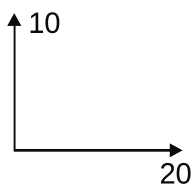
PAPER-1

PART-1 : PHYSICS SOLUTION

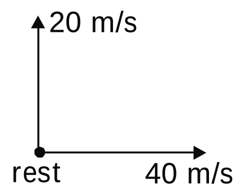
SECTION-I

1. Ans. (D)

Sol.



at t = 1 sec



after explosion

now $n = \left[20(1) - \frac{1}{2}g(1)^2\right] + \frac{(20)^2}{2g}$

= 35 m

2. Ans. (A)

Sol. $h_{max} = \frac{mv_0^2}{2(w+f)}$

now $v_f^2 = 2\left(\frac{w-f}{m}\right)\frac{mv_0^2}{2(w+f)}$

$v_f = \sqrt{\frac{w-f}{w+f}}\,v_0$

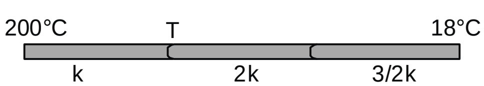
3. Ans. (B)

Sol. Since temperature is same

$\Rightarrow$ same average speed

4. Ans. (B)

Sol.



$$Q = \frac{200-18}{\frac{L}{kA}\left(\frac{1}{1}+\frac{1}{2}+\frac{2}{3}\right)} = \frac{200-T}{\frac{L}{kA}}$$

$$\frac{182}{13/6} = 200 - T$$

T = 116 °C

5. Ans. (A)

Sol. The maximum current is obtained at resonance where the net impedance is only resistive which is the resistance of the coil only. This gives the resistance of the coil as 10 ohm. Now, this coil along with the internal resistance of the cell gives a current of 0.5 A.

$$R = \frac{6}{600 \times 10^{-3}} = 10\,\Omega$$

$$i = \frac{6}{10+2} = \frac{1}{2}\,A$$

6. Ans. (C)

Sol. WD by $\vec{F} = \vec{C} \times \vec{v}$ will be zero

since WD = $\int \vec{f}\cdot d\vec{r}$ = $\int \vec{f}\cdot \vec{v}\,dt$

$= \int \left(\vec{C} \times \vec{V}\right)\cdot \vec{V}\,dt$ = 0

WD = ΔK + ΔU

$0 = \left[\frac{1}{2}m\left(\frac{v_0}{3}\right)^2 - \frac{1}{2}mv_0^2\right] + qEd$]

7. Ans. (B)

Sol. $C_\mu = \frac{R}{\gamma - 1} + \frac{R}{1 - X}$

At A, $X = \gamma$

8. Ans. (C)

Sol. $a_{m_2} = \left(\frac{m_2 g - m_1 g}{m_2}\right)$

9. Ans. (D)

Sol. $V_A - 2 \times 3 - 3 + 2 - 1 = V_B$

$V_B - V_A = -8$ volt

10. Ans. (D)

Sol. Plates are brought closer capacity will increase. As battery is removed charge remain constant. $U = \frac{1}{2}\frac{Q^2}{C}$

$\Rightarrow$ $U \propto 1/C$. Hence stored energy will decrease.

11. Ans. (A,B,C,D)

Sol. $\frac{nR(T_1 - T_2)}{\gamma - 1} = W$

$$\sqrt{\frac{\frac{4}{3} \times \frac{25}{3} \times 400}{40 \times 10^{-3}}} = \frac{100 \times 2 \times 10}{3 \times 2} = \frac{1000}{3} = v$$

$400 \times (10^{-3})^{1/3} = 300 \times V^{1/3}$

$V^{1/3} = \frac{4}{30}$

$V = \frac{8}{3375}\ m^3$

12. Ans. (A,D)

Sol. $2A \sin kx = 3\sqrt{2}$

$2 \times 3 \sin kx = 3\sqrt{2}$

$\sin kx = \frac{1}{\sqrt{2}}$

$\frac{2\pi}{\lambda} x = \frac{\pi}{4}\ ;\ \frac{3\pi}{4}$

Distance between consecutive points

$= \frac{3\lambda}{8} - \frac{\lambda}{8} = \frac{\lambda}{4}$

$\frac{\lambda}{4} = 20$ cm

$\Rightarrow$ $\lambda = 80$ cm

So, $(n + 1)\frac{\lambda}{2} = 240$

$\Rightarrow$ $(n + 1)\frac{80}{2} = 240$

or $n + 1 = 6$

$n = 5$

So string is vibrating in fifth overtone.

13. Ans. (A,B,D)

Sol. $i_{rms} = 5A$

Reading of voltmeter $= i_{rms}\sqrt{R^2 + X_C^2}$

From this $X_C = 20\Omega \Rightarrow \omega = 10^3$ rad/s

$\Rightarrow f = \frac{1000}{2\pi}$

$\therefore X_L = \omega L = 20\Omega$

therefore the circuit is in resonance

$E_{rms} = i_{rms} R = 50$ V also power factor = 1

av. Power $= E_{rms}\, i_{rms} \cos\phi = 250$ W

14. Ans. (B,C)

Sol. $\frac{v_A}{v_B} = \sqrt{\frac{\gamma_A}{\gamma_B}\frac{M_B}{M_A}} = \sqrt{\frac{7 \times 3}{5 \times 5} \times \frac{4}{2}} = \sqrt{\frac{42}{25}}$

(M_A and M_B are molecular weight of A and B respectively)

$$\frac{3v_A}{2l_A} = \frac{3v_B}{4l_B}$$

$$\Rightarrow \frac{l_A}{l_B} = \frac{2v_A}{v_B} = \sqrt{\frac{168}{25}}$$

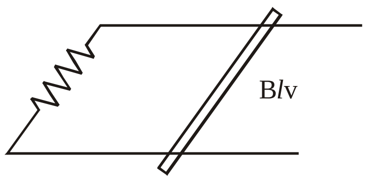
15. Ans. (C,D)

Sol. Since the orientation of the disc is constantly changing the angular velocity is constantly changing the direction, hence the angular acceleration cannot be zero.

SECTION-IV

1. Ans. 8

Sol. $i = \frac{Blv}{R}$



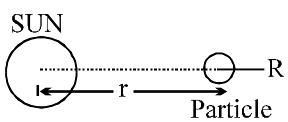
$mv\frac{dv}{dx} = f = Bil = -\frac{B^2l^2v}{R}$

$\int_{v_0}^{0} dv = \frac{-B^2l^2}{mR}\int_0^x dx$

$\Rightarrow x = \frac{mv_0R}{B^2\ell^2} = 8$ m

2. Ans. 6

Sol.



energy incident/sec on the particle

$= \frac{P}{4\pi r^2} \times \pi R^2$

$= \frac{PR^2}{4r^2}$

$\vec{F}$ due to striking of photon

$\vec{F}_1 = \frac{\vec{dp}}{dt} =$ Change in lin. mom. of 1 photon in collision × No. of photons striking per sec.

$= \frac{h\nu}{C} \times \frac{PR^2}{4r^2 \times h\nu}$

$= \frac{PR^2}{4r^2C}$

Gravitational force on the particle

$\vec{F}_2 = \frac{GM_sm}{r^2} = \frac{GM_s}{r^2} \times \frac{4}{3}\pi R^3\rho$

$\left|\vec{F}_1\right| = \left|-\vec{F}_2\right|$

$\frac{PR^2}{4r^2C} = \frac{GM_s \times 4\pi R^3\rho}{3r^2}$

$\Rightarrow \quad R = \frac{3P}{4GM_s\pi\rho C} = 0.6$

3. Ans. 2

Sol. $\frac{1}{v} - \frac{1}{25} = \frac{1}{f} \Rightarrow \frac{1}{25m_1} - \frac{1}{25} = \frac{1}{f}$ ....(1)

$\frac{1}{v} - \frac{1}{40} = \frac{1}{f} \Rightarrow \frac{1}{40m_2} - \frac{1}{40} = \frac{1}{f}$ ....(2)

$m_1 = 4m_2$ ..... (3)

Solving $m_2 = -1$

$\frac{1}{f} = -\frac{1}{40} - \frac{1}{40} = \frac{-1}{20}$

$f = -20$

$x = 2$

4. Ans. 5

Sol. $I_0 = \frac{V_0}{\sqrt{X_L^2 + R^2}}$

$= \frac{220\sqrt{2}}{\sqrt{\left(35 \times 10^{-3} \times 50 \times 2 \times \frac{22}{7}\right)^2 + (11)^2}}$

$= \frac{220}{11\sqrt{2}}\sqrt{2} = 20 = 4n \quad \Rightarrow \quad n = 5$

5. Ans. 4

Sol. $A_1v_1 = A_2v_2$

$V_2 = 16$ m/s

Applying Bernoulli's equation

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad (\rho = 0.9 \times 10^3 \text{ kg/m}^3)$

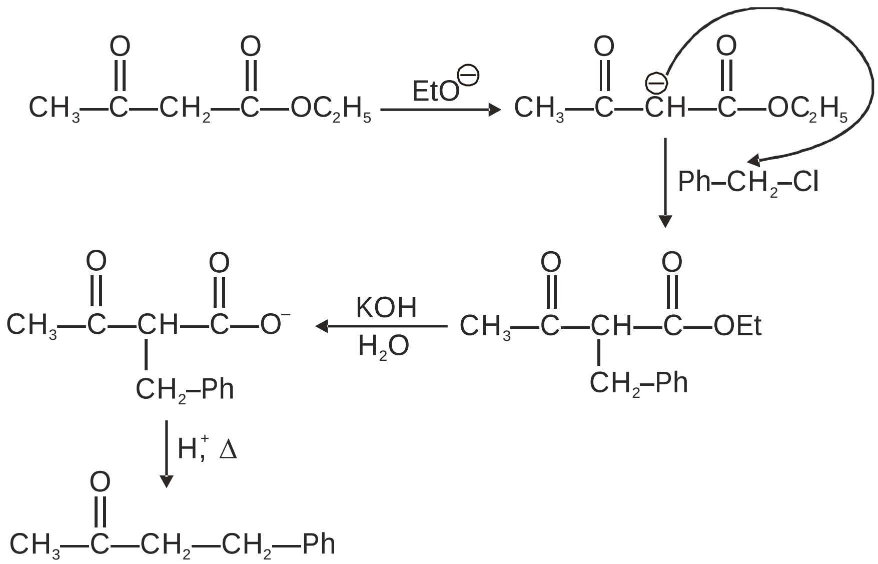
SECTION-I

1. Ans. (D)

Sol. Theory based.

2. Ans. (C)

Sol.



3. Ans. (C)

Sol. P $\rightarrow$ Q – Adiabatic irreversible $\Rightarrow \Delta S_{PQ} > 0$.

P $\rightarrow$ R – Adiabatic reversible $\Rightarrow \Delta S_{PR} = 0$.

Since entropy is state function hence

$$\Delta S_{PQ} + \Delta S_{QR} = \Delta S_{PR}$$

$\Rightarrow \Delta S_{QR} < 0$.

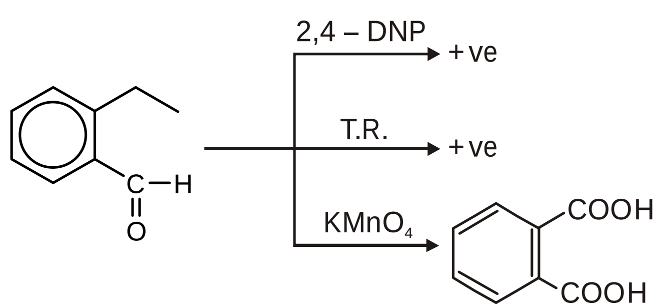
4. Ans. (D)

Sol. Complete hydrolysis of XeF_4 is redox.

$$6XeF_6 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$$

5. Ans. (C)

Sol.



6. Ans. (A)

Sol. Boiling takes place when vapour pressure is equal to external pressure. If we are still heating then vapour pressure will remain same but temperature of the water increases.

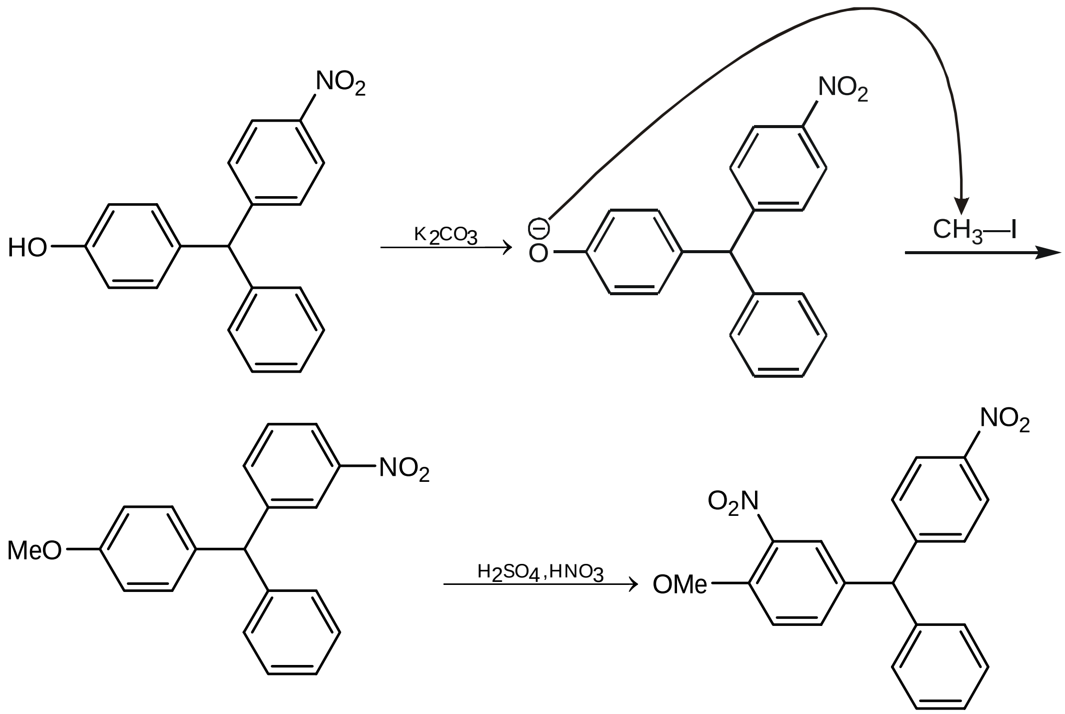
7. Ans. (C)

Sol. $N \equiv N$

C = C $\rightarrow$ It has both bonds as π – bond according to MOT.

8. Ans. (D)

Sol.



9. Ans. (D)

Sol. The cosmic ray generates neutrons in the atmosphere which bombards the nucleus of atmospheric nitrogen to form radioactive ^{14}C hence ^{14}C in the atmosphere has been remaining constant over thousands of years. In living materials, the ratio of ^{14}C to ^{12}C remains relatively constant. When the tissue in an animal or plant dies, assimilation of radioactive ^{14}C ceased to continue. Therefore, in the dead tissue the ratio of ^{14}C to ^{12}C would decrease depending on the age of the tissue.

$$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}p$$

$$^{14}_{6}C + ^{14}_{7}N + ^{0}_{-1}e$$

A sample of dead tissue is burnt to give carbon dioxide and the carbon dioxide is analysed for the ratio of ^{14}C to ^{12}C. From this data, age of dead tissue (plant or animal) can be determined.

10. Ans. (C)

Sol. Natural rubber $\rightarrow$ Natural, Addition, Homopolymer.

Starch $\rightarrow$ Natural, Condensation, Homopolymer.

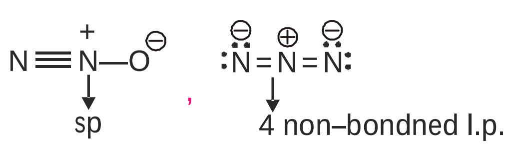
Insulin $\rightarrow$ Natural, Condensation, Co–polymer.

Dacron $\rightarrow$ Synthetic, Condensation, Co–polymer.

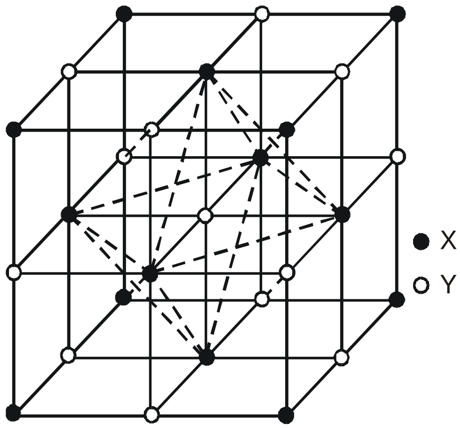
11. Ans. (B)

Sol. $Zn + HNO_3(dil.) \longrightarrow N_2O(g) + Zn(NO_3)_2 + H_2O$

$N_2O + 2NaNH_2 \longrightarrow NaN_3 + NaOH + NH_3$



12. Ans. (A,B,D)

Sol. 

Every small cube center is containing one tetrahedral void.

Clearly from the diagram four corners of small cube is occupied by x and four are occupied by Y.

13. Ans. (A,C,D)

Sol. Acid + Alcohol $\underset{}{\overset{H^+}{\rightleftharpoons}}$ Ester + H_2O

1	10	0	
1 – x	10 – x $\simeq$ 10	x	x

$$\frac{10 \times x}{1-x} = 1$$

$$\frac{x}{1-x} = 0.1$$

10 x + x = 1

$$x = \frac{1}{11} = 0.9$$

In option (B) Catalyst cannot change free energy.

14. Ans. (A,B,C,D)

Sol. This motion is indepdent of the nature of the colloid but depends on the size of the particles and viscosity of the solution. Smaller the size and lesser the viscosity faster is the motion.

The Brownian movement has been explained to be due to the unbalanced bombardment of the particles by the molecules of the dispersion medium. The Brownian movement has a stirring effect which does not permit the particles to settle and thus, is responsible for the stability of sols.

15. Ans. (B,C,D)

Sol. $CoCl_3(aq) + NH_3(aq) \longrightarrow CoCl_3.6NH_3 + CoCl_3.5NH_3 + CoCl_3.4NH_3$

$[Co(NH_3)_6]^{3+}3Cl$ – yellow – d^2sp^3(diamagnetic).

$[Co(NH_5)Cl]^{2+}2Cl$ – purple – d^2sp^3(diamagnetic).

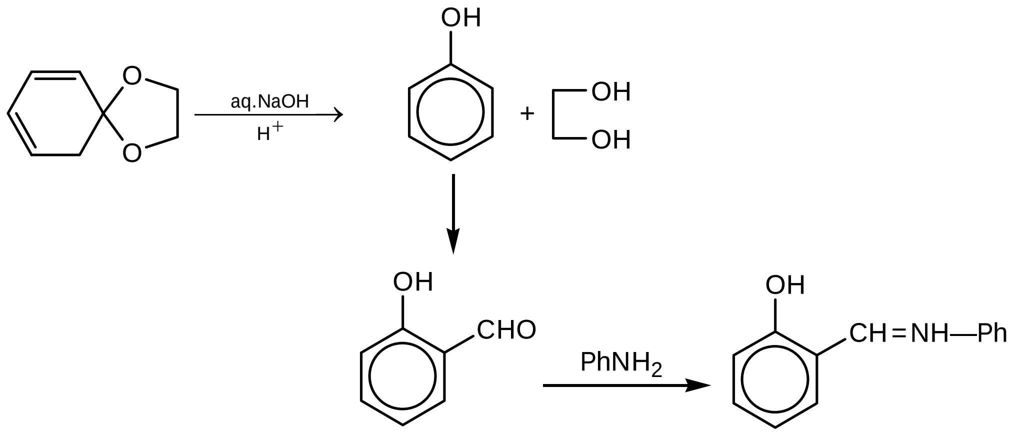
$[Co(NH_3)_4Cl_2]^+Cl^-$ – green and $[Co(NH_3)_4Cl_2]^+Cl^-$ – violet are geometrical isomers.

* Geometrical isomers will produce same amount of AgCl.

SECTION-IV

1. Ans. (4)

Sol.



2. Ans. (9)

Sol. $0 \le \ell\ n + 2$, $\quad 0 \le m < \ell + 1$

for $n = 2$

$\ell = 0, 1, 2, 3, 4$ (In integral steps).

If $\ell = 0$,	$m = 0$
$\ell = 1$,	$m = 0, 1$
$\ell = 2$,	$m = 0, 1, 2$
$\ell = 3$,	$m = 0, 1, 2, 3$
$\ell = 4$,	$m = 0, 1, 2, 3, 4$

Hence total 15 orbitals are possible in 2^{nd} shell.

3. Ans. (5)

Sol. $N(SiH_3)_3$, BF_3, $B_3N_3H_6$, SO_2, $NO_2 \rightarrow sp^2$

SiO_2, $H_3O^{\oplus} \rightarrow sp^3$

$AlCl_3.6H_2O \rightarrow sp^3d^2$.

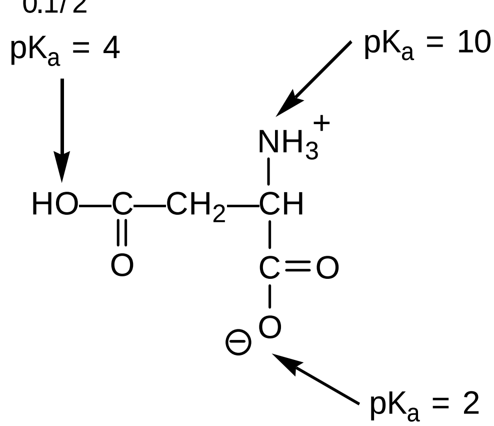
4. Ans. (5)

Sol. It is a buffer solution

$$pH = X = pKa + \log \frac{[salt]}{[acid]}$$

$$= 4 + \log \frac{0.1/2}{0.1/2} = 4.0$$

Zwitter ion form of amino acid



Assorbic acid pK_a values

Hence isoelectric point = $\dfrac{pKa_1 + pKa_2}{2} = \dfrac{2+4}{2} = 3$

Hence at 4.0 pH, it is overall –vely charged and moves towards anode.
Hence answer is (X + 1) = (3.7 + 1) = 4.7

5. Ans. (6)

Sol. Molarity of 1st solution = $\dfrac{0.2 \times M}{\dfrac{0.2M + 18 \times 0.8}{d_1}} \times 1000$

Molarity of 2nd solution = $\dfrac{0.1}{\dfrac{0.1M + 0.9 \times 18}{d_2}} \times 1000$

$\Rightarrow \quad d_2 = 0.8d_1$

$\dfrac{1}{2} \times \dfrac{0.2 \times 1000}{0.2M + 18 \times 0.8} \times d_1 = \dfrac{0.1 \times 0.8d_1}{0.1M + 0.9 \times 18}$

$\Rightarrow \quad \dfrac{5}{0.2M + 18 \times 0.8} = \dfrac{4}{0.1M + 0.9 \times 18}$

M = 78

PART-3 : MATHEMATICS SOLUTION

SECTION-1

1. Ans. (B)

Sol. $\lim\limits_{n\to\infty} \sum\limits_{r=0}^{n} \dfrac{{}^nC_r}{n^r} \cdot \left(\int_0^1 x^{r+3} dx \right)$

$= \int_0^1 \left(\lim\limits_{n\to\infty} \sum\limits_{r=0}^{n} {}^nC_r \cdot \left(\dfrac{x}{n}\right)^r \cdot x^3 \right) dx$

$= \int_0^1 \lim\limits_{n\to\infty} \left(1 + \dfrac{x}{n}\right)^n \cdot x^3 dx \quad = \int_0^1 e^x \cdot x^3 dx = 6 - 2e$

2. Ans. (B)

Sol. $y'(x) \cdot g(x) - y(x) \cdot g'(x) + y^2(x) = 0$

$\Rightarrow \quad -d\left(\dfrac{g(x)}{y(x)}\right) + 1 = 0$

$\Rightarrow \quad -\dfrac{g(x)}{y(x)} + x + c = 0$

$\Rightarrow \quad \dfrac{g(x)}{y(x)} = x + c$

y(–1) = 1 and g(–1) = 0

$\Rightarrow$ c = 1

$\Rightarrow \quad \dfrac{g(x)}{y(x)} = 1 + x$

$= \int_1^2 \dfrac{(1+x)dx}{x^2\sqrt{x^2 + (1+x)^2}}$

$= \int_1^2 \dfrac{(1+x)dx}{x^2\sqrt{2 + \dfrac{2}{x} + \dfrac{1}{x^2}}}$ put $2 + \dfrac{2}{x} + \dfrac{1}{x^2} = t^2$

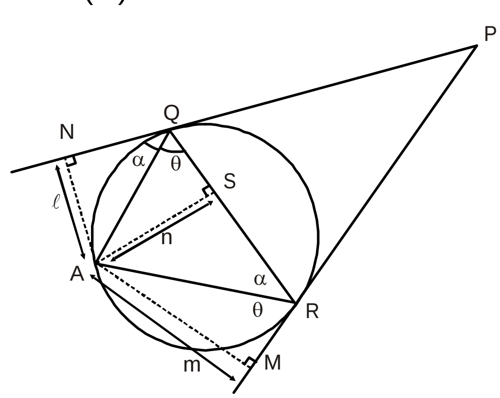
$= \int_{\sqrt{5}}^{\frac{\sqrt{13}}{2}} \dfrac{-tdt}{t}$

$\left(-\dfrac{2}{x^2} - \dfrac{2}{x^3}\right)dx = 2tdt$

$= \sqrt{5} - \dfrac{\sqrt{13}}{2} = \dfrac{2\sqrt{5} - \sqrt{13}}{2} = \dfrac{7}{2\left(2\sqrt{5} + \sqrt{13}\right)}$

3. Ans. (C)

Sol.



ΔANQ and ΔASR are similar

$\Rightarrow \frac{\ell}{n} = \frac{AQ}{AR}$ ....(i)

Similarly ΔAQS and ΔARM are similar

$\frac{n}{m} = \frac{AQ}{AR}$ ....(ii)

$\Rightarrow n^2 = \ell m$

4. Ans. (B)

Sol. Let vertices A_1, A_2, ... A_7 are 7^{th} roots of unity. Let $A_1(1)$, $A_2(\alpha)$, $A_3(\alpha^2)$, $A_4(\alpha^3)$

$A_5 = \alpha^4 = \overline{\alpha^3}$, $A_6 = \alpha^5 = \overline{\alpha^2}$, $A_7 = \alpha^6 = \overline{\alpha}$

$$(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)(1-\alpha^5)(1-\alpha^6) = 7$$

$$\Rightarrow \left(|1-\alpha|\,|1-\alpha^2|\,|1-\alpha^3|\right)^2 = 7$$

$p_1 = (A_1A_2)(A_1A_3)(A_1A_4)(A_1A_5)(A_1A_6)(A_1A_7)$

$$= |1-\alpha|\,|1-\alpha^2|\,|1-\alpha^3|\,|1-\overline{\alpha^3}|\,|1-\overline{\alpha^2}|\,|1-\overline{\alpha}|$$

$$= |1-\alpha|\,|1-\alpha^2|\,|1-\alpha^3|\,|\overline{1-\alpha^3}|\,|\overline{1-\alpha^2}|\,|\overline{1-\alpha}|$$

$$= \left(|1-\alpha|\,|1-\alpha^2|\,|1-\alpha^3|\right)^2 = \left(\sqrt{7}\right)^2$$

Similarly

$$p_2 = |1-\alpha|\,|1-\alpha^2|\,|1-\alpha^3|\,|1-\alpha^3|\,|1-\alpha^2|$$

$$= \sqrt{7}\left(|1-\alpha^3|\,|1-\alpha^2|\right)$$

$p_3 = \sqrt{7}\left(|1-\alpha^3|\right)$

$p_4 = \sqrt{7}$, $p_5 = |1-\alpha|\,|1-\alpha^2|$

and $p_6 = |1-\alpha|$

$$\Rightarrow p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 = \left(\sqrt{7}\right)^7$$

5. Ans. (A)

where s is semiperimeter and Δ is area of triangle

Sol. $\ell_1 = \sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{b-c}{2}\right)^2}$

$$\ell_1^{\,2} = \left(\frac{a+b-c}{2}\right)\left(\frac{a-b+c}{2}\right)$$

$\ell_1^{\,2} = (s-c)(s-b)$

$\ell_2^{\,2} = (s-a)(s-c)$

$\ell_3^{\,2} = (s-a)(s-b)$

$\Rightarrow (\ell_1\ell_2\ell_3)^2 = (s-a)^2(s-b)^2(s-c)^2$

$\ell_1\ell_2\ell_3 = (s-a)(s-b)(s-c)$

$$\ell_1\ell_2\ell_3 = \frac{\Delta^2}{s}$$

6. Ans. (C)

Sol. Let $A_1 H_{11} = A_2 H_{10} = A_3 H_9 = A_4 H_8 = A_5 H_7 = 9$

$G_2G_{10} = G_4G_8 = (G_6)^2 = 9$

$$\prod_{k=1}^{5}\left(A_k \cdot G_{12-2k} \cdot H_{12-k}\right)$$

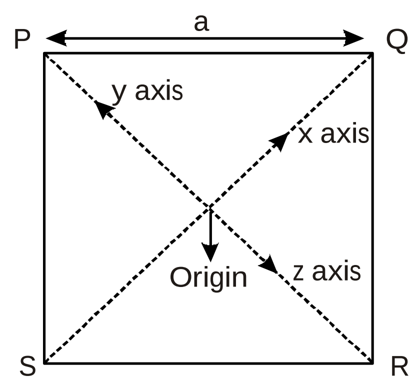
$=(A_1H_{11})(A_2H_{10})(A_3H_9)(A_4H_8)(A_4H_7)(A_5H_7)(G_{10}G_8G_6G_4G_2)$

$= (9^5)(9)^2 \cdot 3$

$= 3^{15}$

7. Ans. (B)

Sol.



$$\overrightarrow{OQ} = \frac{a}{\sqrt{2}}\hat{i}, \overrightarrow{OR} = \frac{a}{\sqrt{2}}\hat{k}, \overrightarrow{OP} = \frac{a}{\sqrt{2}}\hat{j}$$

$$\overrightarrow{SP} = \overrightarrow{OP} - \overrightarrow{OS} = \frac{a}{\sqrt{2}}\left(\hat{j} + \hat{i}\right)$$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = \frac{a}{\sqrt{2}}\left(\hat{k} - \hat{i}\right)$$

$$\cos\theta = \left|\frac{\overrightarrow{QR}.\overrightarrow{SP}}{|\overrightarrow{QR}||\overrightarrow{SP}|}\right| = \left|\frac{-\frac{a^2}{2}}{a^2}\right| = \left|-\frac{1}{2}\right|$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

8. Ans. (C)

Sol. $x < y < z \Rightarrow$ number of numbers $= {}^9C_3$

$x = y < z \Rightarrow$ number of numbers $= {}^9C_2$

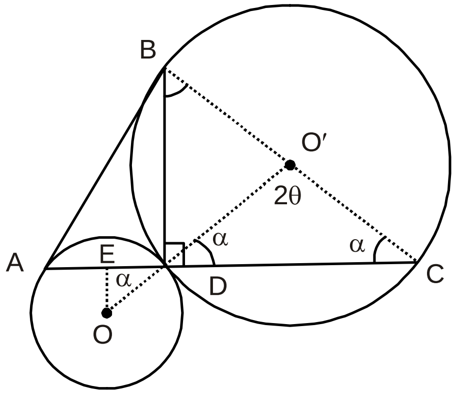
$x < y = z \Rightarrow$ number of numbers $= {}^9C_2$

$x = y = z \Rightarrow$ number of numbers $= 9$

total numbers $= {}^9C_3 + {}^9C_2 + {}^9C_2 + 9 = 165$

9. Ans. (B)

Sol.



$\cos\alpha = \frac{ED}{OD} = \frac{1}{4}$

BD = 4 tan α

$= 4\sqrt{15}$

Area of $\Delta ABC = \frac{1}{2} \times 6 \times 4\sqrt{15} = 12\sqrt{15}$

10. Ans. (D)

Sol. $4^a + 4^b + 3$

$= (5-1)^a + (5-1)^b + 3$

$= 5 \text{ (integer)} + (-1)^a + (-1)^b + 3$

⇒ a and b both must be even

$\Rightarrow \text{prob} = \frac{^{25}C_2}{^{50}C_2} = \frac{25 \times 24}{50 \times 49} = \frac{12}{49}$

11. Ans. (A, B)

Sol. Let $f(x) = (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{2018})$

f(0) = f(1)

$\Rightarrow \alpha_1 \cdot \alpha_2 \ldots\ldots \alpha_{2018} = (1-\alpha_1)(1-\alpha_2) \ldots\ldots (1-\alpha_{2018})$

$\Rightarrow \alpha_1^2 \cdot \alpha_2^2 \ldots\ldots \alpha_{2018}^2 = \alpha_1(1-\alpha_1) \cdot \alpha_2(1-\alpha_2) \ldots\ldots \alpha_{2018}(1-\alpha_{2018})$

Apply A.M. ≥ G.M.

in α_1 and $1-\alpha_1$

$$\frac{\alpha_1 + 1 - \alpha_1}{2} \geq \left(\alpha_1(1-\alpha_1)\right)^{\frac{1}{2}}$$

$$\Rightarrow \left(\alpha_1(1-\alpha_1)\right)^{\frac{1}{2}} \leq \frac{1}{2}$$

$$\Rightarrow \left(\alpha_1(1-\alpha_1)\alpha_2(1-\alpha_2) \ldots\ldots \alpha_{2018}(1-\alpha_{2018})\right)^{\frac{1}{2}} \leq \left(\frac{1}{2}\right)^{2018}$$

$\Rightarrow$ Product of roots $\leq \frac{1}{2^{2018}}$

12. Ans. (A, C)

Sol. If given lines are coplanar then

$(kx - 4y + 7z + 16) + \lambda(4x + 3y - 2z + 3) = 0$

and

$(x - 3y + 4z + 6) + \mu(x - y + z + 1) = 0$

represents same plane

$(4\lambda + k)x + (3\lambda - 4)y + (7 - 2\lambda)z + 3\lambda + 16 = 0$

$(1 + \mu)x - (3 + \lambda)y + (\mu + 4)z + \mu + 6 = 0$

$$\Rightarrow \frac{4\lambda + k}{1+\mu} = \frac{3\lambda - 4}{-(3+\mu)} = \frac{7-2\lambda}{\mu+4} = \frac{3\lambda+16}{\mu+6}$$

$$\Rightarrow \frac{4\lambda + k}{1+\mu} = \frac{\lambda+3}{1} = \frac{5\lambda+9}{2}$$

$\Rightarrow \lambda = -1$ and $\mu = \frac{1}{2}$ and K = 7

equation of plane will be

$3x - 7y + 9z + 13 = 0$

13. Ans. (B, D)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

total symmetre matrices = 3 × 3 × 3 = 27

For singular matrices |A| = 0

i.e. ad – bc = 0

ad = bc

when ad = bc = 1 then no. of matrices = 1

when ad = bc = 4 then no. of matrices = 1

when ad = bc = 9 then no. of matrices = 1

when ab = cd = 2 then no. of matrices = 4

when ab = cd = 3 then no. of matrices = 4

when ab = cd = 6 then no. of matrices = 4

total number of singular matrices = 15

14. Ans. (A, B)

Sol. $f'(x) - 3x^2 f(x) > 0 \quad \forall x \geq 1$

$$\Rightarrow \frac{d}{dx}\left(e^{-x^3} f(x)\right) > 0 \; \forall x \geq 1$$

$$\Rightarrow e^{-x^3} f(x) \geq e^{-1} f(1) \Rightarrow e^{-x^3} f(x) \geq e$$

$$\Rightarrow f(x) \geq e^{x^3+1} \forall x \geq 1$$

$$f(x) \geq e^2$$

15. Ans. (A,C)

Sol. $f(x) = g\left(\frac{1}{x^5}\right) e^{-\frac{1}{x^6}}$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{g\left(\frac{1}{x^5}\right)}{e^{\frac{1}{x^6}}}$$

$$= \lim_{t\to\infty} \frac{g\left(t^5\right)}{e^{t^6}} = 0$$

$$f'(x)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}$$

$$=\lim_{x\to 0}\frac{g\left(\frac{1}{x^5}\right)e^{-\frac{1}{x^6}}}{x}\quad=\lim_{x\to 0}\frac{g\left(\frac{1}{x^5}\right)}{xe^{\frac{1}{x^6}}}$$

$$=\lim_{t\to\infty}\frac{t\,g\left(t^5\right)}{e^{t^6}}=0$$

$\Rightarrow$ f(x) is continuous and differentiable at x = 0

SECTION–IV

1. Ans. (6)

Sol. $\omega^{23}=1$

$$N=\sum_{k=1}^{22}\frac{1}{1+\omega^{8k}+\omega^{16k}}$$

$$=\sum_{k=1}^{22}\left(\frac{1-\omega^{8k}}{1-\omega^{24k}}\right)$$

$$=\sum_{k=1}^{22}\left(\frac{1-\omega^{8k}}{1-\omega^{k}}\right)$$

$$=\sum_{k=1}^{22}\left[1+\omega^{k}+\omega^{2k}+.....\omega^{7k}\right]$$

$$=22+\sum_{k=1}^{22}\left(\omega^{k}\right)+\sum_{k=1}^{22}\left(\omega^{2k}\right)+.....\sum_{k=1}^{22}(\omega^{7k})$$

= 22 – 7 = 15

2. Ans. (6)

Sol. $\frac{x^2}{16}+\frac{y^2}{1}=1$

$a^2 = 16$, $b^2 = 1$

a = 4, b = 1

let $A(x_1, y_1)$ and $B(x_2, y_2)$ then P, Q will be

$P\left(x_1,\frac{y_1}{4}\right)$ and $Q\left(x_2,\frac{y_2}{4}\right)$

mid point of AB will be $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

mid point of PQ will be

$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{8}\right)=(h,k)$

$\Rightarrow$ mid point of AB will be (h, 4k)

let point S(t, t + 8) be a point on y = x + 8

AB is chord of contact of $x^2 + y^2 = 16$

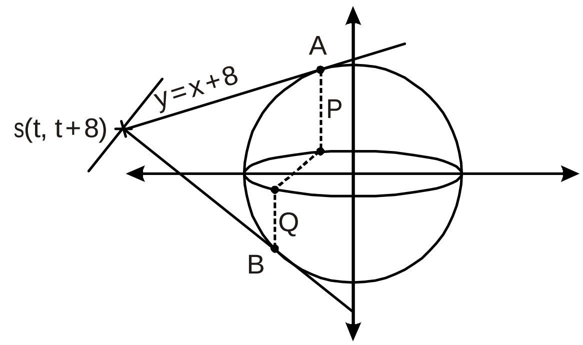
$\Rightarrow$ xt + y(t + 8) = 16 ....(i)

AB as chord whose mid point is (h, 4k)

$xh + y(4k) = h^2 + 16k^2$ ...(ii)

$$\frac{t}{h}=\frac{t+8}{4k}=\frac{16}{h^2+16k^2}$$

$$\Rightarrow\ t=\frac{16h}{h^2+16k^2}\ \text{ and }\ t+8=\frac{64k}{h^2+16k^2}$$



$$\Rightarrow\ \frac{16h}{h^2+16k^2}+8=\frac{64k}{h^2+16k^2}$$

$\Rightarrow$ $2h + (h^2 + 16k^2) = 8k$

$\Rightarrow$ $x^2 + 16y^2 + 2x – 8y = 0$

$x^2 + \alpha y^2 + \beta x + \gamma y = 0$

$\Rightarrow$ $\alpha = 16$, $\beta = 2$, $\gamma = –8$

$\Rightarrow$ $\alpha – \beta + \gamma = 16$

3. Ans. (1)

Sol. $P(at_1 t_2, a(t_1+t_2))$

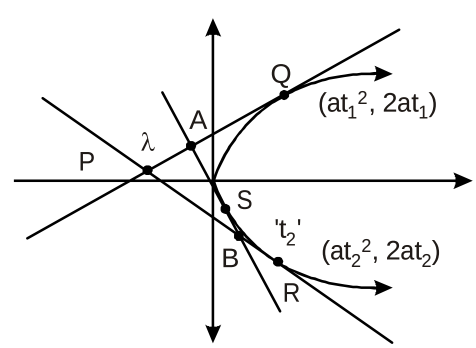
$S(at_3^2, 2at_3)$

$A(at_1t_3, a(t_1+t_3))$

$B(at_2t_3, a(t_2+t_3))$

Let PA : PQ = λ : 1

Using section formula $\lambda=\frac{t_3-t_2}{t_1-t_2}$



$$\Rightarrow\ \frac{PA}{PQ}=\frac{t_3-t_2}{t_1-t_2}$$

Similarly $\frac{PB}{PR}=\frac{t_1-t_3}{t_1-t_2}$

$\Rightarrow \frac{PA}{PQ}+\frac{PB}{PR}=1$

4. Ans. (0)

Sol. $S=\sum_{r=0}^{n}(-1)^r\,{}^nC_r\,\frac{1+rx}{(1+nx)^r}$

(let n = 2018, x = ln 2)

$$=\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(1+nx)^r}+\sum_{r=0}^{n}(-1)^r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1}\frac{rx}{(1+nx)^r}$$

$$=\sum_{r=0}^{n}(-1)^r\frac{{}^nC_r}{(1+nx)^r}-\frac{nx}{(1+nx)}\sum_{r=0}^{n}(-1)^{r-1}\,{}^{n-1}C_{r-1}\frac{1}{(1+nx)^{r-1}}$$

$$=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}$$

$$=\left(\frac{nx}{1+nx}\right)^n-\frac{nx}{1+nx}\cdot\left(\frac{nx}{1+nx}\right)^{n-1}=0$$

5. Ans. (3)

Sol. $I_n=\int_0^{\frac{3\pi}{2}}(\overset{I}{\ln|\sin x|})\,\overset{II}{\cos(2nx)}\,dx$

applying integration by parts

$$I_n=\left\{\ln|\sin x|.\frac{\sin 2nx}{2n}\right\}_0^{\frac{3\pi}{2}}-\int_0^{\frac{3\pi}{2}}\frac{\cot x.\sin 2nx}{2n}dx$$

$$I_n=0-\frac{1}{2n}I_n'$$

$$I_n'=\int_0^{\frac{3\pi}{2}}\frac{\cos x\cdot\sin 2nx}{\sin x}dx$$

$$I_n'-I_{n-1}'=\int_0^{\frac{3\pi}{2}}\frac{\cos x(\sin 2nx-\sin(2n-2))x}{\sin x}dx$$

$$=\int_0^{\frac{3\pi}{2}}\frac{2\cos x\cdot\cos(2n-1)x\sin x}{\sin x}dx$$

$$I_n'-I_{n-1}'=\int_0^{\frac{3\pi}{2}}2\cos(2n-1)x\cdot\cos x\,dx=0$$

$$I_n'=I_{n-1}'=I_{n-2}'=\ldots\ldots\ I_1'$$

$$I_n'=\int_0^{\frac{3\pi}{2}}\frac{\sin 2x\cos x}{\sin x}dx$$

$$=\int_0^{\frac{3\pi}{2}}2\cos^2x\,dx=\int_0^{\frac{3\pi}{2}}(1+\cos 2x)dx$$

$$=\frac{3\pi}{2}$$

$$I_n=-\frac{3\pi}{4n}$$

$12I_3=-3\pi$

$16I_2=-6\pi$

$\Rightarrow 12I_3-16I_2=3\pi$