PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A)

Sol. The diagram shows limiting case for penetration into the liquid.

Snell's law at S_1

$\Rightarrow \quad \sin\phi = \frac{1}{n'} \qquad \ldots(i)$

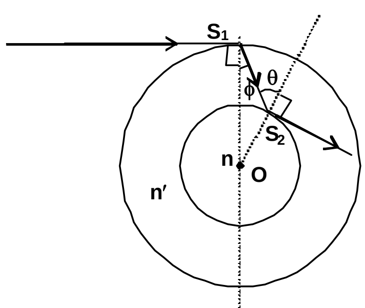
Snell's law for S_2

$\Rightarrow \quad \sin\theta = \frac{n}{n'} \qquad \ldots(ii)$

sin rule in triangle S_1S_2O

$\Rightarrow \quad \frac{\sin\phi}{r} = \frac{\sin(\pi-\theta)}{R} \quad \ldots(i)$

Solving the above equations we get



$r = \frac{R}{n}$

2. Ans. (B)

Sol. $E = 2 \times \frac{60}{75} \times \left(\frac{10000}{100}\right)$ = 160 volt; $E = E_0$

$\frac{\ell_1}{\ell_2}\left(\frac{R_1+R_2}{R_1}\right)$

$$\frac{\Delta E}{E} = \left|\frac{\Delta\ell_1}{\ell_1}\right| + \left|\frac{\Delta\ell_2}{\ell_2}\right| + \left|\frac{\Delta(R_1+R_2)}{R_1+R_2}\right| + \left|\frac{\Delta R_1}{R_1}\right|$$

$\Rightarrow \Delta E = 0.64$ volts

3. Ans. (A)

4. Ans. (D)

Sol. $+2E + E - 4E + I_1R = 0$

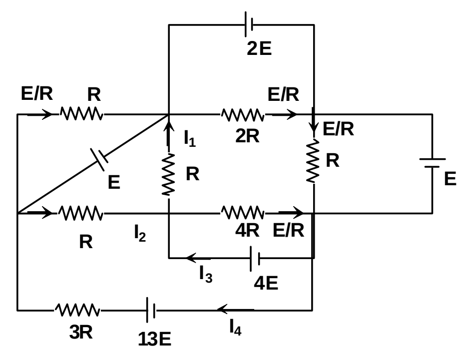
$\Rightarrow \quad I_1 = E/R$

$I_2 = 0$

$I_3 = 2E/R$

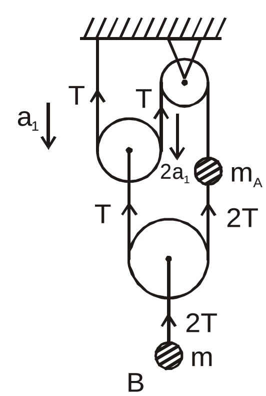
$0 + 4E - 13E + I_4(3R) = 0$

$\Rightarrow \quad I_4 = 3E/R$



5. Ans. (A)

Sol.



If pulley is ideal,

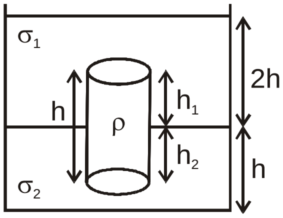
T = 0

$a_B = g\downarrow$

$a_P = \frac{a_1 + a_2}{2}$

6. Ans. (B)

Sol.



For condition of equilibrium :

B = mg

$\rho[A][h_1 + h_2]g = \sigma_1 A.h_1 g + \sigma_2 A h_2 g$

$\rho h_1 + \rho h_2 = \sigma_1 h_1 + \sigma_2 h_2$

$h_2(\rho - \sigma_2) = h_1(\sigma_1 - \rho)$

$$\frac{h_2}{h_1} = \frac{(\sigma_1 - \rho)}{(\rho - \sigma_2)}$$

7. Ans. (A)

Sol. $N = \frac{mv^2}{R}$ $\qquad f_{max} = \mu N = \frac{\mu m v^2}{R}$

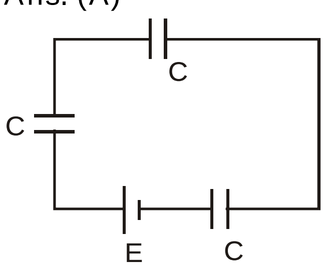
$\Rightarrow$ Retardation $a = \frac{f_{max}}{m} = \frac{\mu v^2}{R}$

$$\Rightarrow -\frac{dv}{dt} = \frac{\mu v^2}{R}$$

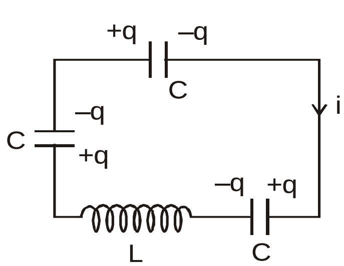
$$\Rightarrow -\int_{v_0}^{v} \frac{dv}{v^2} = \frac{\mu}{R}\int_0^t dt \qquad v = v_0 \Big/ \left(1 + \frac{\mu v_0 t}{R}\right)$$

8. Ans. (A)

Sol.



At steady state, $Q = \frac{CV}{3}$ on each capacitor.



At t = 0

$q = \frac{CV}{3}$, i = 0

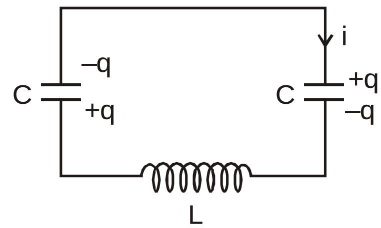
$$q = \frac{CV}{3}\cos(\omega t)$$

$$i = -\frac{da}{dt} = \frac{CV}{3}\omega \sin(\omega t)$$

$$\omega = \sqrt{\frac{1}{LC_{eq}}}$$

$$C_{eq} = \frac{C}{3}$$

At $t = t_0$



For Q_{max}

$$\left(\frac{Q_{max}^2}{2C}\right) \times 2 = \left(\frac{q^2}{2C}\right) \times 2 + \frac{1}{2}Li^2$$

9. Ans. (C)

10. Ans. (D)

11. Ans. (B)

Sol. $h\nu' = h\nu - \frac{GMm}{R} = h\nu - \frac{GMh\nu}{C^2R}$

$$\Rightarrow h\nu' = h\nu\left(1 - \frac{GM}{C^2R}\right)$$

or, $\frac{\nu'}{\nu} = 1 - \frac{GM}{c^2R}$

12. Ans. (C)

Sol. $\frac{\nu'}{\nu} = 1 - \frac{GM}{c^2R}$

$$\Rightarrow \frac{GM}{c^2R} = 1 - \frac{\nu'}{\nu}$$

$$\frac{\nu - \nu'}{\nu} = \frac{\Delta\nu}{\nu} = GRS$$

13. Ans. (B)

Sol. $\Delta x = (\mu_{air}\ t_2 + \mu t_1) \times 2$

$$= \left(1t_2 + \frac{3}{4}t_1\right)2$$

$\Delta x = 2t_2 + 3t_1$

Since both rays suffer a phase difference of π on reflection, condition of constructive interference

$3t_1 + 2t_2 = n\lambda_0$

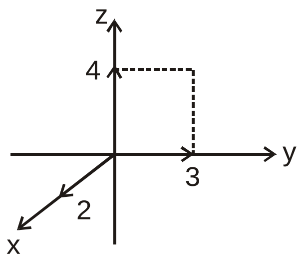
14. Ans. (C)

Sol. $\Delta x = 2 \times 70 + 3 \times 130$

$= 530 \text{ nm} = n\lambda$

15. Ans. (C)

Sol. At t = 0



$$R = \frac{mV}{qB} = \frac{10 \times 5}{\pi \times 10} = \frac{5}{\pi}$$

$$T = \frac{2\pi m}{qB} = \frac{2\pi \times 10}{\pi \times 10} = 2 \sec$$

At t = 0.5 sec,

$x_1 = 2 \times 0.5 = 1\text{m}$

At t = 1 sec

$x_2 = 2 \times 1 = 2\text{m}$

$\sqrt{y_2^2 + z_2^2} = 2R = \dfrac{10}{\pi}$

At t = 2 sec, $x_3 = 2 \times 2 = 4$

$y_3 = 0$

$z_3 = 0$

16. Ans. (C)

Sol. If $\vec{E} = (-40\hat{j} + 30\hat{k})$

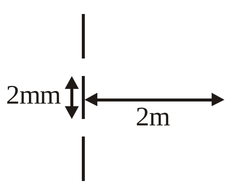
Net force : $q\vec{E} + q(\vec{V} \times \vec{B}) = 0$

Motion will be straight line

17. Ans. (B)

Sol. d = 2mm

D = 2m



$I = I_0 + I_0 + 2I_0 \cos\phi \qquad \left(\Delta x = \dfrac{yd}{D}\right)$

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

18. Ans. (B)

Sol. (1) Charge decays exponentially

(2) Charge on capacitor oscillates

(3) Current through inductor oscillates

(4) Current increases exponentially to reach a steady volve.

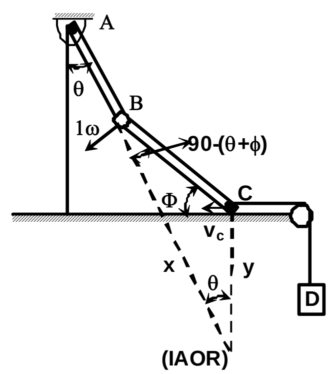
19. Ans. (D)

Sol. Angular velocity of rod BC $\omega' = -\dfrac{d\phi}{dt}$

$$\omega' = \frac{v_c}{y} = \frac{1\omega}{x}$$

$$\frac{x}{\cos\phi} = \frac{y}{\cos(\theta + \phi)} = \frac{1}{\sin\theta}$$

$$\omega' = \omega\frac{\sin\theta}{\cos\phi}$$



Angular acceleration of rod BC = α =

$$\frac{d\omega'}{dt} = -\omega^2\frac{\left(\cos^2\phi\cos\theta + \sin\phi\sin^2\theta\right)}{\cos^3\phi}$$

20. Ans. (C)

Sol. 22.95 eV

$\Rightarrow$ Li^{2++} transitions from n = 4 to n = 2

PART–2 : CHEMISTRY SOLUTION

1. Ans. (B)

Sol. (A) Theory based.

(B) Order of IE_1 is B > Tℓ > Ga > Al > In

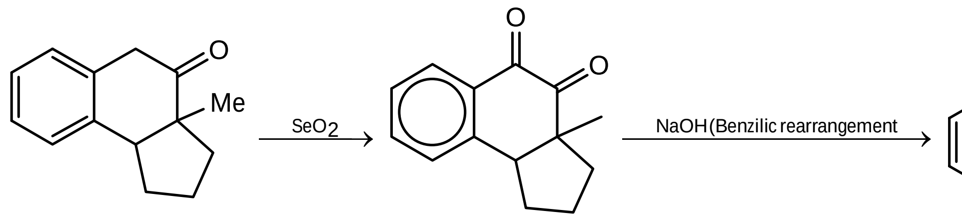
Order of IE_2 will be B > Ga > Tℓ > Al > In

(C) N – N bond energy is lower because of lone pair – lone pair repulsions.

(D) Thermochromic substances change colour on heating shown by both ZnO & S_4N_4.

2. Ans. (C)

Sol.



3. Ans. (B)

Sol. Radial part of any orbital follow the function.

$\Psi_{(r)} = K.e^{-\sigma/K}.\sigma^{\ell}$ (polynomial in σ)

All roots finite & different & non–zero.

(A) does not have "σ^{ℓ}" $\therefore$ s – orbital $\therefore$ θ expression cannot be present, hence incorrect.

(B) Contains "σ^1" $\therefore$ p – orbital which can have sin θ cos ϕ as its angular wavefunction.

(C) The polynomial in σ does not have real roots.

(D) Contains "σ^1" $\therefore$ P-orbital which cannot have ($3\cos^2\theta - 1$) as its angular component.

4. Ans. (C)

Sol. (A) These use distillation as one method of purification.

(B) In partial roasting Cu & SO_2 are obtained.

(C) When H_2 is used as reducing agent

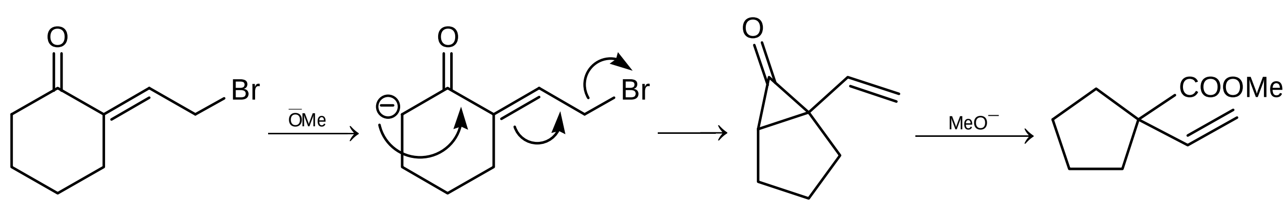
$$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H_2O_{(g)or(\ell)} \qquad \Delta S = -ve$$

$\therefore$ Slope of ΔG^0 vs T is +ve, hence cannot be used at high temperatures.

(D) Lead can be obtained from reduction by Carbon also.

5. Ans. (B)

Sol.



6. Ans. (B)

Sol. $\Delta T_f = i \times k_f \times m$

$\therefore 3.5 \times 10^{-3} = i \times 2 \times 10^{-3}$

$\therefore i = 1.75$

$$\underset{1-\alpha}{HA} \rightleftharpoons \underset{\alpha}{H^+} + \underset{\alpha}{A^-}$$

$1 + \alpha = i$ $\qquad$ $\therefore \alpha = 0.75$

(A) $\alpha = 0.75$ when conc. is 0.001 not 0.01

(B) $0.75 = \dfrac{\lambda_m}{460}$

$$\lambda_m = 345\,\Omega^{-1}cm^2mol^{-1}$$

(C) As conc. $\downarrow \lambda_m \uparrow$

$\therefore \lambda_m$ at 10^{-4} should be greater than 345 $\Omega^{-1}cm^2mol^{-1}$.

7. Ans. (C)

Sol. (A) Theory based true statement.

(B) Borax in aq. solution gives Boric acid & its salt.

(C) B_2H_6 with excess NH_3 at low temperature gives $B_3N_3H_6$ which is aromatic but more reactive than benzene.

(D) Theory based true statement.

8. Ans. (B)

Sol. P/P_C & V_m/V_C are reduced pressure & reduced volume & as per law of corresponding states in such cases reduced temperatures should also be same.

$\therefore \quad \dfrac{T_A}{T_{CA}} = \dfrac{T_B}{T_{CB}} \qquad \therefore \dfrac{T_A}{T_B} = \dfrac{T_{CA}}{T_{CB}}$

9. Ans. (C)

Sol. Distortion will occur when there is unsymmetrical filling of electrons in d – orbital (t_{2g} & e_g).

(A) [↑|↑|↑] [↑|↑] symmetrical (B) [↑↓|↑↓|↑↓] [↑|↑] symmetrical

(C) [↑|↑|↑] [↑|] unsymmetrical (D) [↑|↑|↑] [|] symmetrical

10. Ans. (B)

Sol. (A) Cu^{2+} has d^9 configuration hence there will be distortions.

(B) [e_g at end]. The t_{2g} orbitals are slightly away from ligands as compared to e_g orbitals which are head on to ligand molecules hence e_g orbitals will show significant interactions.

(C) Pt^{2+} will also have significant distortions because of high nuclear charge.

(D) [CrF_6^{4-}] CrF_6^{4-} have Cr^{2+} which is d^4 configuration & F^- is weak ligand.

$\therefore$ There will be distortion & hence Cr – F bond length will not be same.

11. Ans. (C)

Sol. As per Clausius inequality

$$TdS \geq dU - w$$

$$TdS \geq dU + PdV - w_{nPV}$$

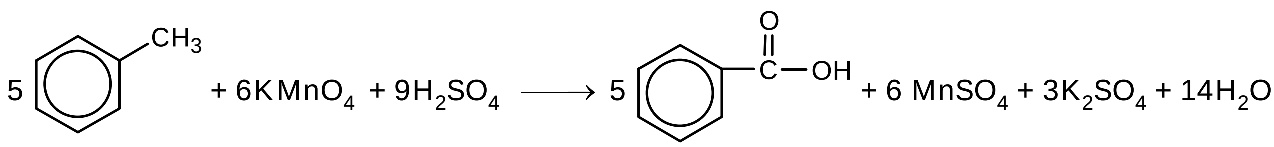
(A) $dU \leq w_{nPV}$ (if $S\rightarrow$ & $V\rightarrow$)

(B) $dH \leq w_{nPV}$ (if $S\rightarrow$ & $P\rightarrow$)

(C) $d(H - TS) \leq w_{nPV}$ (if $P\rightarrow$ & $T\rightarrow$)

$\therefore \Delta G \leq w_{nPV}$ ($P\rightarrow$ & $T\rightarrow$) $\therefore$ True statement.

12. Ans. (C)
Sol. The balanced reaction is :



Number of e^- = 30
$\Delta G^0 = -30 \times 96500 \times 0.1$
$= -289.5$ kJ
$|\Delta G^0|$ gives maximum non PV work which can be extracted.

(A) Per mole of toulene = $\frac{289.5}{5} = 57.9\,\text{kJ}$

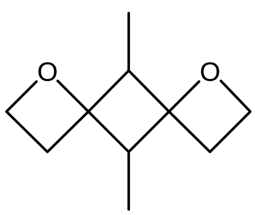
(B) Per mole of $KMnO_4$ = $\frac{289.5}{6} = 48.25\,\text{kJ}$

(C) Per mole of H_2SO_4 = $\frac{289.5}{9} = 32.17\,\text{kJ}$

(D) Per mole of $MnSO_4$ = $\frac{289.5}{6} = 48.25\,\text{kJ}$

$\therefore$ C is correct.

13. Ans. (D)
Sol. Diastereoisomers are stereoisomers, which are not mirror images of each other.

14. Ans. (B)
Sol.  has 5 stereoisomers & all have POS or COS.

15. Ans. (C)
Sol. $AgBr_{(\text{yellow coloured solid})} \Rightarrow A$

$$AgBr_{(g)} + NH_3 \longrightarrow [Ag(NH_3)_2]Br$$

$$AgBr_{(s)} + 2Na_2S_2O_3 \longrightarrow \underset{(B)}{Na_3[Ag(S_2O_3)_2]} + NaBr$$

$$Na_3[Ag(S_2O_3)_2 \xrightarrow{\Delta} \underset{(C)}{Ag_2S}\downarrow_{\text{(Black)}}$$

(A) $S_2O_3^{2-}$ acts as monodentate ligand.
(B) $Ag_2S + AgNO_3 \rightarrow$ +vely charged sol.
(C) $AgBr + CN^- \rightarrow [Ag(CN)_2]^-$
Soluble complex
(D) AgBr has Rock salt structure

16. Ans. (B)
Sol. $$AgBr + \text{conc.HCl} + \text{conc.HNO}_3 \longrightarrow \underset{\text{White}}{AgCl}\downarrow$$

$$AgCl_{(s)} + NH_3 \longrightarrow [Ag(NH_3)_2]Cl$$

$$AgCl_{(g)} + NaCl \longrightarrow \underset{\text{soluble complex}}{AgCl_2^-}$$

(A) Between AgBr & AgCl; lather is more soluble.

(B) The complex is $[Ag(NH_3)_2]^+$ which will have lower standard reduction potential as compared to $E^\circ_{Ag^+/Ag}$.

$$E^\circ_{Ag(NH_3)_2^+/Ag} = E^\circ_{Ag^+/Ag} - \frac{0.059}{1}\log K_f$$

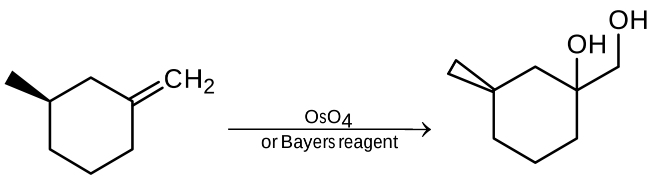
where K_f is formation constant of complex which is very high.

(C) On the basis of reaction.

(D) $AgCl + AgNO_3 \rightarrow$ +vely charge colloid which shows tyndall effect.

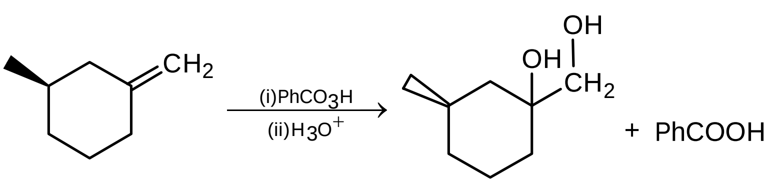
17. Ans. (B)

Sol. (P)

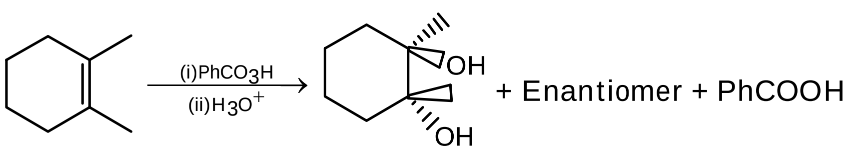


(1 new chiral 'C'
so 2 product)

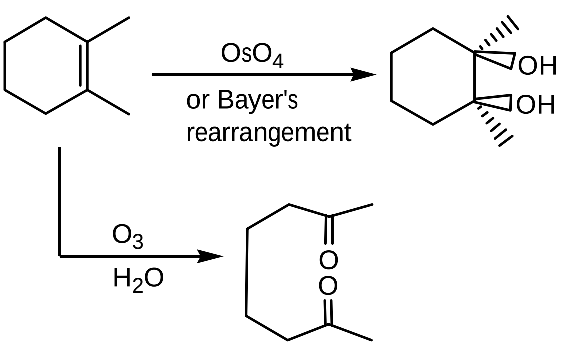
(Q)



(R)



(S)



18. Ans. (C)

Sol. $2A + B \rightarrow C$

$\because$ Elementary reaction

$\therefore R = K[A]^2[B]^1$

$\therefore$ When $[A]_0 = 0.1$ & $[B]_0 = 10^{-9}$

Reaction follows first order & half life of A not defined.

$\therefore P \rightarrow (3)$

When $[A]_0 = 10^{-4}$ M & $[B]_0 = 10^{-1}$ M

Reaction follows second order

$\therefore t_{3/4} = t_{1/2} + 2 \times t_{1/2} = 3t_{1/2}$

and half life of B is not defined.

$\therefore Q \rightarrow (4)$.

When $[A]_0 = 0.1$ M & $[B]_0 = 0.05$ M

$\because [A]_0 = 2[B]_0$

$\therefore$ Reaction will follow 3rd order kinetics & half life of A & B both will be same.

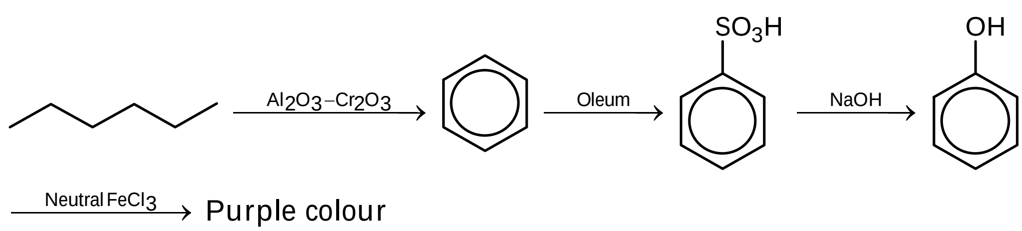
When $[A]_0 = 0.1$ M & $[B]_0 = 0.2$ M

Half life of A & half life of B will be different.

19. Ans. (B)

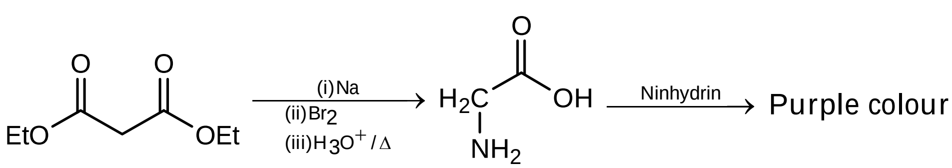
Sol. (P) $Mg_2C_3 \xrightarrow{H_2O} CH_3C \equiv CH \xrightarrow[NH_4OH]{Cu_2Cl_2} \text{Red colour}$

(Q)



(R) Ethanol $\xrightarrow{Et_2SO_4}$ Ether $\xrightarrow{air}$ Peroxide of ether $\xrightarrow{FeSO_4, KCNS}$ Red colour

(S)



20. Ans. (C)

Sol. (P) $AlCl_3$ in aq. solution exist as $Al^{+3}_{(aq)}$ & $Cl^-_{(aq)}$ ions cannot act as an oxidising or reducing agent forms $[Al(OH)_4]^-$ with excess KOH.

(Q) I_2O_5 act an oxidising agent getting converted to I_2.

In aq. solution gets converted to HIO_3 which dissociates to give H^+ & IO_3^-.

No complex formation with KOH.

(R) Generally used as a reducing agent.

Reacts with KOH to form complex.

Gives insoluble basic salt in aq. solution.

$$SnCl_2 + H_2O \rightleftharpoons Sn(OH)Cl_{(s)} + HCl_{(aq.)}$$

(S) Strong oxidising agent

Insoluble in water.

Forms complex with KOH.

SECTION-1

1. Ans. (C)

Sol. $a_0 = 1, a_1 = 0$

$a_n = 3a_{n-1} - 2a_{n-2}$

$(a_n - 2a_{n-1}) = (a_{n-1} - 2a_{n-2})$

$\therefore$ $a_n - 2a_{n-1}$ = constant (k)

$\therefore$ $a_1 - 2a_0 = k \Rightarrow k = -2$

$\therefore$ $a_n = 2a_{n-1} - 2$

$\Rightarrow$ $(a_n - 2) = 2(a_{n-1} - 2)$

$\Rightarrow$ $b_n = 2 \cdot b_{n-1}$

$\therefore$ $b_0, b_1, b_2, b_3, \ldots\ldots., b_n$ are in G.P.

$b_n = b_0 \cdot 2^n$

$b_n = (-1) \cdot 2^n$

$a_n - 2 = -2^n$

$a_n = 2 - 2^n$

2. Ans. (B)

Sol. $10 \sin^4 \alpha + 15 \cos^4 \alpha = 6$

$\Rightarrow$ $10 \sin^4 \alpha + 15(1 - \sin^2 \alpha)^2 = 6$

$\Rightarrow$ $10 \sin^4 \alpha + 15(\sin^4 \alpha - 2 \sin^2 \alpha + 1) = 6$

$\Rightarrow$ $25 \sin^4 \alpha - 30 \sin^2 \alpha + 9 = 0$

$\Rightarrow$ $(5 \sin^2 \alpha - 3)^2 = 0$

$\Rightarrow$ $\sin^2 \alpha = \frac{3}{5} \Rightarrow \cos^2 \alpha = \frac{2}{5}$

$\therefore$ $S = 9 \operatorname{cosec}^4 \alpha + 4 \sec^4 \alpha$

$S = 9 \cdot \left(\frac{25}{9}\right) + 4\left(\frac{25}{4}\right)$

$S = 25 + 25$

$\frac{S}{25} = 2$

3. Ans. (A)

Sol. $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2$, we get,

$$D = \begin{vmatrix} x_1 + y_1 & y_1 & 1 \\ x_2 + y_2 & y_2 & 1 \\ x_3 + y_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} -8 & y_1 & 1 \\ -8 & y_2 & 1 \\ -8 & y_3 & 1 \end{vmatrix} = 0$$

4. Ans. (B)

Sol. Given $a_{n+1} = a_n - 4n + 3$

Now

$$a_k - a_1 = \sum_{n=1}^{k-1}(a_{n+1} - a_n = \sum_{n=1}^{k-1}(4n+3)$$

$$= 4(k-1)\frac{(k)}{2} + 3(k-1) = 2k^2 + k - 3$$

$$\Rightarrow a_k = 2k^2 + k - 3$$

So,

$$\lim_{k\to\infty} \frac{\sqrt{2k^2+k-3} + \sqrt{2(4k)^2+(4k)-3} + \ldots.. + \sqrt{2(4^{10}k)^2+(4^{10}k)-3}}{\sqrt{2k^2+k-3} + \sqrt{2(2k)^2+(2k)-3} + \ldots + \sqrt{2(2^{10}k)^2+(2^{10}k)-3}}$$

$$= \lim_{k\to\infty} \frac{\sqrt{2+\frac{1}{k}-\frac{3}{k^2}} + \sqrt{2(4)^2+\frac{4}{k}-\frac{3}{k^2}} + \ldots. + \sqrt{2(4^{10})^2+\frac{4^{10}}{k}-\frac{3}{k^2}}}{\sqrt{2+\frac{1}{k}-\frac{3}{k^2}} + \sqrt{2(2)^2+\frac{2}{k}-\frac{3}{k^2}} + \ldots. + \sqrt{2(2^{10})^2+\frac{2^{10}}{k}-\frac{3}{k^2}}}$$

$$= \frac{\sqrt{2} + \sqrt{2(4^2)} + \ldots + \sqrt{2(4^{10})^2}}{\sqrt{2} + \sqrt{2(2^2)} + \ldots + \sqrt{2(2^{10})^2}} = \frac{\sqrt{2}(1+4+4^2+\ldots+4^{10})}{\sqrt{2}(1+2+2^2+\ldots+2^{10})}$$

$$= \frac{\frac{1}{3}(4^{11}-1)}{2^{11}-1} = 683$$

5. Ans. (B)

Sol. $y = x^{n-1} \ln x$

$$y_1 = \frac{x^{n-1}}{x} + (n-1)x^{n-2}\ln x$$

$$y_1 = x^{n-2} + (n-1)\cdot\frac{y}{x} \quad \ldots (1)$$

$xy_1 = x^{n-1} + (n-1)y$

$xy_2 + y_1 = (n-1)x^{n-2} + (n-1)y_1 \quad \ldots. (2)$

$xy_2 + y_1 = (n-1)(y_1 - (n-1)\frac{y}{x}) + (n-1)y_1$

$xy_2 + y_1 = (n-1)y_1 - (n-1)^2 \frac{y}{x} + (n-1)y_1$

$x^2y_2 + xy_1 = (n-1)xy_1 - (n-1)^2y + (n-1)xy_1$

$x^2y_2 + xy_1(1 - n + 1 - n + 1) + (n-1)^2y = 0$

$x^2y_2 + xy_1(3 - 2n) + (n-1)^2y$

$f(n) = 3 - 2n$

$g(n) = n^2 - 2n + 1$

$f(n) - g(n) = 2 - n^2$

$f(3) - g(3) = 2 - 9 = -7$

$f(4) + g(4) = -5 + 9 = 4$

$f(5) \cdot g(5) = (-7)(16) = -112$

6. Ans. (D)

Sol. On integrating twice by parts taking e^{-x} as second function we have

$$I_m = \left[-\sin^m x e^{-x}\right]_0^\infty - \int_0^\infty m\sin^{m-1}x\cos x(-e^{-x})dx$$

$$= 0 + m\int_0^{\infty}(\sin^{m-1}x\cos x)e^{-x}dx$$

$$= m\left[\left\{\sin^{m-1}x\cos x\right\}\left(-e^{-x}\right)\right]_0^{\infty}$$

$$+m\int_0^{\infty}[(m-1)\sin^{m-2}x\cos^2 x - \sin^m x]e^{-x}dx$$

$$= 0 + m(m-1)\int_0^{\infty}\sin^{m-2}x(1-\sin^2 x)e^{-x}dx$$

$$-m\int_0^{\infty}e^{-x}\sin^m x dx$$

$$= m(m-1)\int_0^{\infty}e^{-x}\sin^{m-2}x dx$$

$$-m(m-1)\int_0^{\infty}e^{-x}\sin^m x dx - m\int_0^{\infty}e^{-x}\sin^m x dx$$

$$I_m = m(m-1)I_{m-2} - m^2 I_m$$

or $(1+m^2)I_m = m(m-1)I_{m-2}$ ...(i)

To evaluate I_4, put $m = 4,2$, successively in (i), so that

$17I_4 = 12I_2$ or $I_4 = \frac{12}{17}I_2$

and $5I_2 = 2I_0$, i.e., $I_2 = \frac{2}{5}I_0$

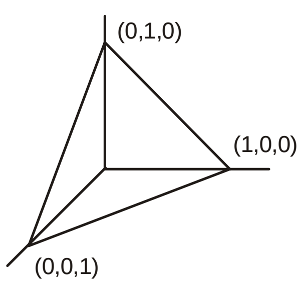
But $I_0 = \left[-e^{-x}\right]_0^{\infty} = 1$

$\therefore\ I_4 = \frac{12}{17}\times\frac{2}{5}\times 1 = \frac{24}{85}$

7. Ans. (D)

Sol. Plane through 3 points x + y + z = 1

For circumcentre of the triangle.



$$x^2 + (y-1)^2 + z^2 = (x-1)^2 + y^2 + z^2$$
$$= x^2 + y^2 + (z-1)^2$$

$\Rightarrow\quad -2x + 1 = -2y + 1 = -2z + 1$

$\Rightarrow\quad x = y = z = \frac{1}{3}$ as x + y + z = 1

8. Ans. (C)

Sol. Let $f(x) = x^{\frac{1}{x}}$

$\Rightarrow f'(x) = x^{\frac{1}{x}}\left(\frac{\ln x - 1}{x^2}\right)$

If x > e then f(x) is decreasing

$\because$ 2018 > 2017 $\Rightarrow$ f(2018) < f(2017)

$\Rightarrow (2018)^{\frac{1}{2018}} < (2017)^{\frac{1}{2017}}$

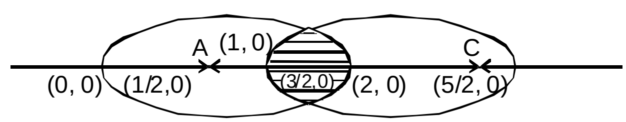
$\Rightarrow (2018)^{2017} < (2017)^{2018}$

$\because \lim_{x\to\infty}\left\{\left((2017)^{2018}\right)^n + \left((2018)^{2017}\right)^n\right\}^{\frac{1}{n}}$

$$= \lim_{n\to\infty}(2017)^{2018}\left\{1 + \left(\frac{(2018)^{2017}}{(2017)^{2018}}\right)^n\right\}^{\frac{1}{n}} = (2017)^{2018}$$

Paragraph for Questions 9 and 10

Sol.



PA + PB < 2 and PB + PC < 2

PA + PB < 2

Region inside ellipse with foci A and B

2a = 2, a = 1

$2ae = \frac{3}{2} - \frac{1}{2} = 1$

$b^2 = a^2 - a^2e^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Equation of ellipse is

$\frac{(x-1)^2}{1} + \frac{y^2}{3/4} = 1$

$PB + PC < 2 \Rightarrow P$ lies inside ellipse with foci B and C

Whose equation is $\frac{(x-2)^2}{1} + \frac{y^2}{3/4} = 1$

Locus of P is shown by shaded region which is symmetric about x-axis

Area of region

$$= 4\int_1^{3/2}\frac{\sqrt{3}}{2}\sqrt{1-(x-2)^2dx} = \sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

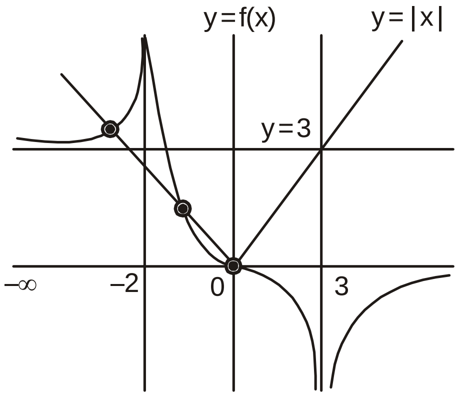
9. Ans. (A)

10. Ans. (B)

Paragraph for Questions 11 and 12

11. Ans. (C)

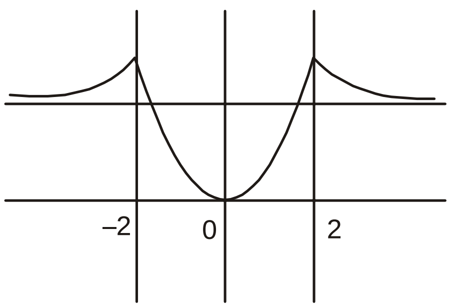
Sol.



Maximum possible number of solutions = 3

12. Ans. (A)

Sol. $y = f(-|x|)$



Clearly non-differentiable at 2 points.

Paragraph for Question 13 to 14

Sol. $\frac{y''}{y'} = \frac{2y'}{y} \Rightarrow \frac{y'}{y^2} = k$

$\int \frac{dy}{y^2} = \int k dx \Rightarrow -\frac{1}{y} = kx + \lambda$ ...(1)

since (2,2) and $\left(8, \frac{1}{2}\right)$ lies on (1), we get C_1 : xy = 4

shortest distance between

C_1 : xy = 4 and C_2 : $x^2 + y^2 = 4$ is

$\left(2\sqrt{2} - 2\right) = \left(\sqrt{8} - 2\right)$

Hence, (a + b) = 10

13. Ans. (C)

14. Ans. (D)

15. Ans. (C)

Sol. $a_{ij} = (k)^{i+j}$

$$A = \begin{bmatrix} k^2 & k^3 & k^4 \\ k^3 & k^4 & k^5 \\ k^4 & k^5 & k^6 \end{bmatrix}$$

$|A| = k^{12}(1 + 1 + 1 - 1 - 1 - 1) = 0$

16. Ans. (B)

Sol. $A^2 = 3A$

$$A^2 = \begin{bmatrix} k^2 & k^3 & k^4 \\ k^3 & k^4 & k^5 \\ k^4 & k^5 & k^6 \end{bmatrix}\begin{bmatrix} k^2 & k^3 & k^4 \\ k^3 & k^4 & k^5 \\ k^4 & k^5 & k^6 \end{bmatrix}$$

$$= \begin{bmatrix} k^4 + k^6 + k^8 & k^5 + k^7 + k^9 & k^6 + k^8 + k^{10} \\ k^5 + k^7 + k^9 & k^6 + k^8 + k^{10} & k^7 + k^9 + k^{11} \\ k^6 + k^8 + k^{10} & k^7 + k^9 + k^{11} & k^8 + k^{10} + k^{12} \end{bmatrix}$$

$$\Rightarrow A^2 = k^2\left(1 + k^2 + k^4\right)\begin{bmatrix} k^2 & k^3 & k^4 \\ k^3 & k^4 & k^5 \\ k^4 & k^5 & k^6 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} k^2 & k^3 & k^4 \\ k^3 & k^4 & k^5 \\ k^4 & k^5 & k^6 \end{bmatrix}$$

Hence $k^2(k^4 + k^2 + 1) = 3$

or $k = 0 \Rightarrow k = 0, \pm 1$

17. Ans. (D)

(P) Required probability $= \frac{1}{2}$

(Q) Required probability

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^7 = \frac{47}{2^7}$$

(R) Required probability $= \frac{4}{2^7}$

(S) There are five cases 0, 1, 2, 3, or 4 wins

Required probability

$$= \frac{1 + 7 + 15 + 10 + 1}{2^7} = \frac{34}{2^7}$$

18. Ans. (B)

Sol. (P) $\bar{a} \times \left(\bar{a} \times \bar{b}\right) = \left(\bar{a} \cdot \bar{b}\right)\bar{a} - \left(\bar{a} \cdot \bar{a}\right)\bar{b}$

$\bar{a} \times \left(\bar{a} \times \left(\bar{a} \times \bar{b}\right)\right) = -\left(\bar{a} \times \bar{b}\right)$

$\bar{a} \times \left(\bar{a} \times \left(\bar{a} \times \left(\bar{a} \times \left(\bar{a} \times \bar{b}\right)\right)\right)\right) = \left(\bar{a} \times \bar{b}\right)$

Similarly if (4n + 1) $\bar{a}$'s are there then product is $(\bar{a}\times\bar{b})$.

Hence m = 1.

(Q) Clearly the value is least when P is centroid of tetrahedron.

(R) Ceva's theorem.

(S) Image point of focus through any tangent lies on directrix.

Image points of focus through two given tangents are $\left(-\frac{7}{5},\frac{4}{5}\right)$ and (–1, 0).

Hence equation is directrix is 2x + y + 2 = 0.

Semilatus rectum = $2\sqrt{3}$

19. Ans. (A)

Sol. (P)

$$\int\frac{2x^7+3x^2}{x^{10}-2x^5+1}dx=\int\frac{x^6\left(2x+\frac{3}{x^4}\right)}{x^6\left(x^4-\frac{2}{x}+\frac{1}{x^6}\right)}dx=\int\frac{2x+\frac{3}{x^4}}{\left(x^2-\frac{1}{x^3}\right)^2}dx$$

Let $x^2-\frac{1}{x^3}=t\Rightarrow\left(3x+\frac{3}{x^4}\right)dx=dt$

$$=\int\frac{dt}{t^2}=-\frac{1}{t}+c=-\frac{x^3}{x^5-1}+c$$

$$=\frac{x^3(x^5-1)}{(x^5-1)^2}+c=\frac{x^3-x^8}{(x^5-1)^2}+c$$

(Q) $\left|f\left((1+i\sqrt{3})^n\right)\right|$

$$\left|f\left(2^n\cos n\frac{\pi}{3}+i\sin\frac{n\pi}{3}\right)\right|=2^n\left|\cos\frac{n\pi}{3}\right|$$

$$\sum_{n=1}^{6}\log_2\left(2^n\left|\cos\frac{n\pi}{3}\right|\right)$$

$$=\sum_{n=1}^{6}n+\log_2\left|\cos\frac{n\pi}{3}\right|$$

$$=\frac{6}{2}(6+1)+\left[-1-1+0-1-1+0\right]$$

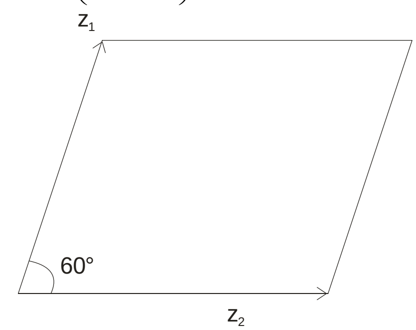
= 3 × 7 – 4

= 17

(R) $|z_1|=2,\ |z_2|=3$

$$\frac{z_1}{z_2}=\frac{2}{3}e^{i\pi/3}$$

$$\frac{z_1}{z_2}=\frac{2}{3}\left(\frac{1+i\sqrt{3}}{2}\right)$$



$$\left|\frac{z_1+z_2}{z_1-z_2}\right|=\left|\frac{\frac{2}{3}\left(\frac{1+i\sqrt{3}}{2}\right)+1}{\frac{2}{3}\left(\frac{1+i\sqrt{3}}{2}\right)-1}\right|=\left|\frac{4+i\sqrt{3}}{-2+i\sqrt{3}}\right|$$

$$=\sqrt{\frac{16+3}{4+3}}=\sqrt{\frac{19}{7}}=\frac{\sqrt{133}}{7}$$

(S) $f(z)=z^4+a_1z^3+a_2z^2+a_3z+a_4=0$

$f(ki)=k^4-a_1k^3i-a_2k^2+a_3ki+a_4=0$

$k^4-k^2a_2+a_4=0$ ...(1)

$k^3a_1-ka_3=0$ ...(2)

$\Rightarrow\quad k^2=\frac{a_3}{a_1}$ as $k\neq0$

$\therefore\quad \frac{a_3^2}{a_1^2}-\frac{a_3a_2}{a_1}+a_4=0$

$$\frac{a_3^2}{a_1^2}+a_4=\frac{a_3a_2}{a_1}\Rightarrow\frac{a_3}{a_1a_2}+\frac{a_1a_4}{a_2a_3}=1$$

20. Ans. (D)

Sol. (P) ${}^9C_4+{}^9C_4+{}^9C_4+{}^9C_3$

$={}^{10}C_5+{}^{10}C_4={}^{11}C_5={}^{11}C_6$

(Q) $={}^{10}C_5+{}^{10}C_4={}^{11}C_5={}^{11}C_6$

(R) ${}^9C_5+{}^9C_5=2.{}^9C_5$

$={}^9C_5+{}^9C_4={}^{10}C_5$

(S) ${}^{10}C_5=2.{}^9C_5$