HINTS & SOLUTIONS

CHEMISTRY :

1. (D) In the reaction

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In this reaction three moles (or volumes) of reactants are converted into two moles (or volumes) of products i.e. there is a decrease in volume and so if the volume of the reaction vessel is halved the equilibrium will be shifted to the right i.e. more product will be formed and the rate of forward reaction will increase i.e. double that of reverse reaction.

2. (A) [Concept Code : C110507/504]

$H_3AsO_4 + OH^- \longrightarrow H_2AsO_4^- + H_2O$

	H_3AsO_4	OH^-	$H_2AsO_4^-$
Initial milimols	30	10	0
After reaction	20	0	10

$H_3AsO_4 + HAsO_4^{2-} \longrightarrow 2H_2AsO_4^-$

	H_3AsO_4	$HAsO_4^{2-}$	$H_2AsO_4^-$
Initial milimols	20	30	10
After reaction	0	10	50

So final pH = $pKa_2 + \log\frac{10}{50} = 6.3$

3. B

Both the NO_2 groups exert –I as well as –R effect.

4. B

Free radical mechanism takes place in anti-Markownikoff reaction.

5. (ABC)

$N_2 + PCl_5 \rightleftharpoons PCl_3 + Cl_2$

	N_2	PCl_5	PCl_3	Cl_2
Moles at t = 0	1	3	0	0
Moles at equilibrium	1	(3 – x)	x	x

Total moles present at equilibrium = 4 + x

Given total pressure at equilibrium = 2.05

Total volume = 100 Litre

$\because$ PV = nRT

$\therefore$ 2.05 × 100 = (4 + x) × 0.082 × 500

$\therefore$ x = 1.0

Degree of dissociation 'α' for PCl_5

$= \frac{\text{Moles dissociated}}{\text{Total moles present}} = \frac{1}{3}$ = 0.3333 = 33.33%

Also $K_P = \frac{x^2}{(3-x)}\left[\frac{P}{\sum n}\right]^2 = \frac{x^2}{(3-x)} \times \frac{2.05}{(4+x)}$

$= \frac{(1)^2 \times 2.05}{(3-1)(4+1)}$

K_P = 0.205 atm.

6. (CD)

7. (BCD)

8. (ACD)

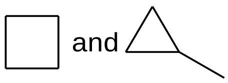
9. AC

10. AC
NO_2 and $CONH_2$ are more deactivating than $NHCOCH_3$.

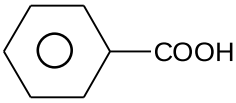
MATRIX MATCHING :

1 QST, PQST, RST, PRST
2 RST, QRS, RS, RQ
3 P, QS, RS, T
4 QR, QS, PR, P

(Integer Type)

1. 2
The isomers are 

2. (4)
$x^2+y^2+2z^2 = 2yz + 2x + 4z - 5$
=> $x = 1; z = y = 2$

3. 7
The product is 

4. 8

PCl_3	+ Cl_2	$\rightleftharpoons$	PCl_5
0.20	0.08		0.40 moles/litre

Kc = 25
If 0.22 mole of Cl_2 is added then at equilibrium

0.20 – x	0.30 – x	0.40 + x

$$25 = \frac{0.40 + x}{(0.20 - x)(0.30 - x)}$$

or x = 0.1
$[PCl_3] = 0.2 - 0.1 = 0.1$ moles
$[Cl_2] = 0.3 - 0.1 = 0.2$ moles
$[PCl_5] = 0.4 + 0.1 = 0.5$ moles
Sum of conc. = 0.8

5. (8)
P is $[BH_2(NH_3)_2]^+[BH_4]^-$ Q is borazine

6. 9
Double bond equivalent = No of double bond + No. of rings

MATHS

1. $(2a+3b)^2 = (2c)^2$
$2a+3b = \pm 2c$

$\Rightarrow a+\frac{3}{2}b+c=0$

$-a-\frac{3b}{2}+c=0$

So ax + by + c = 0 represent family of lines passing through either of two points $\left(1,\frac{3}{2}\right)$ or $\left(-1,\frac{-3}{2}\right)$

So greatest distance between any two lines of family

= the distance between the points $\left(1,\frac{3}{2}\right)$ and $\left(-1,\frac{-3}{2}\right)$

$=\sqrt{4+9}=\sqrt{13}$

2. Centre of the circles will lie on a bisector of the angles between two given lines

$\frac{5x+12y-10}{13}=\pm\left(\frac{5x-12y-40}{13}\right)$

$\Rightarrow 4y+5=0$ or $x-5=0$

Centre lies in the first quadrant. So it must lie on x – 5 = 0

Let centre be (5, k)

So $\frac{|25+12k-10|}{13}=3$

$\Rightarrow k=2$ or $\frac{-11}{3}$

So centre is (5, 2) $r=\sqrt{3^2+4^2}=5$

So circle is $x^2+y^2-10x-4y+4=0$

3. $\int_0^1 (4x^6+(4x^3-f(x))f(x)-4x^6)\,dx=\frac{4}{7}$

$\int_0^1 (f(x)-2x^3)^2\,dx=0$

$f(x) = 2x^3$

4. f(0) = 2

$f(x)=(e^x+e^{-x})\cos x-2x-\left[x\int_0^x f'(t)dt-\int_0^x t\,f'(t)\,dt\right]$

$f(x)=\left(e^x+e^{-x}\right)\cos x-\int_0^x f(t)\,dt$

Differentiating

$f'(x)+f(x)=\cos x(e^x-e^{-x})-(e^x+e^{-x})\sin x$

5. $\frac{dy}{dx}-y\cot x=-\frac{\sin x}{x^2}$

$I.F.=\frac{1}{\sin x}$

Solution is

$y.\frac{1}{\sin x}=\frac{1}{x}+c$

As $x\to\infty, y\to 0\Rightarrow c=0$

$y=\frac{\sin x}{x}$

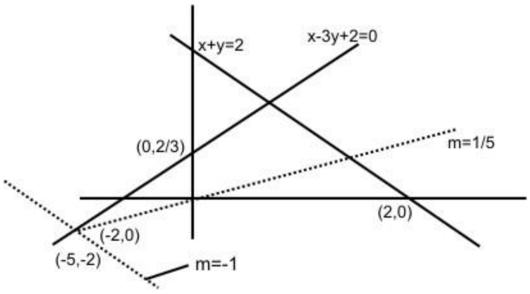
$I=\int_0^{\pi/2}\frac{\sin x}{x}\,dx$

Since, $\frac{\sin x}{x}$ is decreasing

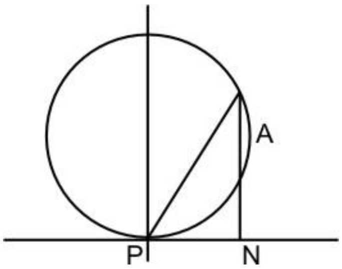
$x>0\Rightarrow f(x)<f(0)$

$\int_0^{\pi/2} f(x) < \frac{\pi}{2}$

6. $m \in \left(-1, \frac{1}{5}\right)$ for origin to lie inside the triangle



7. Let L be the x-axis and S be $x^2 + (y - 1)^2 = 1$ touches the line L at P(0, 0)
$A(\cos\theta, 1+\sin\theta)$



Area of $\Delta PAN = \frac{1}{2}|\cos\theta(1+\sin\theta)| = \frac{1}{2}|f(\theta)|$

$f'(\theta) = \cos\theta(\cos\theta) - \sin\theta(1+\sin\theta) = 0$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$f''(\theta) = -2\sin 2\theta - \cos\theta$

$f''(\theta) < 0$ if $\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$f\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{4}, f\left(\frac{5\pi}{6}\right) = \frac{-3\sqrt{3}}{8}$

Maximum area of ΔPAN is $= \frac{3\sqrt{3}}{8}$

So area of $\Delta PAN \le \frac{3\sqrt{3}}{8}$

8. $P_1P_2 = a[(t_1+t_2)^2 + 4]$ $\quad (\because t_1t_2 = -1)$
$Q_1Q_2 = a[(t_3+t_4)^2 + 4]$
$\Rightarrow \quad |(t_1+t_2)(t_3+t_4)| = 4$

Area of $P_1Q_1P_2Q_2 = \frac{1}{2}d_1d_2 = \frac{1}{2}a^2\left[(t_1+t_2)^2+4\right]\left[(t_3+t_4)^2+4\right]$

$A \ge \frac{1}{2}a^2.2\sqrt{4(t_1+t_2)^2}.2\sqrt{4(t_3+t_4)^2}$

$\ge 32a^2$

$A_{min} = 2(4a)^2$

$(t_1+t_2)^2 = 4 = (t_3+t_4)^2$

Or $t_1+t_2 = 2, t_3+t_4 = -2 \Rightarrow m_{p_1p_2} = 1, m_{Q_1Q_2} = -1$

Or $t_1+t_2 = -2, t_3+t_4 = 2 \Rightarrow m_{p_1p_2} = -1, m_{Q_1Q_2} = 1$

9. Put $(a - r\cos\theta, 2 - r\sin\theta)$ to ellipse

$$\Rightarrow (PA)(PD) = \frac{\frac{a^2}{9}}{\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{4}} = \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta}$$

Put $(a - r\cos\theta, 2 - r\sin\theta)$ to xy = 0

$\Rightarrow 2a + \ell^2 \sin\theta\cos\theta - (a\sin\theta + 2\cos\theta)r = 0$

$(PD)(PC) = \frac{2a}{\sin\theta\cos\theta}$

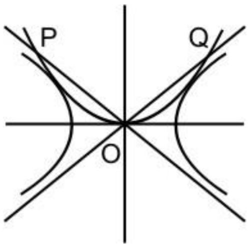
PA.PD = PB.PC

$a = \frac{4\cot\theta + 9\tan\theta}{2}$

$|a| \geq \sqrt{4\times 9}$

$|a| \geq 6$

10.



(α, α^2) lies on parabola y = x² meets the asymptotes $y = \pm 2x$ in P and Q, then the point can not lie on the portion POQ of the parabola so $\alpha < -2$ or $\alpha > 2$

1. (a) $2a^2 + a - 3 = 0$

$a \in (0,1)$

Number of integral values of a = 0

(b) $a^2 + a - 2 = 0$ $\quad 4a^2 + 4a - 3 = 0$

$a = -2, 1$ $\quad a = \frac{1}{2}, \frac{-3}{2}$

$a \in \left(-2, \frac{-3}{2}\right) \cup \left(\frac{1}{2}, 1\right)$

(c) Slope of line = $\frac{2t + 2 - t}{t - 1 - 2t - 1} = -1$

So slope of perpendicular bisectors of point is 1

(d) Images of A with respect to y = x and y = 0 lies on BC

Which are (2, 1), (1, -2)

Equation of BC is y = 3x – 5

$d(A, BC) = \frac{4}{\sqrt{10}}$

2. $(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2 = 0$

$\Rightarrow x_1 + x_2 + x_3 = 0$

$y_1 + y_2 + y_3 = 0$

$\Rightarrow$ Circum centre and centroid of triangle ABC coincide

$\Rightarrow \Delta ABC$ is equilateral

(a) $(PA)^2 + (PB)^2 + (PC)^2 = 3(x^2 + y^2 + 1) = 6$

(b) $\angle OBQ = \frac{\pi}{3}, K = 3$

(c) Maximum distance of R from S, $d = \frac{2}{\sqrt{2}} + 1 = \sqrt{2} + 1$

(d) I and G coincide with origin

IA = IB = IC = GA = GB = GC = 1

3. (a) Slope of tangent $= \frac{-b}{a}\cot\theta \Rightarrow \frac{-2}{3}\cot\theta = \frac{8}{9}$

(b) The tangent at the point should be parallel to the given line $\Rightarrow \frac{-2}{3}\cot\theta = \frac{-1}{2}$

(c) $\frac{a^2h}{x_1} - \frac{b^2k}{y_1} = a^2 - b^2$

$\Rightarrow$ P,Q,R,S lie on $a^2hy - b^2kx = (a^2 - b^2)xy$

$a^2 = 3, b^2 = 1$ and $h = \frac{4}{15}, k = \frac{6}{15}$, so the curve is

$5xy + x - 2y = 0$ whose centre is $\left(\frac{2}{5}, \frac{-1}{5}\right)$

(d) Slope of normal $= \frac{a}{b}\tan\theta \Rightarrow \frac{3}{2}\tan\theta = -2$

4. (a) Let $P\left(ct, \frac{c}{t}\right)$ be common point, then the tangents at P are $x + t^2y = 2ct$ and $c + x - \frac{c}{t}y = a^2$

Slopes are $\frac{-1}{t^2}$ and t^2

(b) Slope of PQ is $\frac{-1}{t_1t_2}$ and slope of RS is $\frac{-1}{t_3t_4}$

$t_1t_2t_3t_4 = -1$

Product of CP, CQ, CR and CS is $= \frac{1}{t_1^2t_2^2t_3^2t_4^2} = 1$

(c) S(5, 12) S'(24, 7)

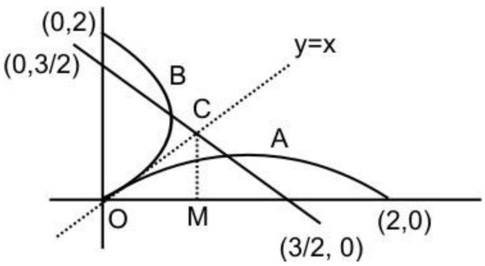
$SS' = \sqrt{386}$, $SO = 13$ $S'O = 25$

If conic is ellipse, $e = \frac{SS'}{SO + S'O} = \frac{\sqrt{386}}{38}$

If conic is hyperbola, $e = \frac{SS'}{S'O - SO} = \frac{\sqrt{386}}{12}$

(d) Asymptotes are $y = \pm\frac{x}{2}$, which intersect x = 1 at $\left(1, \frac{1}{2}\right)$ and $\left(1, \frac{-1}{2}\right)$

1.



$f(x) = \frac{2x - x^2}{2}$

Area of OAB = 2[area of OCM + area CMNA – area ONA]

$= 2\left[\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} + \frac{1}{2}\left(\frac{3}{4} + \frac{1}{2}\right) \times \frac{1}{4} - \frac{1}{2}\int_0^1 (2x - x^2)dx\right]$

$= \frac{5}{24}$

2. $\frac{1}{y^2}\frac{dy}{dx} - \frac{2}{y}\frac{1}{x} = x^3$

$\frac{-2}{y} = t \Rightarrow \frac{dt}{dx} + \frac{2t}{x} = 2x^3$

I.F. = x^2

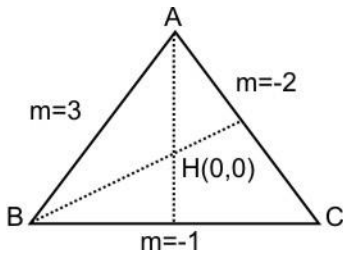
$$x^2 t = \frac{2x^6}{6} + C \Rightarrow \frac{-2}{y} = \frac{x^4}{3} + \frac{C}{x^2}$$

If $x = 1, y = -6 \Rightarrow C = 0$

$$f(x) = \frac{-6}{x^4}$$

$$\frac{dy}{dx} = 24x^{-5} = \frac{24}{x^5} \Rightarrow \left.\frac{dy}{dx}\right|_{x=3^{1/5}} = 8$$

3.



$m_{AH} = 1 \Rightarrow A = (x_1, y_1)$

$$m_{BH} = \frac{1}{2} \Rightarrow B = \left(x_2, \frac{1}{2}x_2\right)$$

$$m_{CH} = \frac{-1}{3} \Rightarrow C = \left(x_3, \frac{-x_3}{3}\right)$$

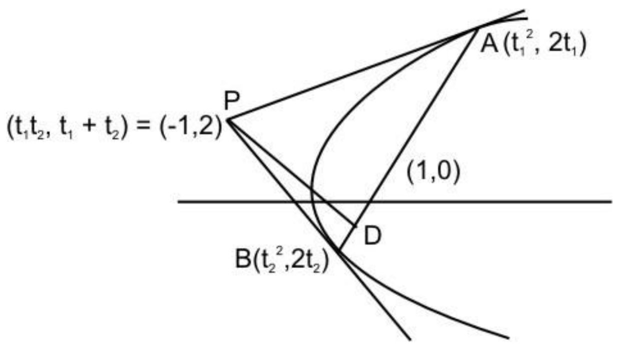
$$m_{BC} = \frac{\frac{x_2}{2} + \frac{x_3}{3}}{x_2 - x_3} = \frac{3x_2 + 2x_2}{6(x_2 - x_3)} = -1 \Rightarrow 9x_2 = 4x_3$$

$$m_{CA} = \frac{x_1 + \frac{x_3}{3}}{x_1 - x_3} = -2 \Rightarrow 9x_1 = 5x_3$$

$$G = (h,k) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{x_1 + \frac{x_2}{2} - \frac{x_3}{3}}{3}\right)$$

$3h = 2x_3 \qquad 3k = \frac{4x_3}{9} \Rightarrow \frac{k}{h} = \frac{2}{9}$

4.



$2y = 2(x - 1)$

$y = x - 1$

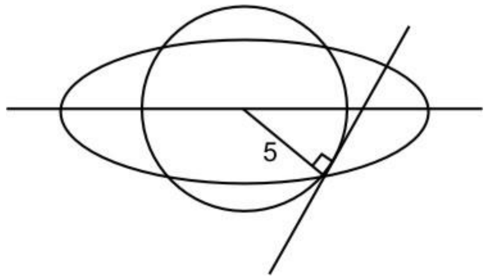
$$PD = \frac{|-1-1-2|}{\sqrt{2}} = 2\sqrt{2}$$

$$AB = (1 + t_1^2)(1 + t_2^2) = (t_1 + t_2)^2 + 4 = 4 + 4 = 8$$

$$Ar(\Delta PAB) = \frac{1}{2} \times 2\sqrt{2} \times 8 = 8\sqrt{2}, \quad N = 8$$

5. Equation of normal at P is

$$ax\sec\theta - \beta y \operatorname{cosec}\theta = \alpha^2 - \beta^2$$



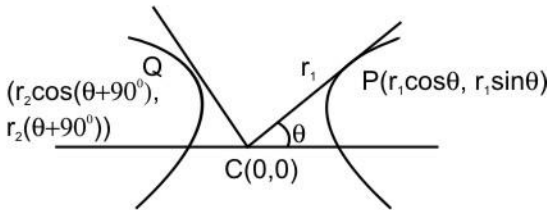
$$r = \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2 \sec^2\theta + \beta^2 \operatorname{cosec}^2\theta}} = \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2 + \alpha^2\tan^2\theta + \beta^2 + \beta^2\cot^2\theta}}$$

$$r \le \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta}}$$

$$r_{max} = |\alpha - \beta| = (a^2 + 2a + 2) - (a^2 + 1) = 2a + 1$$

$2a + 1 = 5 \Rightarrow a = 2$

6.



$$\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{r_1^2} + \frac{1}{r_2^2}$$

P lies on $9x^2 - 5y^2 = 1$

$$\frac{1}{r_1^2} = 9\cos^2\theta - 5\sin^2\theta$$

$$\frac{1}{r_2^2} = 9\sin^2\theta - 5\cos^2\theta$$

$$\Rightarrow \frac{1}{r_1^2} + \frac{1}{r_2^2} = 9 - 5 = 4$$

SOLUTIONS (PHYSICS)

1. Current through the capacitor will be zero.

3. Charges would be placed at the corner of tetrahedron

$$P.E = \frac{{}^4C_2 q^2}{4\pi\varepsilon_0 R}$$

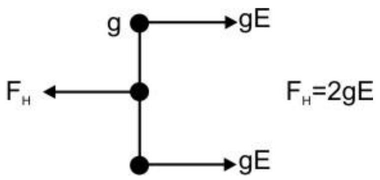
4. $V_{C_8} = \vec{V}_C - \vec{V}_D = 2V \downarrow$

6. $I_\lambda = I_y = I_2$

$I_n, I_y < 7_2$

7. Voltage drop across parallel branches is equal

9.



10. Capacitors of the circuits will change from 2C/3 to 2C. Since the capacitance has increased more charge will flow from the battery

$q_{Initial} = \frac{2CV}{3} \qquad = q_{final} = 2CV$

$\Delta q = V\left(2C - \frac{2C}{3}\right) = \frac{4CV}{3}$

11. $\because$ battery is disconnected charge on the plates will not charge and thus electric field will remain same.
$V = Ed$ $\because$ is decreased V will also decrease.

MATRIX MATCHING :

1. Q, PRT, Q, Q
2. Q S R P

$q_B = -q\left(1 - \frac{1}{K}\right)$

$q_A = +q$

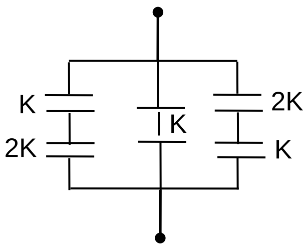
$q_C = +q\left(1 - \frac{1}{K}\right)$

$q_D = -q$

3. A –PQ, B –QR , C –S, D – P
4. S, PR, PR, Q

NUMERICALS :

1. **6**



Given system can be considered as

$C_{eq} = \frac{11 KC_0}{9}$

$C_0 = \frac{9a^2\varepsilon_0}{d}$

$K = 2$

2. **6**

$U_1 = \frac{1}{2}C_1\varepsilon_1^2 = \frac{1}{2} \times 12 \times 4^2 = 96\ \mu H$

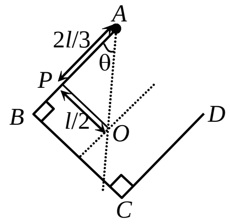
$U_2 = \frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times 16 \times 9 = 72\ \mu H$

$\Delta H = U_1 - U_2 = 24\ \mu J$

$\frac{\Delta H}{4} = 6\ \mu J$

3. 5
4. 8
5. **6**

Sol. In equilibrium vertical line passes through the centre of mass of the system of three rods.
Let O be the centre of mass.
In triangle AOP, $\tan\theta = \frac{l/2}{2l/3} = \frac{3}{4}$



$\therefore \quad \sin\theta = \frac{3}{5}$

6. **5**

Sol. $I_1 = \frac{2}{3}mR^2$

$I_2 = \frac{2}{3}mR^2 + mR^2 = \frac{5}{3}mR^2$

$\frac{I_2}{I_1} = \frac{5}{2} = \frac{x}{2} \quad \therefore \quad x = 5$