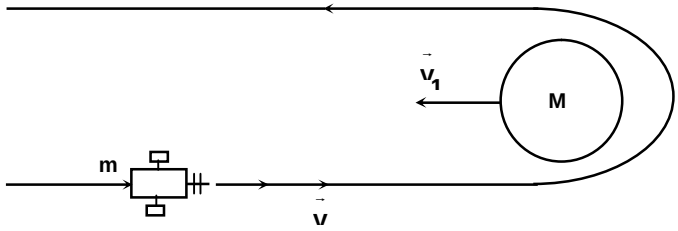
(One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

11. Two blocks, of masses M = 3 kg and 2M, are connected to a spring of spring constant k = 200 N/m that has one end fixed, as shown in figure. The horizontal surface and the pulley are frictionless, and the pulley has negligible mass. The blocks are released from rest with the spring relaxed. The tension in the connecting string when the block 2M has fallen to the maximum extent is 20 k newton, where k is an integer. Find the value of k.



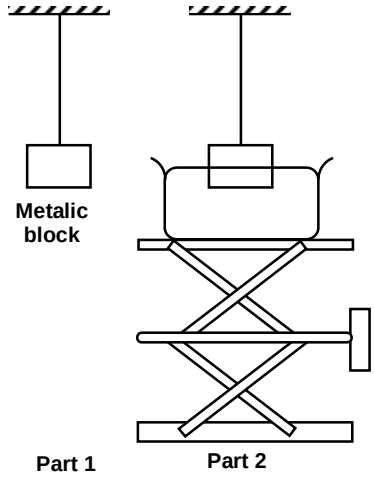
12. An unmanned space probe (of mass m) and speed v = 2 km/s relative to the Sun, approaches the planet Jupiter (of mass M >>m) and speed V_J relative to the Sun as shown in the figure. The spacecraft rounds the planet and departs in the opposite direction. The mass of Jupiter is very much greater than the mass of the spacecraft (M >>m), and it orbits the sun at a distance of 5.29 AU, where 1 AU is the orbital radius of the earth around the sun: 1 AU = 1.5×10^8 km, 1 earth year = $\pi \times 10^7$ s, nearly. The speed of the space probe after it leaves the solar system, relative to the sun is nearly (20 + n) km/s, where n is an integer. Find the value of n.



Space for rough work

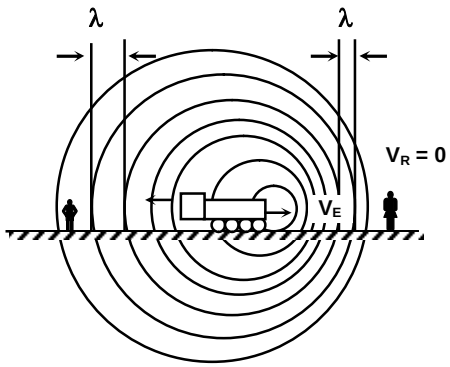
13. A series LCR circuit containing a resistance of 120 Ω has angular frequency 4×10^5 rad/sec. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. At angular frequency ω the current in the circuit lags the voltage by $\pi/4$. If $\omega = n \times 10^5 \frac{\text{rad}}{\text{sec}}$; the value of n is

14. A metallic block is suspended from a wire, as in part 1 of the drawing. A container of mercury is then raised up around the block, as in part 2, so that 50% of the block's volume is submerged in the mercury. The specific gravities of metal is 9.1, and that of mercury is 13.6. Find the ratio of the 1st overtone frequency of the wire in part 1 to the fundamental frequency of the wire in part 2 to the nearest integer.

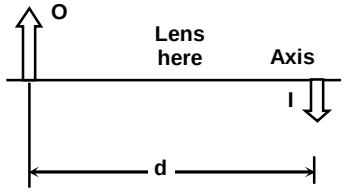


Space for rough work

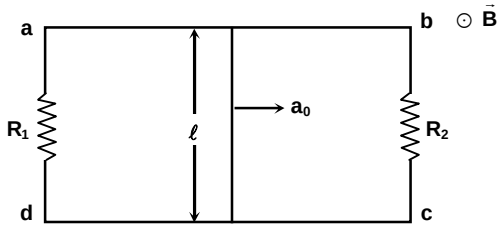
15. Consider sound waves emitted by a moving emitter like the whistle of a moving train as shown in the figure. If the observer were standing on the ground so that sound waves reach him at an angle 60° with the direction of the moving train then:
Let the change in frequency due to the Doppler effect be Δf, and the change in wavelength due to Doppler effect be $\Delta\lambda$. Assume that the speed of sound in air is 1200 km/h, the speed of the engine (emitter) is 400 km/h. The product $\Delta f.\Delta\lambda = \left(\frac{200}{n}\right)$ km/h, where n is an integer. Find the value of n



16. In figure, a real inverted image *I* of an object *O* is formed by a particular lens (not shown); the object–image separation is d = 40 cm, measured along the central axis of the lens. The image is just half the size of the object.
Find the focal length of the lens in cm, to the closest integer.

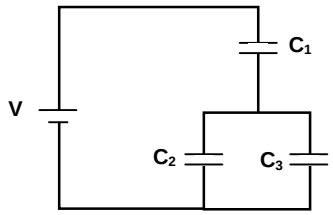


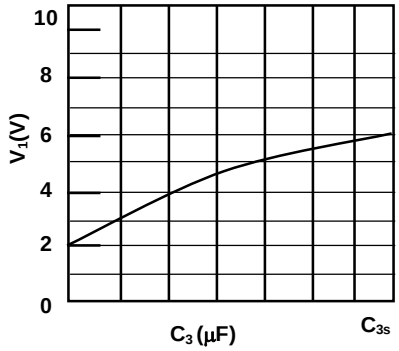
17. A rectangular loop with a sliding conductor of length $\ell = 0.5$ meter is located in a uniform magnetic field B = 0.5 T perpendicular to the plane of loop. The part ad and bc has electrical resistance $R_1 = 50\Omega$ and $R_2 = 50\Omega$ respectively. The conductor of mass m = 1 kg starts moving with constant acceleration $a_0 = 1\,m/s^2$. Neglecting the self inductance of the loop and resistance of conductor, the external force required to move the conductor with given acceleration after n sec. of start is 1.01N. n is



Space for rough work

18. Capacitor 3 in figure (a) is a variable capacitor (its capacitance C_3 can be varied). figure (b) gives the electric potential V_1 across capacitor 1 versus C_3. The horizontal scale is set by C_{3s} = 12 mF. Electric potential V_1 approaches an asymptote of 10 V. Find the ratio C_1/C_2 to the nearest integer.

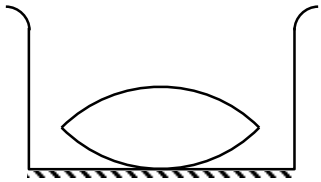




19. The potential energy of a particle of mass 1 gm is given by $U(x)=\begin{cases} a & \text{for } 0 \le x \le 1 \quad \text{Region 1} \\ 0 & \text{for } x > 1 \quad \text{Region 2} \end{cases}$

λ_1 and λ_2 are the de Broglie wavelengths of the particle in Region 1 and Region 2 respectively. If total energy of the particle is 3a and $\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{6}{n}}$. Then n is

20. A thin biconvex lens of refractive index $\frac{9}{5}$ is placed on a horizontal plane mirror as shown. The space between the lens and the mirror is then filled with a liquid of refractive index $\frac{5}{4}$. It is found that when a point object is placed 20 cm above the lens on its principal axis, the object concides with its own image. On repeating with another liquid, the object and the image again concide at a distance 40 cm from the lens. Refractive index of the liquid is $\frac{11n}{40}$ then n is



Space for rough work

SECTION – D

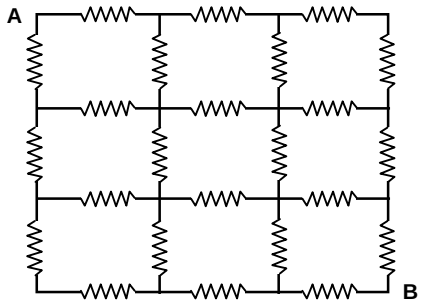
(Numerical Answer Type)

This section contains **3 questions**. Each question, when worked out will result in **numerical answer type** (XXXXX.XX)

21. A uniform circular ring of radius R = 2.5 cm and mass 10 gm is made of an elastic material. Symmetrical radially outward forces are applied on the ring to increase its radius from R = 2.5 cm to 2.7 cm. Young's modulus of material of the ring is $2\times10^{11}\frac{N}{m^2}$ and radius of cross section of the ring is 1 mm. If all the external forces are removed; in how much time will the ring come to its original radius for the first time (in mili second).

22. An ant with negligible mass is standing peacefully on top of a horizontal stretched rope. The rope has mass per unit length μ = 0.01 kg/meter and is under tension of 0.1 Newton. Without warning a boy starts a sinusoidal transverse wave of wavelength λ = 5 cm propagating along the rope. The motion of the rope is in the vertical plane. Assume that the mass of ant is so small that the presence of the ant has no effect on the propagator of the wave. What minimum wave amplitude measured in micrometer will make the ant become momentarily weightless (acceleration due to gravity is g = π^2 m/s^2).

23. 24 resistors of $\frac{28}{65}\Omega$ are arranged in a square as shown. Equivalent resistance of the network between points A and B is



Space for rough work

Chemistry PART – II

SECTION – A

Straight Objective Type

This section contains **3 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which only **ONE** is correct

24. At 48°C, the vapour pressure of pure CS_2 is 850 torr. A solution of 2 gm of sulphur in 100 gm of CS_2 has a vapour pressure 844.9 torr. Determine the atomicity of sulphur molecule
(A) 1 (B) 2
(C) 4 (D) 8

25. Which of the following will act as the best protective.
(A) Gelatin (Gold number = 0.005) (B) Starch (Gold number = 25)
(C) Gum Arabic (Gold number = 0.15) (D) Egg Albumin (Gold number = 0.08)

26. Which of the following is the correct statement?
(A) Physical adsorption takes place due to covalent bonding between adsorbate and adsorbent.
(B) Physical adsorption increases at high temperature.
(C) Physical adsorption is irreversible.
(D) Adsorption enthalpy is generally greater for chemical adsorption than for physical adsorption.

Space for rough work

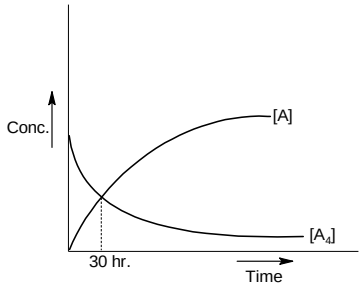
Multiple Correct Choice Type

This section contains **5 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE OR MORE THAN ONE** is/are correct

27. Consider the following 1st order reaction and the accompanying concentration time plot.

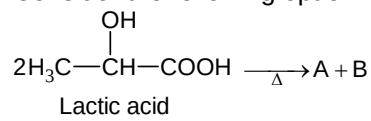
$A_4(g) \longrightarrow 4A(g)$

Which of the following regarding decomposition reaction is/are correct?



(A) At 30 hr, 20% reaction is complete
(B) Half-life of the reaction is 90 hr (approximate)
(C) Rate of decomposition decreases linearly with time.
(D) Changing initial concentration will change the time at which two curve intersect. [Given ln 1.25 = 0.223]

28. Consider the following option is correct



Lactic acid

Which of the following option is correct?
(A) A and B are geometrical isomers. (B) A and B are enantiomers.
(C) A is optically active and B is optically inactive. (D) A is a chiral dissymmetric molecule.

29. Which of the following option is correct?
(A) White phosphorus produce PH_3 with conc. NaOH.
(B) $\bar{S}CN$ is a pseudo halide ion which produces blood red colouration with Fe^{3+}.
(C) He is mixed with O_2 for scuba divers.
(D) Standard hypo solution is used for iodine titrations.

30. An aqueous solution of 6.3 gm of a hydrated oxalic acid ($H_2C_2O_4.xH_2O$) is made up to 250 ml. The 40 ml of 0.1 N NaOH was required to completely neutralize 10 ml of the above prepared stock solution. Which of the following statement(s) are correct?
(A) The acid is dihydrated.
(B) Equivalent weight of the hydrated salt is 45.
(C) Equivalent weight of anhydrous acid is 45.
(D) 20 ml of the same stock solution would require 40 ml 0.1 M $Ca(OH)_2$ for complete neutralization.

31. Which of the following statement is correct?
(A) For any cyclic process, $\Delta U = 0$.
(B) $\left(\frac{\delta U}{\delta V}\right)_T = \frac{a}{V^2}$ for 1 mole of a gas obeying van der Waal's equation.
(C) $\bar{C}_P = \infty$ for water when it is in equilibrium with vapour at 1 atm pressure and 373 K.
(D) For any real gas, the molar internal energy can be given as $\frac{3}{2}R$.

Space for rough work

Comprehension Type

This section contains **1 group of questions.** The group has 2 multiple choice question based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE** is correct

Paragraph for Question Nos. 32 to 33

An inorganic salt (A) is treated with KI solution which gives a white precipitate of compound (R) which further dissolve in excess of KI solution with the formation of soluble complex (S). 'A' gives white precipitate with KCN also which again dissolve in excess of KCN and the resulted solution give no precipitate with Na_2S. 'A' gives green precipitate (T) with $K_3\left[Fe(CN)_6\right]$. 'A' dissolve in excess of NH_4OH solution giving deep blue colouration. It also give chocolate brown precipitate (U) with $K_4\left[Fe(CN)_6\right]$ in acetic acid. Anionic part of salt A gives yellow precipitate with $Hg(NO_3)_2$, and its silver salt is soluble; calcium salt is partially soluble while its Barium salt is insoluble even in HNO_3, $HClO_4$ and HCl etc. solution also.

32. What are R and S respectively?

(A) Cu_2I_2, CuI_4^{3-} (B) CuI_2, CuI_4^{3-}

(C) CuI_2, CuI_3^{2-} (D) Cu_2I_2, CuI_3^{2-}

33. Anionic part (acidic radial) of salt A and compound T are .....

(A) SO_4^{2-}, $Cu_2\left[Fe(CN)_6\right]$ (B) S^{-2}, $Cu_2\left[Fe(CN)_6\right]$

(C) $SO_4^{2-}Cu_3\left[Fe(CN)_6\right]_2$ (D) S^{-2}, $Cu_3\left[Fe(CN)_6\right]_2$

Space for rough work

SECTION – C

(One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

34. How many number of moles of $CH_3 - CH = CH_2$ is consumed by 1 mole of B_2H_6 in the following reaction?

35. The general formula of polythionate ion is $S_{n+2}O_6^{2-}$. If the average oxidation state of 'S' atom in any polythionate ion is equal to the bond order of 'S – O' bond, then calculate the value of 'n' for the corresponding polythionate ion.

36. Equivalent conductance of 0.2 M aqueous solution of a weak monobasic acid [HA] is 10 S cm^2 eqv^{-1}. and that at infinite dilution is 200 S cm^2 eqv^{-1}. Hence, what is the pH of the solution?

37. A definite amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.1 atm pressure. NH_4HS decomposes to give NH_3 and H_2S and at equilibrium, the total pressure in flask is 1.1 atm. If the equilibrium constant K_P for the reaction $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$ is represented on z × 10^{-1}, then z = ?

38. How many optically active isomers of dichlorocyclopentane are possible

Space for rough work

39. $CO + Cl_2 \longrightarrow COCl_2$

The reaction takes place in the following steps.

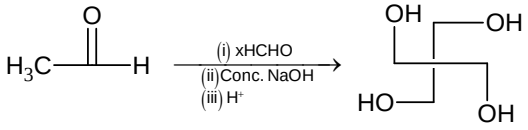
$Cl_2 \underset{}{\overset{K_{eq1.}}{\rightleftharpoons}} 2Cl.(\text{fast})$

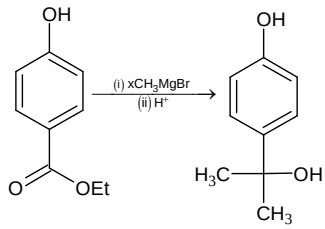
$Cl + CO \underset{}{\overset{K_{eq2.}}{\rightleftharpoons}} COCl.(\text{fast})$

$COCl + Cl_2 \xrightarrow{\text{Slow}} COCl_2 + Cl$

If the order of the reaction is n, what is the value of 2n?

40. If x is the number of moles of formaldehyde consumed in the following reaction, find out the value of x?



41. 

How many moles (x) of Grignard reagent is consumed in the above reaction?

42. Out of the given ions, how many give precipitate with H_2S in acidic medium?
Cr^{3+}, Zn^{2+}, Cd^{2+}, Hg^{2+}, Fe^{2+}, Br^{3+}, Pb^{2+}

43. Ti is purified by following method:

$Ti(\text{impure}) + I_2 \xrightarrow[\Delta]{} \underset{\text{vapour}}{TiI_x} \xrightarrow[\Delta]{} \text{Pure } Ti + I_2$

Find the value of x.

Space for rough work

SECTION – D

(Numerical Answer Type)

This section contains **3 questions**. Each question, when worked out will result in **numerical answer type** (XXXXX.XX)

44. If the density of vapour of a substance (X) at 1 atm pressure and 500 K is 0.8 kg/m^3. The vapour effuses through a small hole at a rate of $\frac{4}{5}$ times slower than oxygen under same condition. What is the compressibility factor of the vapour?

45. If the radiation corresponding to the second line of 'Balmer Series" of Li^{2+} ion, knocked the electron from the 1st excited state of H atom, what is the kinetic energy of electron in eV?

46. Determine the degree of association (for the reaction in the aqueous solution.)
$6HCHO \rightleftharpoons C_6H_{12}O_6$
If the observed molar mass of HCHO and $C_6H_{12}O_6$ is 150.

Space for rough work

Mathematics PART – III

SECTION – A

Straight Objective Type

This section contains **3 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which only **ONE** is correct

47. If y is a function of x satisfies $\frac{dy}{dx} = \frac{y}{x} + \int_1^2 ydx$ and f(1) = 2 then $\int_1^2 ydx$ is equal to

(A) $\frac{12}{7+4\ln 2}$ (B) $\frac{12}{7-8\ln 2}$

(C) $\frac{12}{7+8\ln 2}$ (D) none of these

48. Let line x + 2y = 3 intersects a circle S = 0 at A and B. Let point of intersection of tangents to circle at A and B meet at P(3, 5). If S = 0 passes through origin then radius of circle S = 0 is

(A) $\frac{\sqrt{83}}{8}$ (B) $\frac{\sqrt{85}}{8}$

(C) $\frac{\sqrt{79}}{8}$ (D) $\frac{\sqrt{87}}{8}$

49. A parabola having directrix 2x + y = 3 touches a line x + y = 2 at (3, –1). If focus of parabola is (α, β), then $\alpha + 2\beta$ is equal to

(A) 1 (B) 2

(C) 3 (D) 4

Space for rough work

Multiple Correct Choice Type

This section contains **5 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE OR MORE THAN ONE** is/are correct

50. $\vec{a}$, $\vec{b}$, $\vec{c}$ are three unit vectors such that $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=\frac{1}{3}$. If $\vec{a}\times\vec{b}=p\vec{a}+q\vec{b}+r\vec{c}$ where p, q, r are scalars then

(A) $p^2=\frac{1}{10}$ (B) $p^2=\frac{1}{15}$

(C) $q^2=\frac{1}{15}$ (D) $\frac{r^2}{q^2}=16$

51. $\int_0^{\pi/4} 2^{100}\left(\sec^{100}x+\operatorname{cosec}^{100}\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to

(A) $\int_0^{\ln\left(1+\sqrt{2}\right)} 2\left(e^u+e^{-u}\right)^{99}du$ (B) $\int_0^{\pi/4} 2^{101}\sec^{101}x\,dx$

(C) $\int_0^{\ln\left(\sqrt{2}+1\right)} 4\left(e^u+e^{-u}\right)^{99}du$ (D) $\int_0^{\pi/4} 2^{101}\sec^{100}x\,dx$

52. Let $f:\left[0,\frac{1}{2}\right]\to R$, $e^{-2x}f(x)$ is twice differentiable function having local minima at $x=\frac{1}{4}$ and $\frac{d^2}{dx^2}\left(e^{-2x}f(x)\right)>0\ \forall\ x\in\left(0,\frac{1}{2}\right)$. If $f(0)=f\left(\frac{1}{2}\right)=0$ then which of the following is/are correct?

(A) $\frac{f'\left(\frac{3}{8}\right)}{f\left(\frac{3}{8}\right)}>2$ (B) $\frac{f'\left(\frac{3}{8}\right)}{f\left(\frac{3}{8}\right)}<2$

(C) $\frac{f'\left(\frac{1}{8}\right)}{f\left(\frac{1}{8}\right)}>2$ (D) $\frac{f'\left(\frac{1}{8}\right)}{f\left(\frac{1}{8}\right)}<2$

Space for rough work

53. Let a, b and c be positive real numbers such that a + b + c = 1 then which of the following is/are true

(A) $a^2 + b^2 + c^2 \geq a^a b^b c^c$ (B) $a^2 + b^2 + c^2 \leq a^a b^b c^c$

(C) $a^2 + b^2 + c^2 \leq 1 - 18abc$ (D) $a^2 + b^2 + c^2 \geq 1 - 18abc$

54. If n_1 = number of ways of outcomes if 2 alike and 4 different dice are thrown, n_2 = number of ways of outcomes if 6 alike dice are thrown then n_3 = number of divisors of n_1 of form 4k + 1, $k \in N$

(A) $n_1 + n_2 = 27678$ (B) $n_1 + n_2 = 27578$

(C) $n_3 = 5$ (D) $n_3 = 6$

Comprehension Type

This section contains **1 group of questions.** The group has 2 multiple choice question based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only **ONE** is correct

Paragraph for Question Nos. 55 to 56

Read the following write up carefully and answer the following questions:

A triangle ABC is such that a circle passing through vertex C, centroid G touches side AB at B. If AB = 6, BC = 4 then

55. The length of median through A is equal to

(A) $\frac{13}{2}$ (B) $\sqrt{42}$

(C) $2\sqrt{42}$ (D) none of these

56. Length of AC is equal to

(A) $\sqrt{17}$ (B) $2\sqrt{14}$

(C) $2\sqrt{17}$ (D) $\sqrt{14}$

Space for rough work

SECTION – C

(One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in **one integer** from 0 to 9 (both inclusive).

57. For a function $f(x)=\frac{\ln\left(\{\sin x+3\}\{\cos x+2\}+1\right)}{\{\sin x+1\}\{\cos x+1\}}$ (where {.} denote the fractional part of x), then $f\left(0^-\right)+f\left(\frac{\pi^-}{2}\right)-f\left(\frac{\pi^+}{2}\right)$ is equal to _____

58. $\int_0^{\pi/4} e^{\sec x}\left(\frac{\cos x+\sin x}{2\cos x-\sin 2x}\right)dx=\frac{\left(1+\sqrt{2}\right)}{2}e^{\sqrt{2}}+k$, then 3 ln(2|k|) is equal to _____

59. Dice A has 3 red, 3 white faces where as dice B has 2 red 4 white faces. A fair coin is tossed once. If it shows a head the game continues by dice A, if it shows tail then dice B is to be used. If the probability that dice B is used is $\frac{16}{25}$ where it is given that white throws up every time in first n throws then n is equal to _____

60. For the function f(x) = |x| + |x + 1| + |x – 2| – ||x| + |x + 1| – |x – 2||, the total number of critical points is _____

61. Let a complex number z = x + iy satisfies equation $|z|^4 - 16|z|^2 - 3z^2 - 3\bar{z}^2 + 9 = 0$. If a and b are the maximum and minimum value of |z| then ab is equal to _____

62. Let f: R $\to$ R f(x) = 3^{x-1} and g: R $\to$ R, g(x) = $2x^2 - 4x + 3$, then sum of all roots of equation f(x) = g(x) is _____

Space for rough work

63. If the perpendicular distance of A(1, 4,–2) from the line $\frac{x-2}{2}=\frac{y-1}{6}=\frac{z+2}{3}$ is p then $\frac{7p}{\sqrt{26}}$ is equal to _____

64. A ball moving around the circle $x^2 + y^2 – 2x – 4y – 20 = 0$ in anticlockwise direction leaves it tangentially at P(–2, –2). After getting reflected from a straight 'L' it passes through the centre of circle. If distance of P from line L is $\frac{5}{2}$ and 2α be the angle between incident ray and reflected say then $10 \cot 2\alpha \cot \alpha$ is equal to _____

65. If the line $3x + 4y = \sqrt{7}$ touches the ellipse $3x^2 + 4y^2 = 1$ at (x_1, y_1) then the value of $7x_1^2 + 14y_1^2 + 3$ is _____

66. The number of solution of trigonometric equation $\sin 5x = 16 \sin^5 x$ in $[0, 2\pi]$ is _____

SECTION – D

(Numerical Answer Type)

This section contains **3 questions**. Each question, when worked out will result in **numerical answer type** (XXXXX.XX)

67. Let ABC be right angle triangle, $\angle B = 90^o$. The median through A and C are $y = 3x + 1$ and $y = x + 1$ respectively. If AC = 8 and then area of triangle ABC is _____

68. Consider the cubic polynomial $P(x) = x^3 – ax^2 + bx + c$. If the equation $P(x) = 0$ has integral roots such that $P(6) = 3$ then sum of all possible values of a is _____

69. Six cards and six envelops are numbered 1, 2, 3, 4, 5, 6 and cards are placed in envelops so that each envelop contains exactly one card and no card is placed in the envelop bearing the same number. If card 1 and 2 is always placed in envelope numbered 3 and 4 respectively then number of ways is k then $\frac{k}{3}$ is equal to _____

Space for rough work

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FULL TEST – IX

PAPER-2

Q. No.	PHYSICS	Q. No.	CHEMISTRY	Q. No.	MATHEMATICS
1.	**C**	24.	**D**	47.	**B**
2.	**B**	25.	**A**	48.	**B**
3.	**C**	26.	**D**	49.	**C**
4.	**A, B, C, D**	27.	**A, B**	50.	**B, C, D**
5.	**B, C**	28.	**A, C, D**	51.	**C, D**
6.	**B, C**	29.	**A, B, C, D**	52.	**B, C**
7.	**A, B, C**	30.	**A, C, D**	53.	**A, C**
8.	**B, C, D**	31.	**A, B, C**	54.	**A, C**
9.	**B**	32.	**A**	55.	**B**
10.	**D**	33.	**C**	56.	**B**
11.	**5**	34.	**6**	57.	**1**
12.	**1**	35.	**4**	58.	**3**
13.	**8**	36.	**2**	59.	**2**
14.	**4**	37.	**3**	60.	**4**
15.	**5**	38.	**4**	61.	**3**
16.	**9**	39.	**5**	62.	**6**
17.	**4**	40.	**4**	63.	**3**
18.	**4**	41.	**3**	64.	**5**
19.	**4**	42.	**4**	65.	**6**
20.	**7**	43.	**4**	66.	**7**
21.	**00001.25**	44.	**00001.52**	67.	**00010.66**
22.	**00062.50**	45.	**00019.55**	68.	**00051.00**
23.	**00001.60**	46.	**00000.96**	69.	**00004.66**

Physics

PART – I

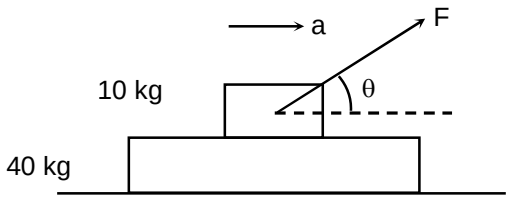
SECTION – A

1. An induced current will be developed in the loop due to change in flux.

2. Centre of mass will be at rest

3. Speed of pulley Y is 10 m/s and speed of block A is 10 m/s so speed of block B will be 10 m/s.

4. $F\cos\theta = 50a$ …(1)
$N + F\sin\theta = 10g$ …(2)
$F\cos\theta - 0.6N = 10a$ …(3)
Solving for a, N from (1), (2) and Putting in (3), we get,
$F = \dfrac{30g}{4\cos\theta + 3\sin\theta}$
$\therefore \quad F_{max} = 60 = 15K \ \& \ K = 4.$



5. Segment S_1 has lower wavelength photons, so $E_1 > E_2$ and Intensity is equal, so $n_2 > n_1$.

6. Liquid will experience a force in downward direction always. So, N will be more than Mg. Magnitude of velocity will increase on downward journey and will decrease in upward journey.

7. By energy conservation
$\Rightarrow v = \sqrt{\dfrac{2}{15}}(9.8)\ m/s$
$\omega = 140\sqrt{\dfrac{2}{15}}\ rad/sec$
at bottom point $N - mg = \dfrac{mv^2}{(R-r)}$, where R is radius of curvature at (0, 0)
$\Rightarrow$ N = 49 Newton (approx)

8. voltage sensitivity = $\dfrac{\text{deflection}}{\text{voltage measured}}$

9. $Q = \sigma\left[\pi r^2 + 2\pi rh\right]$
$= 2\times10^{-6}\times\pi\left[\left(\dfrac{2}{10}\right)^2 + 2\left(\dfrac{2}{10}\right)\left(\dfrac{2}{10}\right)\right]$
$= \dfrac{24\pi}{100}\times10^{-6}$
$\simeq 0.72\ \mu C$

10. $U = \dfrac{Q^2}{2C} = \dfrac{\left(72\times10^{-8}\right)^2}{2\times36\times10^{-12}}$
$= 72\times10^{-4}\ J = 7.2\ mJ$

SECTION – C

11. At mean F = kx and at maximum extension spring energy = $\frac{1}{2}ky^2$.

12. Use momentum conservation principle.

13. At resonance $X_L = X_C$ and $Z = R$

So $I_{rms} = \frac{[V_R]_{rms}}{R} = \frac{60}{120} = \frac{1}{2}$ Amp;

$(V_L)_{rms} = I_{rms}X_L \Rightarrow 40 = \frac{1}{2}X_L \Rightarrow X_L = 80\Omega$

$\Rightarrow L = 0.2mH = 0.2\times10^{-3}H$; and $C = 3.125\times10^{-8}F$;

For the current to lag behind the voltage by $\frac{\pi}{4}$;

$$\tan\frac{\pi}{4} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$\Rightarrow \omega = 8\times10^5 \frac{rad}{sec}$;

14. Use buoyancy principle for tension then calculate frequency.

15. Use formula for Doppler's effect in oblique case.

16. Based on lens formula.

17. $I = \frac{B\ell v}{R_{eq}} = \frac{B\ell a_0 n(R_1+R_2)}{R_1R_2}$ (n is time in second)

So $F_{ext} - F_B = ma_0$

$\Rightarrow F_{ext} = a_0\left[m + \frac{B^2\ell^2(R_1+R_2)n}{R_1R_2}\right]$

$\Rightarrow F_{ext} = \left[1 + \frac{10^{-2}n}{4}\right]$

$\Rightarrow n = 4$;

18. Use the graph to find the value of V_1 for C_3.

19. In Region 1 $K = 2a \Rightarrow \lambda_1 = \frac{h}{\sqrt{4ma}}$

In Region 2 $K = 3a \Rightarrow \lambda_2 = \frac{h}{\sqrt{6ma}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{6}{4}}$

20. According to given condition

$$\left[\frac{9}{5}-1\right]\left[\frac{1}{R}+\frac{1}{R}\right]+\left[\frac{5}{4}-1\right]\left[-\frac{1}{R}-\frac{1}{\infty}\right]=\frac{1}{20}$$

$$\Rightarrow \quad \frac{8}{5R}-\frac{1}{4R}=\frac{1}{20}$$

$$\Rightarrow \quad \frac{27}{20R}=\frac{1}{20} \Rightarrow R=27 \text{ cm}$$

and $\left(\frac{9}{5}-1\right)\left[\frac{1}{R}+\frac{1}{R}\right]+(\mu-1)\left(-\frac{1}{R}-\frac{1}{\infty}\right)=\frac{1}{40}$

$$\Rightarrow \quad \frac{8}{5R}-\frac{\mu-1}{R}=\frac{1}{40}$$

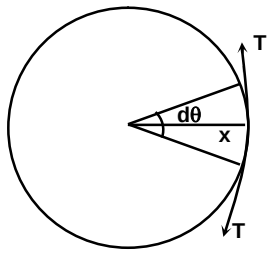
$$\Rightarrow \quad \frac{13-5\mu}{5R}=\frac{1}{40}$$

$$\Rightarrow \quad 13-5\mu=\frac{27}{8}$$

$$\Rightarrow \quad 5\mu=\frac{77}{8} \Rightarrow \mu=\frac{77}{40};$$

SECTION – D

21. Let us suppose that mass per unit length of ring initially is λ; In given condition $0-2T\sin\frac{d\theta}{2}=\lambda R d\theta a$.



$$\Rightarrow \quad a=\frac{-T}{\lambda R} \ \& \ T=\frac{YA}{R}(x-R)$$

$$\Rightarrow \quad a=\frac{-YA}{\lambda R^2}x+\frac{YA}{\lambda R}$$

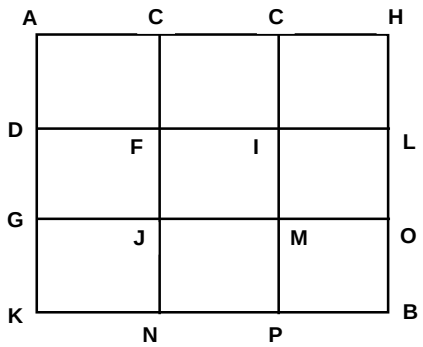
$$\Rightarrow \quad t=\frac{1}{4}\,2\pi\sqrt{\frac{R^2\lambda}{AY}}$$

= 1.25 milisec

22. $A\omega^2 > g$

$$\Rightarrow \quad A>\frac{g}{\omega^2}=\frac{g\lambda^2\mu}{4\pi^2F}=62.5\ \mu m;$$

23. By symmetry of network H; I; J and K are equipotential points.
Besides it potential of D and C; G and E are also same.



Chemistry PART – II

SECTION – A

24. $\frac{P^o - P}{P} = \frac{n}{N} \Rightarrow \frac{W}{M \times N}$

$\Rightarrow \frac{850 - 844.9}{844.9} = \frac{2 \times 76}{M \times 100} \Rightarrow 252$

$n = \frac{252}{32} = 8$

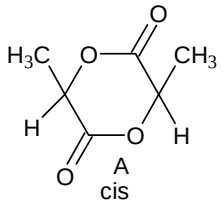
25. Lesser be the gold number, greater be the protective power of the colloid.

27. 4. $\underset{a-x}{A_4(g)} \longrightarrow \underset{4x}{4A(g)}$

At 30 hr, $a - x = 4x \Rightarrow x = \frac{a}{5}$, i.e. 20% A_4 decomposed.

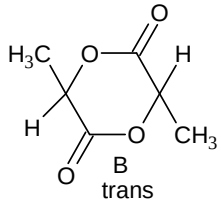
$K \times 30 = \ell n \frac{a}{a-x} = \ell n \frac{5}{4} \Rightarrow t_{1/2} = \frac{30 \ell n 2}{\ell n \frac{5}{4}} \approx 90$ hr

28. $2H_3C-\underset{\text{Lactic acid}}{\underset{OH}{\underset{|}{CH}}-COOH} \longrightarrow$



A
cis

B is optically inactive due to the presence of a point of symmetry.



B
trans

A is optically active, but it is having a C_2 axis, thus, it is dissymmetric.

30. Meq. of oxalic acid $= 40 \times 0.1 \times \frac{250}{100} = 100$

$\therefore$ Eq. weight $= \frac{6.3 \times 1000}{100} = 63$ (hydrated salt.)

Molar mass = 63 × 2 = 126 = 90 + 18 x.

$\Rightarrow$ x = 2.

Equivalent weight of anhydrous acid $= \frac{90}{2} = 45$

Also 20 ml of stock solution $2 \times 40 \times 0.1$

= 8 meq. acid

8 meq. $Ca(OH)_2$ will be present in $= \frac{8}{0.2} = 40$ ml

31. (i) As U is a state function, for any cyclic process, $\Delta U = 0$.

(ii) $\left(\frac{\delta U}{\delta V}\right)_T = \frac{a}{V^2}$.

(iii) During phase transition, T is constant, $dT = 0, \bar{C}_P = \left(\frac{\delta H}{\delta T}\right)_P = \infty$.

(iv) Total molar kinetic energy $= \frac{3}{2}R$.

Internal energy = Total kinetic energy + Potential energy

33. $Cu^{2+} + KI \longrightarrow CuI_2\,(\text{unstable}) \longrightarrow \underset{(\text{white ppt R})}{Cu_2I_2} + I_2 \xrightarrow{\text{excess KI}} CuI_4^{3-}\,(\text{clear solution})(S)$

$$Cu^{2+} + KCN \longrightarrow \underset{\text{unstable}}{Cu(CN)_2} \longrightarrow \underset{\text{white}}{CuCN}\downarrow + \frac{1}{2}(CN)_2\uparrow$$

$$\xrightarrow[\text{excess KCN}]{} \underset{(\text{clear solution})}{\left[Cu(CN)_4\right]^{3-}} \xrightarrow{Na_2S} \text{No ppt}$$

$$Cu^{2+} + K_3\left[Fe(CN)_6\right] \longrightarrow \underset{(\text{Green})(T)}{Cu_3\left[Fe(CN)_6\right]_2}\downarrow + K^+$$

$$Cu^{2+} + \underset{\text{excess}}{NH_4OH} \longrightarrow \left[\underset{\text{deep blue}}{Cu(NH_3)_4}\right]^{2+}$$

$U = Cu_2\left[Fe(CN)_6\right](\text{chocolate brown})$

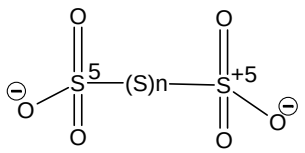
$SO_4^{2-} + Hg^{2+} + H_2O \longrightarrow \underset{\text{yellow}}{HgSO_4.2H_2O}\downarrow (\text{specific test of } SO_4^{2-})$

SECTION – C

34. $CH_3 - CH = CH_2 \xrightarrow[(ii)\ H_2O_2/\bar{O}H]{(i)\ B_2H_6/THF} CH_3 - CH_2 - CH_2OH$

$B_2H_6 \longrightarrow 2BH_3$

$$\underset{(3\text{ mole})}{CH_3 - CH = CH_2} \xrightarrow{BH_3} (CH_3CH_2CH_2)_3B \xrightarrow{H_2O_2/\bar{O}H} 3CH_3CH_2CH_2OH$$

35. 

Bond order of S – O bond $= \frac{5}{3}$

Average oxidation state of 'S' atom $\frac{10}{2+n}$.

$\frac{10}{2+n} = \frac{10}{6} \Rightarrow 2 + n = 6 \Rightarrow n = 4.$

36. $\alpha = \frac{10}{200} = 0.05$

$HA \rightleftharpoons H^+ + A^-$

$\left[H^+\right] = C\alpha = 0.2 \times 0.05 = 0.01$

pH = 2

37. $NH_4HS(s) \rightleftharpoons \underset{0.1+P}{NH_3(g)} + \underset{P}{H_2S(g)}$

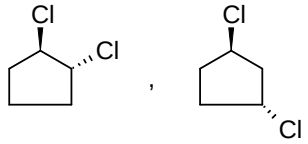
0.1 + 2P = 1.1, thus P = 0.5

$K_P = P_{NH_3}.P_{H_2S} = (0.1+P) \times P$

$= 0.6 \times 0.5 = 0.3$

$= 3 \times 10^{-1}$

Thus, z = 3.

38.  and their enantiomers

39. $\text{Rate} = K[Cl_2][COCl]$

$K_{eq_2} = \dfrac{[COCl]}{[Cl][CO]} \Rightarrow [COCl] = K_{eq_2}[Cl][CO]$

$\dfrac{[Cl]^2}{[Cl_2]} = K_{eq1} \Rightarrow [Cl] = \left\{K_{eq_1}[Cl_2]\right\}^{\frac{1}{2}}$

i.e. $[COCl] = K_{eq_2}K_{eq_1}^{\frac{1}{2}}[CO][Cl_2]^{\frac{1}{2}}$

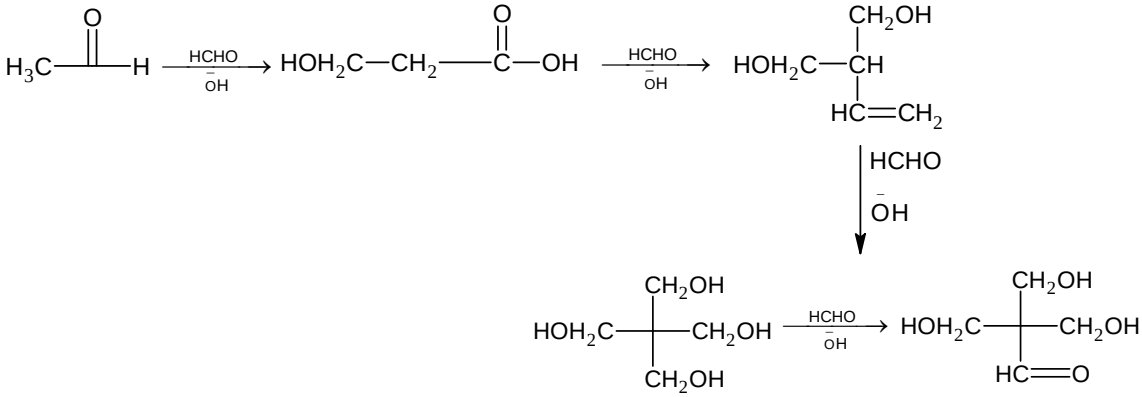
$\text{Rate} = K[Cl_2][COCl]$

$= K[Cl_2]K_{eq_2}K_{eq_1}[CO][Cl_2]^{\frac{1}{2}}$

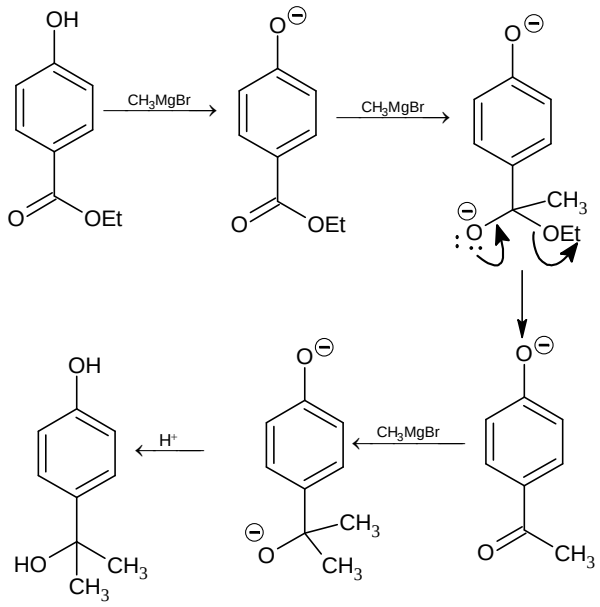
$= K'[CO][Cl_2]^{\frac{3}{2}}$

$\text{Total order} = \dfrac{3}{2} + 1 = \dfrac{5}{2}$

2n = 5

40. 

41.



43. $Ti + 2I_2 \xrightarrow[\Delta]{} TiI_4$

Mathematics PART – III

SECTION – A

47. $\frac{dy}{dx}-\frac{y}{x}=A$ when $A=\int_1^2 ydx$

I.F = $\frac{1}{x}$

So, $y\cdot\frac{1}{x}=\int A\cdot\frac{1}{x}dx+c=A\ln x+c$

$y = A x \ln x + cx$

Now $f(1) = 2 \Rightarrow 2 = c$

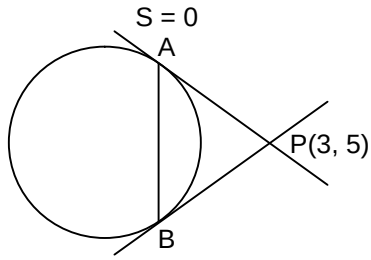
$y = A x \ln x + 2x$

Also, $A=\int_1^2 ydx=\int_1^2 (Ax\ln x+2x)dx$

$= A\left[\ln x\cdot\frac{x^2}{2}-\frac{x^2}{4}\right]_1^2+\left[x^2\right]_1^2 = A\left[\ln 2\cdot 2-1-\left(0-\frac{1}{4}\right)\right]+4-1 = A\left[2\ln 2-\frac{3}{4}\right]+3$

$A\left[\frac{7}{4}-2\ln 2\right]=3;\ A=\frac{12}{7-8\ln 2}$

48. Let circle S = 0 is $x^2 + y^2 + 2gx + 2fy = 0$



Equation of AB as chord of contact

T = 0

$3x + 5y + g(x + 3) + f(y + 5) = 0$

$x(3 + g) + y(5 + f) + 3g + 5f = 0$ ..... (1)

Also, given equation of AB $x + 2y = 3$ ..... (2)

So, $\frac{3+g}{1}=\frac{5+f}{2}=\frac{3g+5f}{-3}$

$6 + 2g = 5 + f$ and $-9 - 3g = 3g + 5f$

$2g - f + 1 = 0$ and $6g + 5f + 9 = 0$

$g=-\frac{7}{8}$ and $f=-\frac{3}{4}$

49. Let foot of perpendicular from (3, –1) on the directrix be (x_1, y_1)

So, $\frac{x_1-3}{2}=\frac{y_1+1}{1}=\frac{-(6-1-3)}{5}$

$\frac{x_1-3}{2}=\frac{y_1+1}{1}=\frac{-2}{5}$

$x_1=3-\frac{4}{5};\ y_1=-1-\frac{2}{5}$

$x_1=\frac{11}{5};\ y_1=-\frac{7}{5}$

Now image of (x_1, y_1) about tangent is focus, which is (x_2, y_2)

$\frac{x_2-\frac{11}{5}}{1}=\frac{y_2+\frac{7}{5}}{1}=\frac{-2\left(\frac{11}{5}-\frac{7}{5}-2\right)}{2}=\frac{6}{5}$

$x_2 = \frac{17}{5}$; $y_2 = \frac{6}{5} - \frac{7}{5} = -\frac{1}{5}$

Focus is $\left(\frac{17}{5}, -\frac{1}{5}\right)$

50. $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}$

Taking dot product with $\vec{a}$, $\vec{b}$, $\vec{c}$

$0 = p + \frac{q}{3} + \frac{r}{3}$ ..... (1)

$0 = \frac{p}{3} + q + \frac{r}{3}$ ..... (2)

$\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \frac{p}{3} + \frac{q}{3} + r$ ..... (3)

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = \begin{vmatrix} 1 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 1 \end{vmatrix} = 1\left(1 - \frac{1}{9}\right) + \frac{1}{3}\left(\frac{1}{9} - \frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{9} - \frac{1}{3}\right)$$

$= \frac{8}{9} - \frac{2}{27} - \frac{2}{27} = \frac{8}{9} - \frac{4}{27} = \frac{20}{27}$

From (1) and (2), we get p = q, r = –4q

From (3) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \frac{q}{3} + \frac{q}{3} - 4q = -\frac{10q}{3}$

$\frac{20}{27} = \frac{100}{9}q^2 \Rightarrow q^2 = \frac{1}{15}$

51. $I = 2\int_0^{\pi/4} 2^{100} \sec^{100} x\,dx = \int_0^{\pi/4} (2\sec x)^{100}\,dx$

Put sec x + tan x = e^u

sec x(sec x + tan x)dx = e^u du

sec x dx = du

Also, 2 sec x = $e^u + e^{-u}$

$I = 4\int_0^{\ln(\sqrt{2}+1)} \left(e^u + e^{-u}\right)^{99} du$

52. Let $\phi(x) = e^{-2x}f(x)$

$\phi'(x) = e^{-2x}(f'(x) - 2f(x)$

Given $\phi''(x) > 0\ \forall\ x \in \left(0, \frac{1}{2}\right)$ and $\phi'\left(\frac{1}{4}\right) = 0$

$\Rightarrow f'\left(\frac{1}{4}\right) = 2f\left(\frac{1}{4}\right)$; $\phi(0) = \phi\left(\frac{1}{2}\right) = 0$

$\Rightarrow f(x) < 0\ \forall\ x \in \left(0, \frac{1}{2}\right)$

So, $f'\left(\frac{3}{8}\right) - 2f\left(\frac{3}{8}\right) > 0$

$\Rightarrow f'\left(\frac{3}{8}\right) > 2f\left(\frac{3}{8}\right);\ f'\left(\frac{1}{8}\right) < 2f\left(\frac{1}{8}\right)$

53. $\frac{a^2+b^2+c^2}{a+b+c} \geq \left(a^a\ b^b\ c^c\right)^{\frac{1}{a+b+c}}$

$a^2 + b^2 + c^2 \geq a^a\ b^b\ c^c$

Also, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq 9$

$ab + bc + ac \geq 9abc$

Now $(a + b + c)^2 = 1$

$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = 1$

$2ab + 2ac + 2bc = 1 - (a^2 + b^2 + c^2)$

$1 - (a^2 + b^2 + c^2) \geq 18\ abc$

$a^2 + b^2 + c^2 \leq 1 - 18\ abc$

54. $n_1 = {}^{2+6-1}C_2 \times 6^4 = {}^{7}C_2 \times 36 \times 36 = 21 \times 36 \times 36 = 27216$

$n_2 = {}^{6+6-1}C_6 = 462$

Now $n_1 = 27216 = 2^4\ 3^5\ 7^1$

n_3 = number of divisors of from $(4k + 1)$ $(k \in N)$ will be 3^2, 3^4, 3×7, $3^3 \times 7$, $3^5 \times 7$ i.e., total 5

55.-56. Let median through A meet BC at D and circle at F

Let GD = x, DF = y then $AG \cdot AF = AB^2$

$2x(3x + y) = 36$ ..... (1)

$xy = 4$

$3x^2 + 4 = 18$

$x^2 = \frac{14}{3}$

So, $AD = 3x = \sqrt{42}$

Also, $AC^2 + AB^2 = 2(AD^2 + BD^2)$

$AC^2 + 36 = 2(42 + 4) = 2 \times 46 = 92$

$AC^2 = 56$

$AC = 2\sqrt{14}$

SECTION – C

57. $f(x) = \frac{\ln\left(\{\sin x\}\{\cos x\}+1\right)}{\{\sin x\}\{\cos x\}}$

$f\left(0^-\right) = \frac{\ln(1\times 1+1)}{1\times 1} = \ln 2$

$f\left(\frac{\pi^+}{2}\right) = \frac{\ln(1\times 1+1)}{1\times 1} = \ln 2$

$f\left(\frac{\pi^-}{2}\right) = \lim_{h\to 0} \frac{\ln\left(\{\cosh\}\{\sinh\}+1\right)}{\{\cosh\}\{\sinh\}} = \lim_{h\to 0} \frac{\ln(\cosh\sinh+1)}{\cosh\sinh} = 1$

58. $I = \int_0^{\pi/4} e^{\sec x} \frac{1}{2}\left[\frac{1+\tan x}{1-\sin x}\right]dx = \frac{1}{2}\int_0^{\pi/4} e^{\sec x}\left(\frac{1+\sin x}{\cos^2 x}+\frac{\tan x(1+\sin x)}{\cos^2 x}\right)dx$

$$= \frac{1}{2}\int_0^{\pi/4} e^{\sec x}\left(\sec^2 x + \tan x \sec x + \tan x \sec^2 x + \tan^2 x \sec x\right)dx$$

$$= \frac{1}{2}\int_0^{\pi/4} e^{\sec x}\left[\sec x \tan x\left(\sec x + \tan x\right) + \left(\sec x \tan x + \sec^2 x\right)\right]dx$$

$$= \frac{1}{2}\left[e^{\sec x}\left(\sec x + \tan x\right)\right]_0^{\pi/4} = \frac{1}{2}\left[e^{\sqrt{2}}\left(\sqrt{2}+1\right) - e\right] = \frac{e^{\sqrt{2}}\left(\sqrt{2}+1\right)}{2} - \frac{e}{2}$$

Here $k = -\frac{e}{2} \Rightarrow 3 \ln (2|k|) = 3$

59. $$\frac{P(T)P\left(\frac{W}{B}\right)}{P(T)P\left(\frac{W}{B}\right) + P(H)P\left(\frac{W}{A}\right)} = \frac{16}{25}$$

$$\frac{\frac{1}{2}\times\left(\frac{4}{6}\right)^n}{\frac{1}{2}\times\left(\frac{4}{6}\right)^n + \frac{1}{2}\left(\frac{3}{6}\right)^n} = \frac{16}{25}$$

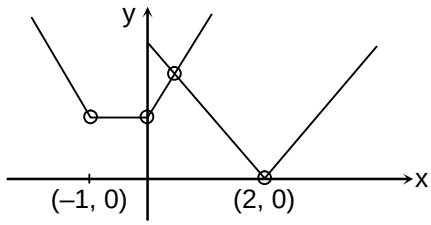
$$\frac{4^n}{4^n + 3^n} = \frac{16}{25} \Rightarrow n = 2$$

60. $f(x) = 2 \min\{|x| + |x+1|, |x-2|\}$

Now, $|x| + |x+1| = 2x + 1, \quad x \geq 0$

$= 1, \quad -1 \leq x < 0$

$= -2x - 1, \quad x < -1$



61. $$|z|^2 - 16 - 3\frac{z^2}{|z|^2} - 3\frac{\bar{z}^2}{|z|^2} + \frac{9}{|z|^2} = 0$$

$$|z|^2 + \frac{9}{|z|^2} - 3\left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right) = 16$$

$$\left(z - \frac{3}{z}\right)\left(\bar{z} - \frac{3}{\bar{z}}\right) = 16$$

$$\left|z - \frac{3}{z}\right| = 4$$

$$\left||z| - \frac{3}{|z|}\right| \leq \left|z - \frac{3}{z}\right| \leq |z| + \frac{3}{|z|}$$

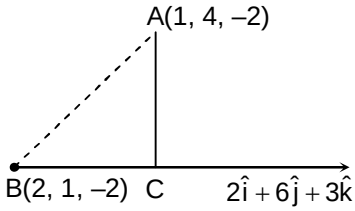
$$\left||z| - \frac{3}{|z|}\right| \leq 4$$

$-4|z| \leq |z|^2 - 3 \leq 4|z|$

$\sqrt{7} - 2 \leq |z| \leq \sqrt{7} + 2$

62. $3^{x-1} = 2x^2 - 4x + 3 = 2(x-1)^2 + 1$ has total 3 solutions
i.e. $x = 1, 2, 3$
Sum of roots = 1 + 2 + 3 = 6

63. $\overrightarrow{BA} = -\hat{i} + 3\hat{j}$



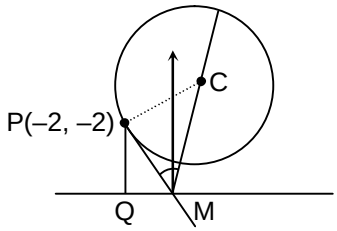
$$AC = \frac{\left|\overrightarrow{BA} \times \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)\right|}{\left|2\hat{i} + 6\hat{j} - 3\hat{k}\right|} = \frac{\left|-6\hat{k} + 3\hat{j} - 6\hat{k} + 9\hat{i}\right|}{\sqrt{4 + 36 + 9}}$$

$$= \frac{\left|9\hat{i} + 3\hat{j} - 12\hat{k}\right|}{\sqrt{49}} = \frac{\sqrt{81 + 9 + 144}}{7} = \frac{\sqrt{234}}{7} = \frac{3\sqrt{26}}{7}$$

$$p = \frac{3\sqrt{26}}{7}$$

$$\Rightarrow \frac{7p}{\sqrt{26}} = 3$$

64. $\tan 2\alpha = \frac{PC}{PM} = \frac{5}{PM}$ ..... (1)



$$\sin(90 - \alpha) = \frac{PQ}{PM}$$

$$PM = \frac{5/2}{\cos\alpha} = \frac{5}{2\cos\alpha} \quad \text{..... (2)}$$

$$5\cot 2\alpha = \frac{5}{2\cos\alpha}$$

$\Rightarrow 10 \cot 2\alpha \cos\alpha = 5$

65. The equation of tangent at (x_1, y_1)
$\Rightarrow 3xx_1 + 4yy_1 = 1$ ..... (1)
Comparing equation (1) with given equation
$3x + 4y = \sqrt{7}$

$$\frac{x_1}{1} = \frac{y_1}{1} = \frac{1}{\sqrt{7}}$$

$$x_1 = \frac{1}{\sqrt{7}}, y_1 = \frac{1}{\sqrt{7}}$$

66. $16\sin^5 x - 20\sin^3 x + 5\sin x = 16\sin^5 x$

Either $\sin x = 0$ or $\sin^2 x = \frac{1}{4}$

$$\Rightarrow \sin x = \pm\frac{1}{2}$$

$$\Rightarrow x = n\pi \text{ or } x = n\pi \pm \frac{\pi}{6},\ n \in I$$

$0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Total number of solution = 7

SECTION – D

67. $\tan\theta_1 = \frac{AB}{BD} = \frac{2c}{a}$

$\tan\theta_2 = \frac{BC}{BE} = \frac{2a}{c}$

$\tan\theta_1, \tan\theta_2 = 4$

Angle between median = $\tan^{-1}\frac{3-1}{1+3} = \tan^{-1}\frac{1}{2}$

$\theta = \pi - \tan^{-1}\frac{1}{2}$

Now $\theta_1 + \theta_2 + \theta = \frac{3\pi}{2}$

$\theta_1 + \theta_2 + \pi - \tan^{-1}\frac{1}{2} = \frac{3\pi}{2}$

$\theta_1 + \theta_2 = \frac{\pi}{2} + \tan^{-1}\frac{1}{2}$

$\tan(\theta_1 + \theta_2) = -\cot\tan^{-1}\frac{1}{2} = -2$

$\frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2} = -2$

$\tan\theta_1 + \tan\theta_2 = -2(1-4) = 6$

$\frac{2c}{a} + \frac{2a}{c} = 6 \Rightarrow \frac{c^2 + a^2}{ac} = 3 \Rightarrow \frac{64}{ac} = 3 \Rightarrow ac = \frac{64}{3}$

Area = $\frac{1}{2}ac = \frac{32}{3}$

68. Let α, β, γ are roots then $x^3 - ax^2 + bx + c = (x-\alpha)(x-\beta)(x-\gamma)$

Put $x = 6$

$(6-\alpha)(6-\beta)(6-\gamma) = 3$

Case-I: $6-\alpha = 1, 6-\beta = 1, 6-\gamma = 3$

$\Rightarrow \alpha = 5, \beta = 5, \gamma = 3$

So, $a = 5 + 5 + 3 = 13$

Case-III: $(6-\alpha) = -1, 6-\beta = -1, 6-\gamma = 3$

$\alpha = 7, \beta = 7, \gamma = 3$

$a = 17$

Case-III: $6-\alpha = -1, 6-\beta = 1, 6-\gamma = -3$

$\alpha = 7, \beta = 5, \gamma = 9$

$a = 21$

69. The number of ways = D(4) + 2D(3) + D(2) = 9 + 4 + 1 = 14 = k then $\frac{k}{3} = 4.66$