FIITJEE SANKALP TEST

(Batches: SANKALP – 46R & 46W)

IIT – JEE, 2016

(CLASS XII)

Paper Code
0000.0

ANSWERS

PHYSICS (PART-I)

SECTION – A

1.	A	(P120405)
2.	B	(P120404)
3.	C	(P120406)
4.	B	(P120406)
5.	A	(P120423)
6.	A	(P120405)
7.	C	(P120406)
8.	A	(120401)
9.	B, D	(P120414)
10.	A, C	(P120418)
11.	A, B, C	(P120406)
12.	B, C	(P120413, P120414)

SECTION – B

1.	(A) → (r, s), (B) → (r, s), (C) → (q, s), (D) → (p, s)	(P120406)
2.	(A) → (r), (B) → (r), (C) → (s), (D) → (r)	(P120409)

SECTION – C

1.	5	(P120401)
2.	5	(P120406)
3.	4	(P120406)
4.	4	(P120413, P120414)
5.	2	(P120414)
6.	8	(P120414)

CHEMISTRY (PART-II)

SECTION – A

1. **D (C120303)**
2. **B (C120303)**
3. **D (C120304)**
4. **B (C120209)**
5. **A (C120209)**
6. **A (C120211)**
7. **B (C120303)**
8. **D (C120302)**
9. **A, C, D (C120205)**
10. **B, C (C120305)**
11. **A, B, D (C120209)**
12. **A, C, D (C120303)**

SECTION – B

1. **(A → p, r, s, t); (B → p, q, s, t); (C → q); (D → p, t) (C120304)**
2. **(A → p,q); (B → r, s); (C → q); (D → t) (C120207)**

SECTION – C

1. **4 (C120207)**
2. **9 (C120304)**
3. **7 (C120207)**
4. **5 (C120304)**
5. **4 (C120304)**
6. **6 (C120209)**

MATHEMATICS (PART-III)

SECTION – A

1. **B (M121001)**
2. **C (M121001)**
3. **A (M121001)**
4. **B (M121001)**
5. **D (M120901)**
6. **C (M120904)**
7. **B (M120908)**
8. **A (M120912)**
9. **A, B, D (M120902)**
10. **B, C (M120903)**
11. **A, B, C (M120907)**
12. **A, C (M121003)**

SECTION – B

1. **(A → q); (B → p); (C → s); (D → r) (M121005); (M121005); (M121001); (M121003)**
2. **(A → p), (B → q), (C → r), (D → s) (M120904); (M120905); (M120911); (M120908)**

SECTION – C

1. **2 (M120912)**
2. **2 (M120904)**
3. **1 (M120912)**
4. **7 (M120905)**
5. **2 (M121001)**
6. **1 (M121001)**

HINTS AND SOLUTION

PHYSICS

SECTION – A

1. **A (P120405)**
2. **B (P120404)**

$$\int_a^{3a}\frac{\mu_0 IL}{2\pi}\left(\frac{dx}{x}+\frac{dx}{4a-x}\right) = \frac{\mu_0 IL}{2\pi}\left[\ln x-\ln(4a-x)\right]_a^{3a} = \frac{\mu_0 IL\ln 3}{\pi}$$

3. **C (P120406)**

$y = 2A\sin kx\cos\omega t$

Velocity $V = \frac{dy}{dt} = -2A\omega\sin kx\sin\omega t$

$V_{max} = -2A\omega\sin kx$

emf (maximum) = $e = \left|\int_0^{\ell} Bv_{max}\cdot dx\right| = \frac{4BA\omega\ell}{3\pi}$

4. **B (P120406)**

When all particles on wire at mean position, the induced emf will be maximum because velocity is maximum at mean position.

= B (aω) dx

where $a = A\sin\frac{\pi x}{L}$

∴ net maximum induced emf = $\int_0^L B\omega A\sin\frac{\pi x}{L}dx = \frac{2BA\omega L}{\pi}$.

5. **A (P120423)**

$2(k2x_0) = mg + iBL$

$mg = 2kx_0$

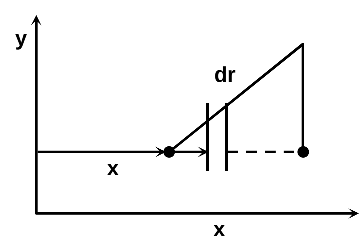
$i = \frac{\varepsilon}{R}$

∴ $B = \frac{mgR}{\varepsilon L}$.

6. **A (P120405)**

7. **C (P120406)**

emf along the path ACD must be equal to rate of change of flux through ΔACD, because no emf will be produced in the straight segment AD.



$d\phi = -B_0(x + r)(rdr)$

$\Rightarrow \phi = -B_0\int_0^{L\cos(\pi/4)}(xr + r^2)dr$

$= -B_0\left[\frac{xr^2}{2}+\frac{r^3}{3}\right]_0^{L/\sqrt{2}}$

$\Rightarrow \frac{d\phi}{dt} = -B_0\frac{L^2}{4}\frac{dx}{dt} = -\frac{B_0vL^2}{4}$

$\Rightarrow |E| = \frac{B_0vL^2}{4}$

8. **A (120401)**

Rate of change of area of the loop $\frac{dA}{dt} = A.\beta\frac{dT}{dt} = A.(2\alpha)\frac{dT}{dt} = \frac{3}{4}\times 2\times 10^{-6}\times 1$

$= 1.5 \times 10^{-6}\ m^2/s$

emf = $-\frac{d\phi}{dt} = -\frac{B.dA}{dt} = -1.5 \times 10^{-6}$ V

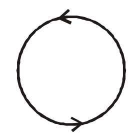
Current in the loop = 1.5×10^{-7} A

The direction will be anticlockwise as the induced current will try to negate the increase in flux due to increase in area.

9. **B, D (P120414)**

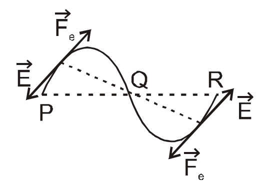
The changing magnetic field inside the plane produces electric lines of forces in anticlockwise direction.

There is no direct connection in the shown conductors, so electrons, experiencing electric force, try to accumulate as shown.



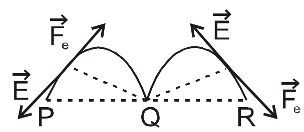
All electrons accumulate at Q symmetrically

$V_P = V_R$

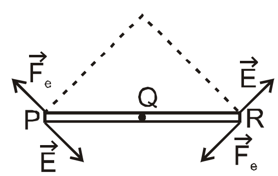


All electrons accumulate at R

$V_P > V_R$

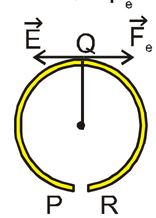


electrons accumulate at P $V_P < V_R$



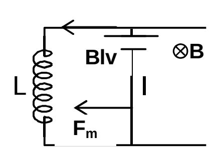
electrons accumulate at R

$V_P > V_R$



10. **A, C (P120418)**

$L\frac{di}{dt} = Bvl$



i = $\frac{Bl}{L}x$

and

$$d = \sqrt{\frac{3v_0^2 mL}{4B^2l^2}}$$

where $v_0 = \frac{J}{M}$

11. **A, B, C (P120406)**

Due to rotation emf = $\frac{Br^2\omega}{2}$

Due to translation indeed emf = Bvr

Where r is the separation.

12. **B, C (P120413, P120414)**

SECTION – B

1. **(A) → (r, s), (B) → (r, s), (C) → (q, s), (D) → (p, s) (P120406)**
2. **(A) → (r), (B) → (r), (C) → (s), (D) → (r) (P120409)**

(A) As ring rolls, velocity of point B is 0,

$\therefore$ emf across AB is, $\varepsilon = \int_0^{2R} B_0(dx)\,x\,\omega$

or, $\varepsilon = 2B_0\omega R^2 = 2B_0\,VR$

(B) Current in the circuit is,

$i = \frac{\varepsilon}{R_0} = \frac{2B_0VR}{R_0}$

(C) Magnetic force on the ring is , $F_B = i\ell B$

$= \left(\frac{2B_0\,VR}{R_0}\right)(2R)(B_0)$

$= \frac{4B_0^2VR^2}{R_0}$

(D) As ring tries to slip on the surface towards left, static friction will act towards right,

$\therefore \quad F_B - f_S = ma$

$f_SR = (mR^2)\alpha = (mR^2)\frac{a}{R}$

$\Rightarrow \quad f_S = \frac{F_B}{2}$ or $\frac{2B_0^2VR^2}{R_0}$.

SECTION – C

1. **5 (P120401)**

Using Faraday's Law

$d\phi$ = Badx

$d\phi$ = 100 a xt dx

$\phi = -50at\left[x^2\right]_{x_0}^{x_0+a}$

$= -50a^2[2tx_0 + at]$

$\Rightarrow -\frac{d\phi}{dt} = \varepsilon_1 = 50a^2\left[2t\frac{dx_0}{dt} + 2x_0 + a\right] \Rightarrow i = \frac{\varepsilon_1}{R} = \frac{\varepsilon}{50} = a^2[2vt + 2vt + a] = a^2[4vt + a]$

$\Rightarrow$ i = 5

2. **5 (P120406)**

Current is 2Ω = 0.1 A

current in circuit = 0.2 A

equivalent resistance about AB = 5Ω

e = I R = 1 volts

and also

e = B ℓ V

$$\therefore \quad V = \frac{e}{B\ell} = \frac{1}{2\times 0.1} = 5\,m/s\,.$$

3. **4 (P120406)**

Force on the rod

$F - F_m = ma$

$$F - \frac{B\ell}{r}(B\ell V - E) = ma$$

$$F + \frac{B\ell E}{r} - \frac{B^2\ell^2 V}{r} = \frac{mdV}{dt}$$

solving $F = -mV_0\omega\sin(\omega t) + \frac{B^2\ell^2}{r}V_0\cos\omega t - \frac{B\ell E}{r}$

Power expended by force is

$$FV = -mV_0^2\omega\sin(\omega t)\cos(\omega t) + \frac{B^2\ell^2}{r}V_0^2\cos^2\omega t - \frac{B\ell E}{r}V_0\cos\omega t$$

Its average over the cycle is

$$\frac{B^2\ell^2V_0^2}{2r}\,.$$

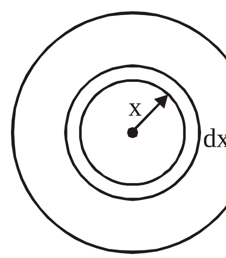
4. **4 (P120413, P120414)**

The half of the maximum current is equal to

$i_1 = \frac{B\omega\ell^2}{4R}$ at $t = \frac{L}{R}\ln 2$

The torque at this instant = $\frac{B^2\omega\ell^4}{8R}$ = 4 N-m

5. **2 (P120414)**



$$E = \frac{x}{2}\frac{dB}{dt}$$

$$E = \frac{3Kxt^2}{2}$$

$$d\tau = \frac{3Kxt^2}{2}\times\frac{2\pi xdx}{\pi r^2}q.x$$

$$\tau = \frac{3Kt^2q}{r^2}\int_0^r x^3dx$$

$$\tau = \frac{3Kq.t^2}{4}.r^2 \qquad \ldots(i)$$

torque due to friction force

$d\tau = \mu dmgx$

$$\tau = 2\mu g\frac{qm}{r^2}\int_0^r x^2dx = \frac{2}{3}\mu mgr \qquad \ldots(ii)$$

$$\frac{3Kq.t^2r^2}{4} = \frac{2}{3}\mu mgr$$

$$t = \sqrt{\frac{8\mu mg}{9Kqr}}\,.$$

= 2 seconds.

6. **8 (P120414)**
If I = kt
Magnetic flux enclosed by trajectory of charged particle is $\phi = \left(2R^2 + r^2\right)\pi\mu_0 n\left(kt\right)$
If induced electric field is E
$$E\,2\pi r = \left(2R^2 + r^2\right)\pi\mu_0\left(nk\right)$$
$$E = \frac{\left(2R^2 + r^2\right)}{r}\left(\frac{\mu_0 nk}{2}\right)$$
Tangential acceleration $a_t = \frac{qE}{m}$
$$v = \frac{qE}{m}t$$
and $\frac{mv^2}{r} = q\,v\,B$
$$\Rightarrow \quad \frac{m}{r}\left(\frac{2R^2 + r^2}{r}\right)\frac{\mu_0 nk}{2}\frac{qt}{m} = q\mu_0 n\,kt$$
$$\Rightarrow \quad r = \sqrt{2}R \Rightarrow r^2 = 8\,cm^2$$

CHEMISTRY

SECTION – A

1. **D (C120303)**

2. **B (C120303)**

3. **D (C120304)**

4. **B (C120209)**
Cu^{+2} + $2CN^-$ → $Cu(CN)_2$ ↓ yellow
$2Cu(CN)_2$ $2CuCN + (CN)$ $\rightarrow_2$
cyanogen

5. **A (C120209)**

6. **A (C120211)**

7. **B (C120303)**
Salt (A) contains (Mn^{2+}) as explained below:
Mn^{2+} + NaOH → $Mn(OH)_2$
$$Mn\left(OH_2\right)\xrightarrow{O_2(dir)} MnO_2$$
(black)
$$MnO_2 + HNO_3 \longrightarrow Mn\left(NO_3\right)_2 + H_2O + \frac{1}{2}O_2$$
$$Mn\left(NO_3\right)_2 + PbO_2 + HNO_3 \longrightarrow HMnO_4 + H_2O + Pb\left(NO_3\right)_2$$
(permanganic acid)
(purple)

8. **D (C120302)**

$\frac{r_{A^+}}{r_{B^-}} = \frac{1}{2} = 0.5$. As it lies in the range 0.414 to 0.732. AB has octahedral structure like that of NaCl.

$\therefore a = 2(r_{A^+} + r_{B^-}) = 2(1+2) = 6$ pm

Volume = a^3 = (6 pm)^3 = 216 pm^3

9. **A, C, D (C120205)**
Radicals: $NH_4^+, Cl^-, Fe^{+2}, SO_4^{--}$
Blue colour is $Fe_3[Fe(CN)_6]_2$ in which Fe(II) and Fe(III) are present.
$BaSO_4$ is soluble in conc. H_2SO_4 or HCl
Red vapours formation is due to Cl^- in chromyl chloride

10. **B, C (C120305)**

11. **A, B, D (C120209)**

12. **A, C, D (C120303)**

SECTION – B

1. **A → (p, r, s, t); B → (p, q, s, t); C → (q); D → (p, t) (C120304)**
For cubic crystal system, there are 6 two fold axis of symmetry exist.
In Rock salt structure, the distance between octahedral and tetrahedral void along the body diagonal is $\frac{a\sqrt{3}}{4}$.
In zinc blende structure, the cation (Zn^{+2}) occupy half of the tetrahedral site.
In Wurzite structure, the cations (Zn^{2+}) occupy half of the tetrahedral sites.

2. **(A) → p,q; (B) → r, s; (C) → q; (D) → t (C120207)**

$$Hg_2I_2 \xrightarrow[\Delta]{H_2O} \underset{(Red)}{HgI_2} + \underset{(Black)}{Hg}$$

$$Pb^{2+} + K_2CrO_4 \longrightarrow PbCrO_4\,(yellow)$$

$$Pb^{2+} + 2KI \longrightarrow \underset{(yellow)}{PbI_2} + 2K^+$$

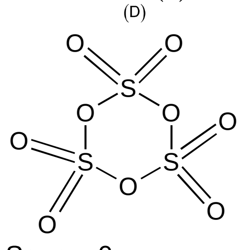
ZnO + CoO → $CoZnO_2$ (Rinnman's green)

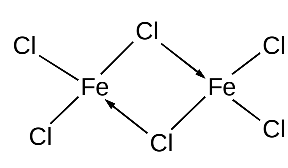
SECTION – C

1. **4 (C120207)**

$$FeSO_4 \xrightarrow{\Delta} \underset{(D)}{Fe_2O_3} + \underset{(B)}{SO_2(g)} + \underset{(C)}{SO_3(g)}$$

$$\underset{(D)}{Fe_2O_3} + HCl(g) \longrightarrow Fe_2Cl_6(g) + H_2O_3$$





So, n = 0 m = 4

2. **9 (C120304)**
A present in corners and face centre.
B present in alternate tetrahedral voids.
No. of A per unit cell = $6\times\frac{1}{8}+6\times\frac{1}{2}=\frac{15}{4}$
No. of B per unit cell = 4 – 1 = 3
$x:y=\frac{15}{4}:3=5:4$
x + y = 5 + 4 = 9

3. **7 (C120207)**
$K_2Cr_2O_7, CuSO_4, H_2O_2, Cl_2, O_3, FeCl_3, HNO_3$
$K_2Cr_2O_7+7H_2SO_4+6KI\longrightarrow 4K_2SO_4+Cr_2(SO_4)_3+3I_2+7H_2O$
$2CuSO_4+4KI\longrightarrow Cu_2I_2+I_2+2K_2SO_4$
$H_2O_2+2KI\longrightarrow I_2+2KOH$
$Cl_2+2KI\longrightarrow 2KCl+I_2$
$O_3+2KI+H_2O\longrightarrow 2KOH+I_2+O_2$
$2FeCl_3+2KI\longrightarrow 2FeCl_2+I_2+2KCl$
$8HNO_3+6KI\longrightarrow 6KNO_3+2NO+4H_2O+3I_2$
$2KMnO_4+KI+H_2O\longrightarrow KIO_3+2MnO_2+2KOH$

4. **5 (C120304)**
A = $2y^{1/3}$ nm. M = 6.023 × y × 4 amu
Density = m/a^3 = $\frac{6.023\times4\times1.66\times10^{-27}}{8y\times10^{-27}}$ kg / m³
$=\frac{40}{8}=5$ kg / m³

5. **4 (C120304)**

6. **6 (C120209)**
Black coloured sulphides $\{PbS, CuS, HgS, Ag_2S, NiS, CoS\}$
Bi_2S_3 in its crystalline form is dark brown but Bi_2S_3 precipitate obtained is black in colour.

MATHEMATICS

SECTION – A

1. p(|x|) has two roots only when P(|b|) = 0
$\Rightarrow b^2 = 4 \Rightarrow |b| = 2$
$P(x) = x^3 - 12x + 16$
$\therefore$ the roots of P(x) are –4, 2
$\int_{-4}^{2}P(x)dx=\int_{-4}^{2}(x^3-12x+16)dx$
$=\frac{1}{4}(2^4-4^4)-\frac{12}{2}(2^2-4^2)+16(6)$
= – 60 + 72 + 96 = 108.

2. $y = 2x^4 - x^2$

$y' = 8x^3 - 2x = 0 \Rightarrow x = 0, \frac{1}{2}, -\frac{1}{2}$

$y'' = 24x^2 - 2$

curve has local minim at $-\frac{1}{2}$ and $\frac{1}{2}$

$\therefore$ area = $\int_{-1/2}^{1/2} (2x^4 - x^2)dx = \frac{7}{120}$.

3. Area = $\left|\int_0^{\pi/2} (1-2^{1+\sin x})dx\right| = \left|\frac{\pi}{2} - 2\int_0^{\pi/2} 2^{\sin x}dx\right| = \frac{\pi}{2} - 2k$

4. $\frac{2\sqrt{2}}{2\pi} < \int_{\pi}^{2\pi} \frac{|\sin x + \cos x|}{x}dx < \frac{2\sqrt{2}}{\pi}$ …(i)

$\because \pi < x < 2\pi \Rightarrow \frac{1}{2\pi} < \frac{1}{x} < \frac{1}{\pi}$

$\frac{2\sqrt{2}}{3\pi} < \int_{2\pi}^{3\pi} \frac{|\sin x + \cos x|}{x}dx < \frac{2\sqrt{2}}{2\pi}$ ….(ii)

adding (i) and (ii)

$\frac{5\sqrt{2}}{3\pi} < A < \frac{3\sqrt{2}}{\pi}$ 0.75 < A < 1.3.

5. Clearly circle will be $(x + r)^2 + y^2 = r^2$; on differentiating both sides, we get

$2(x + r) + 2y\frac{dy}{dx} = 0 \Rightarrow r = -y\frac{dy}{dx} - x$

substituting in $(x + r)^2 + y^2 = r^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + y^2 = \left(x + y\frac{dy}{dx}\right)^2$

$\Rightarrow 2xy\frac{dy}{dx} = y^2 - x^2 \Rightarrow$ order = 1, degree = 1.

6. Clearly we have

$\frac{xdy + ydx}{e^{xy}} = \frac{xdy - ydx}{x^2}$

$\Rightarrow \int d(xy)e^{-xy} = \int d\left(\frac{y}{x}\right) \Rightarrow \frac{y}{x} + e^{-xy} = c$

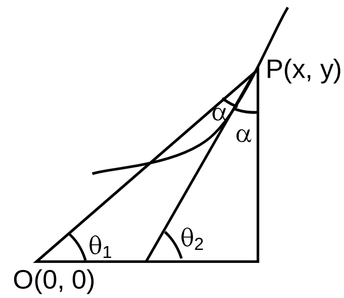
7. $\frac{dy}{dx} + y^3(1 + \cos x) - \frac{y}{x} = 0$

$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{y^2 x} = -(1 + \cos x)$

Let $\frac{-1}{y^2} = u \quad \therefore \frac{1}{2}\frac{du}{dx} + \frac{u}{x} = -(1 + \cos x)$

$IF = e^{\int \frac{2}{x}dx} = x^2 \quad \therefore ux^2 = -2\int x^2(1 + \cos x)dx \Rightarrow \frac{x^2}{2y^2} = \frac{x^3}{3} + x^2\sin x + 2x\cos x - 2\sin x + k$

8. $\theta_1 + \alpha = \theta_2$



$\theta_1 + 2\alpha = \frac{\pi}{2}$

$\theta_1 + 2(\theta_2 - \theta_1) = \frac{\pi}{2}$

$2\theta_2 - \theta_1 = \frac{\pi}{2}$

$\frac{2\tan(\theta_2)}{1-(\tan(\theta_2))^2} = -\frac{1}{\tan\theta_1}$

$\frac{2\frac{dy}{dx}}{1-\left(\frac{dy}{dx}\right)^2} = -\frac{x}{y}$

$2y\frac{dy}{dx} = +x\left(\frac{dy}{dx}\right)^2 - x \Rightarrow x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = x$

9. y = c sinx …(1)

$\frac{dy}{dx}$ = c cosx …(2)

from (2)

$\left(\frac{dy}{dx}\right)^2 = c^2\cos^2 x$ …(3)

Putting c = $\frac{y}{\sin x}$ from (1), $\left(\frac{dy}{dx}\right)^2 = y^2\cot^2 x$

Eliminating c from (1) and (2),

$\frac{dy}{dx}$ = y cotx.

Finding value of c from (2) and eliminating with help of (3), we get

$\left(\frac{dy}{dx}\right)^2 - \left(\sec x\frac{dy}{dx}\right)^2 + y^2 = 0$.

10. (e^x+1) y dy = (y+1)e^x dx

$\Rightarrow \frac{e^x}{e^x+1}dx = \frac{y}{y+1}dy \Rightarrow$ ln (e^x+1) = y — ln |y+1|+ln c

$\Rightarrow$ $(e^x + 1)$ | (y+1)| = ce^y

which can also be written as

$(e^x + 1)$ (y+1) = ± ce^y

11. Given $\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \left(\frac{3x}{1+x^2}\right)$

I.F. = $e^{\ln(1+x^2)}$

So general solution is $y(1+x^2) = \int\left(\frac{3x}{1+x^2}\right)(1+x^2)dx + c$

$y(1+x^2) = \frac{3x^2}{2} + c$

at x = 0, y = 2, c = 2

$$y(1+x^2) = \frac{3x^2}{2} + 2$$

$$y = \frac{3x^2+4}{2(x^2+1)} = \frac{3}{2} + \frac{1}{2(x^2+1)}$$

Range of f(x) = $\left(\frac{3}{2}, 2\right]$

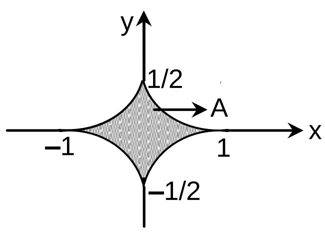
12. Area (T) = $\frac{c.c^2}{2} = \frac{c^3}{2}$

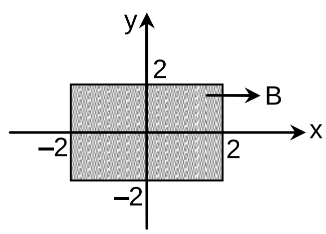
Area (R) = $\frac{c^3}{2} - \int_0^c x^2 dx = \frac{c^3}{2} - \frac{c^3}{3} = \frac{c^3}{6}$

$\therefore \lim_{c\to 0^+} \frac{\text{area (T)}}{\text{area (R)}} = \lim_{c\to 0^+} \frac{c^3}{2} \cdot \frac{6}{c^3} = 3$

SECTION – B

1.





SECTION – C

1. $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ …(1)

Let y = mx + c

m + xm^2 = mx + c

$\Rightarrow$ m = c, m^2 – m = 0

$\Rightarrow$ m = 0, 1

y = 0, x + 1.

2. $x^2y^2\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right) + x^2y^3\left(\frac{1}{y} - \frac{1}{x}\right)dy = 0$

$\Rightarrow d\left(\frac{1}{x} - \frac{1}{y}\right) + y\left(\frac{1}{y} - \frac{1}{x}\right)dy = 0$

$\Rightarrow \frac{d\left(\frac{1}{x} - \frac{1}{y}\right)}{\frac{1}{x} - \frac{1}{y}} = ydy$

$\Rightarrow \ln\left|\frac{1}{x} - \frac{1}{y}\right| = \frac{y^2}{2} + c \Rightarrow$ k = 2.

3. Equation of normal at the point $p(x,y)$ is $Y-y=-\frac{dx}{dy}(X-x)$

Let, $m=\frac{dy}{dx}$ $\Rightarrow X+mY-(x+my)=0$

Distance of perpendicular from the origin to line (i) is $\frac{|x+my|}{\sqrt{1+m^2}}=|y|$ $\Rightarrow \frac{dy}{dx}=\frac{y^2-x^2}{2xy}$

This is homogeneous equation

Let, $y=zx$

$\Rightarrow \frac{dy}{dx}=z+z\frac{dz}{dx}$ $\Rightarrow \frac{2z}{1+z^2}dz=-\frac{dx}{x}$

Integrating $\int \frac{2z}{1+z^2}dz=-\int \frac{dx}{x}$

$\Rightarrow \log(1+z^2)=-\log x+c$

$\Rightarrow (x^2+y^2)=x.e^c$

This curve passes through (1, 1)

$\Rightarrow 1+1=1.e^c$

$e^c=2$

The required equation of the curve is

$\Rightarrow x^2+y^2=2x$

4. $S_k=\int_0^1 x^2(1-x)^k\,dx=\int_0^1(1-x)^2x^k dx = \int_0^1(x^k-2x^{k+1}+x^{k+2})dx = \frac{1}{k+1}-\frac{2}{k+2}+\frac{1}{k+3}$

Now $\lim_{n\to\infty}\sum_{k=1}^{n}S_k=\sum_{k=1}^{\infty}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)-\sum_{k=1}^{\infty}\left(\frac{1}{k+2}-\frac{1}{k+3}\right) = \frac{1}{2}-\frac{1}{3}=\frac{1}{6}=\frac{p}{q}$

$\Rightarrow$ p + q = 7

5. f(x) is an odd function

$\Rightarrow$ g(x) is $f^{-1}(x)$

$\Rightarrow$ Area = $\int_1^{2\pi} f^{-1}(y)dy=\int_0^{2\pi} f'(x)dx = \left[x(f(x))\Big|_0^{2\pi}-\int_0^{2\pi}(x+\sin x)dx\right]$ = $2\pi^2$.

6. Area = n + 1