

HINTS & SOLUTIONS

PART-I (Physics)

1. The displacement

Ans. (A)

2. Find the natural

Sol. (D)

Let mass 'm' falls down by x so spring extends by 4x ;

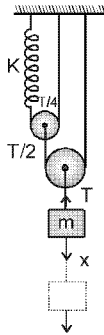
$$\therefore \frac{T}{4} = k(4x)$$

$$T = (16k) x$$

Where T is the restoring force on mass m

$$\therefore f = \frac{1}{2\pi} \sqrt{\frac{16k}{m}}$$

$$f = \frac{2}{\pi} \sqrt{\frac{k}{m}} = \frac{2}{\pi} \times \sqrt{\frac{25}{1}} = \pi \text{ Hz}$$



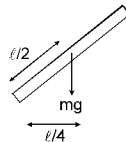
3. A body is performing

Ans. (D)

4. A uniform rod of

Sol. (D)

At the instant string is cut :



$$mg \cdot \frac{l}{4} = \frac{ml^2}{3} \cdot \alpha \Rightarrow \alpha = \frac{3g}{4l}$$

$$a_{cm} = \alpha \cdot \frac{l}{2} = \frac{3g}{8} \Rightarrow a_{horizontal}$$

$$= a_{cm} \sin \theta = \frac{3g}{8} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}g}{16} \text{ m/s}^2$$

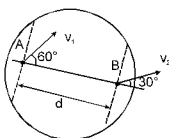
5. Two points A & B

Sol. (D)

For rigid body separation between two point remains same.

$$v_1 \cos 60^\circ = v_2 \cos 30^\circ$$

$$\frac{v_1}{2} = \frac{\sqrt{3} v_2}{2} \Rightarrow v_1 = \sqrt{3} v_2$$



$$\omega_{disc} = \left| \frac{v_1 \sin 60^\circ - v_2 \sin 30^\circ}{d} \right| = \left(\frac{\sqrt{3}v_1 - v_2}{2d} \right)$$

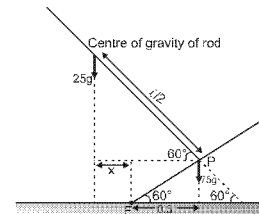
$$= \left| \frac{\sqrt{3} \times \sqrt{3}v_2 - v_2}{2d} \right| = \frac{2v_2}{2d} = \frac{v_2}{d}$$

$$\Rightarrow \omega_{disc} = \frac{v_2}{d} = \frac{v_1}{\sqrt{3}d}$$

all the options are correct

6. A 75 kg man holds

Sol. (D)



Consider (man + rod) as system,
Torque on system about F is zero

$$\tau = 75g(0.5) - 25g(x) = 0$$

$$\Rightarrow x = 1.5$$

$$\Rightarrow \frac{l}{2} \cos 60^\circ = x + 0.5 \Rightarrow \frac{l}{4} = 1.5 + 0.5$$

7. A rod of mass

Sol. (D)

$$W_F = KE \uparrow \quad \text{where } W_F = \tau \theta = (F\ell) \left(\frac{\pi}{2} \right)$$

$$(F\ell) \frac{\pi}{2} = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2$$

$$\omega = \sqrt{\frac{3F\pi}{m\ell}}$$

$$P_F = (\tau) (\omega)$$

$$P_F = (F\ell) \sqrt{\frac{3F\pi}{m\ell}}$$

$$P_F = \sqrt{\frac{3F^3\pi\ell}{m}} = \sqrt{\frac{3 \times (3)^3 \times \pi \times 1}{\pi}} = 9 \text{ watt}$$

8. Consider a uniform

Sol. (C)

T.K.E. of disc

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{mr^2}{2} \times \left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2 = 75 \text{ J}$$

Velocity of particles of upper half is more than that of lower half

hence kinetic energy of upper half will be more than $\frac{3}{8} mv^2 = 37.5 \text{ J}$

9. Select correct statement

Ans. (D)

10. A standing wave

Ans. (A)

Sol. Total mechanical energy of vibration = $\int \frac{1}{2} dm v^2$

$$dm = \mu dx, \quad \mu = 10^3 \text{ kg/m}^3 (0.04) \text{ m}^2$$

$$\mu = 40 \text{ kg/m}$$

$$v(x) = \frac{ds}{dt} = 5 \text{ mm} \sin \pi x (-\sin (200t))$$

$$\text{Total mechanical energy} = \frac{1}{2} \mu \int v^2(x) dx = 10 \text{ joules.}$$

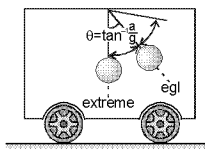
11. A simple pendulum

Sol. (BC)

bob will oscillate about equilibrium position

$$\text{with amplitude } \theta = \tan^{-1} \left(\frac{a}{g} \right) \text{ for any value of } a.$$

If $a \ll g$, motion will be SHM, and then



$$\text{time period will be } 2\pi \sqrt{\frac{\ell}{a^2 + g^2}}$$

12. Which of the

Sol. (ABCD)

Work done by kinetic friction may be positive when it acts along motion of the body.

Friction on rigid body rolling on inclined plane is along upward because tendency of slipping is downwards.

13. A plank with a

Ans. (BCD)

14. A steel wire is

Sol. (ABC)

The mechanical strain

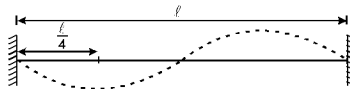
$$= \frac{\Delta \ell}{\ell} = \alpha \Delta T = 1.21 \times 10^{-5} \times 20 = 2.42 \times 10^{-5}$$

The tension in wire

$$= T = Y \frac{\Delta \ell}{\ell} A = 2 \times 10^{11} \times 2.42 \times 10^{-5} \times 10^{-6} = 48.4 \text{ N}$$

∴ speed of wave in wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{0.1}} = 22 \text{ m/s}$$



Since the wire is plucked at $\frac{\ell}{4}$ from one end

The wire shall oscillate in 1st overtone (for minimum number of loops)

$$\lambda = \ell = 1 \text{ m}$$

$$\text{Now } V = f \lambda \quad \text{or } f = \frac{V}{\lambda} = 22 \text{ Hz.}$$

15. The vibrations

Sol. (ABCD)

$$y = 4 \sin \left(\pi \frac{x}{15} \right) \cos 96 \pi t$$

$$\text{At } x = 5 \text{ cm, } y = 4 \sin \frac{\pi}{3} \cos (96 \pi t) \text{ and}$$

$$y_{\text{max}} = 2\sqrt{3} \text{ cm}$$

Positions of nodes is given by equation

$$\sin \left(\frac{\pi x}{15} \right) = 0 \Rightarrow \frac{\pi x}{15} = n\pi$$

$$\Rightarrow x = 15n$$

At $x = 7.5 \text{ cm}$ and $t = 0.25 \text{ sec.}$

$$\text{Velocity of the particle} = \frac{\partial y}{\partial t}$$

$$= -344 \pi \sin \left(\frac{\pi x}{15} \right) \sin (96 \pi t) = 0$$

16. The equation

Sol. (A)

The equation of wave moving in negative x-direction, assuming origin of position at $x = 2$ and origin of time (i.e. initial time) at $t = 1 \text{ sec.}$

$$y = 0.1 \sin (4\pi (t - 1) + 8x)$$

Shifting the origin of position to left by 2m, that is, to $x = 0$. Also shifting the origin of time backwards by 1 sec, that is to $t = 0 \text{ sec.}$

$$y = 0.1 \sin [(4\pi (t - 1) + 8(x - 2))]$$

17. The speed of

Sol. (C)

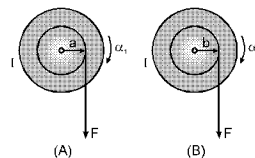
As given the particle at $x = 2$ is at mean position at $t = 1 \text{ sec.}$

∴ its velocity $v = \omega A = 4\pi \times 0.1 = 0.4 \pi \text{ m/s.}$

18. A person pulls

Sol. (9)

$$\text{For A } F \cdot a = I \alpha_1$$



$$\alpha_1 = \frac{Fa}{I}$$

$$\alpha_2 = \frac{Fb}{I}$$

$$\omega_f = \omega_0 + \alpha t$$

$$\omega_{f2} = \frac{Fbt}{I} \quad \omega_{f1} = \frac{Fat}{I}$$

$$W_1 = \Delta k = \frac{1}{2} I \omega_1^2$$

$$= \frac{1}{2} \cdot I \cdot \frac{F^2 a^2 t^2}{I^2} = \frac{F^2 a^2 t^2}{2I}$$

$$W_2 = \frac{F^2 b^2 t^2}{2I} ; \quad \frac{W_2}{W_1} = \frac{b^2}{a^2} = 9.$$

19. A rigid structure

Sol. (8)

When the structure becomes inverted, there is no decrease in the potential energy of the ring. therefore,
Decrease in PE of the rod = Gain in rotational kinetic energy of the structure

$$\Rightarrow M.g. 4R = \frac{1}{2} I_{sy} \omega^2$$

(As COM of the rod comes down by distance 4R)

$$\text{Now } I_{sy} = \frac{MR^2}{2} + \left[\frac{M \cdot 4R^2}{12} + M \cdot 4R^2 \right]$$

$$= \frac{MR^2}{2} + \frac{13MR^2}{3} = \frac{29MR^2}{6}$$

$$\omega = \sqrt{\frac{8M.gR}{\left(\frac{29}{6}\right)MR^2}} = \sqrt{\frac{48g}{29R}} = 8 \text{ rad/s}$$

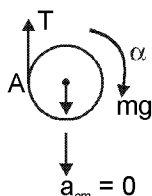
20. In the figure shown

Sol. (2)

$$a_A = a = \alpha \cdot R \quad \dots(i)$$

$$T - mg = 0 \quad \dots(ii)$$

$$T.R = \frac{mR^2}{2} \cdot \alpha \quad \dots(iii)$$



$$\therefore g = \frac{a}{2}$$

Ans. 2

21. A non-uniform

Sol. (2)

$$\mu = Kx = \frac{dM}{dx}$$

$$\int_0^M dM = \int_0^{\ell} Kx dx \text{ and } K = \frac{2M}{\ell^2}$$

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{Kx}} = \frac{dx}{dt} \int_0^{\ell} \sqrt{x} dx = \sqrt{\frac{F}{K}} \int_0^t dt$$

$$\therefore t = \sqrt{\frac{4\ell^3}{9} \cdot \frac{K}{f}} = \sqrt{\frac{4\ell^3}{9F} \cdot \frac{2m}{\ell^2}}$$

$$= \sqrt{\frac{8M\ell}{9F}} = \sqrt{\frac{8 \times 45 \times 1.5}{9 \times 15}} = 2.$$

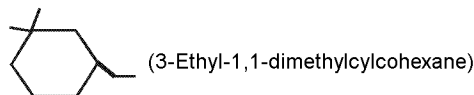
PART-II (Chemistry)

22. IUPAC name of the compound

Sol. (B)

23. Which of the following cycloalkanes

Sol. (A)



24. Which is correct about

Sol. (C)

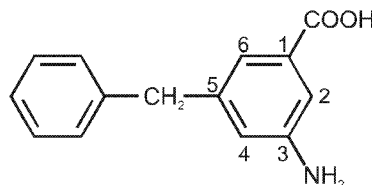
1°, 2° and 3° amines are functional isomers

25. Identify (X) and (Y) respectively

Sol. (C)

26. The IUPAC name of the

Sol. (B)



27. The correct statement

Sol. (D)

28. What is the relation between

Sol. (C)

29. What is correct about following

Sol. (D)

30. The smallest aldehyde and its

Sol. (C)

31. An optically pure compound X

Sol. (C)

$$\% \text{ optical purity of mixture} = \frac{[\alpha]_{\text{obs}}}{[\alpha]_{\text{pure}}} \times 100$$

$$= \frac{10}{20} \times 100 = 50 \%$$

so $d - \ell = 50$


and $d + \ell = 100$

$$2d = 150$$

$\therefore d = 75\%$ and $l = 25\%$

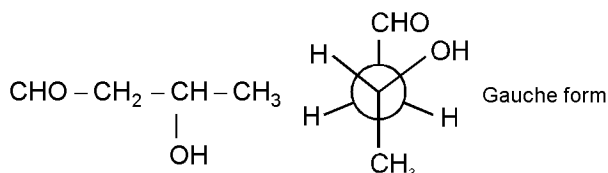
32. Which is / are the correct IUPAC

Sol. (ACB)

The correct IUPAC name of  is Cyclohexanecarbaldehyde

33. Consider the Newmann projection

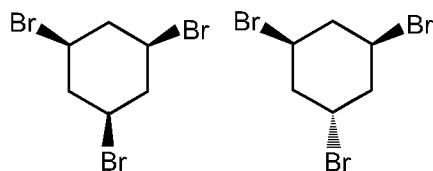
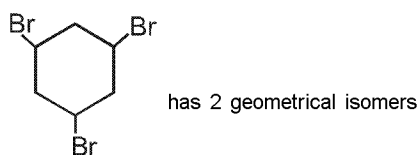
Ans. (AB)



(minimum T.S. and intramolecular H bonding).

34. Which of the following

Sol. (ACD)



35. The correct statement(s)

Sol. (AD)

36. Which of the following is/are

Sol. (ABCD)

37. The correct names for the

Sol. (B)

In naming the above compound, are follow the rules for priority under (E,Z) and (R,S) system. (Read rules in the reading material in case of doubt).

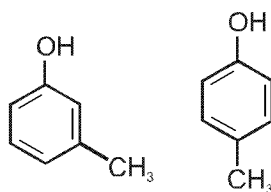
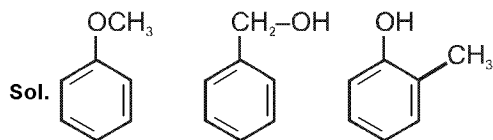
38. The above structure can have

Sol. (B)

Other stereoisomers of above (2R,3E) stereoisomer are (2R,3Z), (2S,3E), (2S,3Z) out of this (2S,3E) is enantiomers. Hence number of diastereomers = 2.

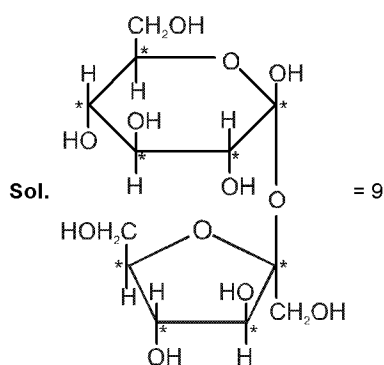
39. How many structural isomers

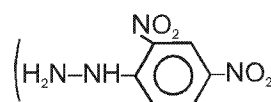
Sol. (5)



40. The number of chiral centers

Ans. 9



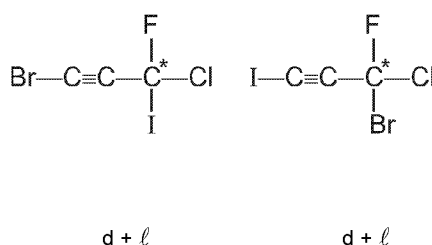
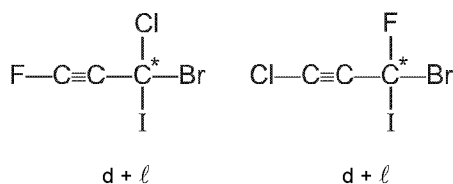
41. 2,4 DNP () reacts with

Ans. 3

Sol. Benzaldehyde is unsymmetrical and gives 2 product but formaldehyde is symmetrical and gives only one product.

42. Total number of alkynes

Sol. (8)



PART-III (Mathematics)

43. Given that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$

Sol. (C)

$$\vec{a} \times \vec{b} = \vec{c}$$

$$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times \vec{c}$$

$$(\vec{a} \cdot \vec{b}) \vec{a} - (\vec{a} \cdot \vec{a}) \vec{b} = \vec{a} \times \vec{c}$$

$$3(\hat{i} + \hat{j} + \hat{k}) - 3\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$3\vec{b} = 3(\hat{i} + \hat{j} + \hat{k}) - \{(-1-1)\hat{i} - \hat{j}(-1) + \hat{k}(1)\}$$

$$= 3\hat{i} + 3\hat{j} + 3\hat{k} + 2\hat{i} - \hat{j} - \hat{k}$$

$$= 5\hat{i} + 2\hat{j} + 2\hat{k}$$

44. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector

Sol. (B)

$$\text{Given } \vec{d} = \lambda\vec{a} + \mu\vec{b} + \nu\vec{c}$$

$$\text{Forming dot product with } \vec{b} \times \vec{c}$$

$$(\vec{b} \times \vec{c}) \cdot \vec{d} = \lambda [\vec{b} \times \vec{c} \cdot \vec{a}] + \mu [\vec{b} \times \vec{c} \cdot \vec{b}] + \nu [\vec{b} \times \vec{c} \cdot \vec{c}]$$

$$[\vec{b} \times \vec{c} \cdot \vec{d}] = \lambda [\vec{a} \cdot \vec{b} \times \vec{c}] = \lambda [\vec{b} \times \vec{c} \cdot \vec{a}]$$

$$\Rightarrow \lambda = \frac{[\vec{b} \times \vec{c} \cdot \vec{d}]}{[\vec{b} \times \vec{c} \cdot \vec{a}]}$$

45. A plane whose equation is $2x - y + 3z + 5 = 0$

Sol. (A)

$$\text{The new equation is } 2x - y + 3z + 5 + \lambda (5x - 4y - 2z + 1) = 0$$

$$\text{or } (2 + 5\lambda)x - (1 + 4\lambda)y + (3 - 2\lambda)z + 5 + \lambda = 0$$

$$\text{It is perpendicular to the plane } 2x - y + 3z + 5 = 0$$

$$\therefore 2(2 + 5\lambda) + 1 + 4\lambda + 3(3 - 2\lambda) = 0$$

$$\text{or } 7 + 4\lambda = 0$$

$$\Rightarrow \lambda = -\frac{7}{4}$$

$$\text{Substituting for } \lambda \text{ we get } 27x - 24y - 26z = 13$$

46. In a cricket championship there.....

Sol. (A)

Let n be the number of teams

$$\therefore {}^n C_2 = 36$$

$$\Rightarrow \frac{n(n-1)}{1.2} = 36$$

$$\Rightarrow n(n-1) = 72 = 9 \times 8$$

$$\Rightarrow n = 9$$

47. Three straight lines L_1, L_2, L_3

Sol. (D)

Total number of points on a three lines are $m + n + k$.

\therefore Maximum number of triangles

$$= {}^{m+n+k} C_3 - {}^m C_3 - {}^n C_3 - {}^k C_3$$

(subtract those triangles in which point on the same line)

48. If $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$ then,.....

Sol. (A)

We have

$$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{2n} x^{2n}$$

On differentiating both sides, we get

$$n(1 + x + x^2)^{n-1} (1 + 2x) = a_1 + 2a_2 x + 3a_3 x^2 + \dots + 2na_{2n} x^{2n-1}$$

On putting $x = -1$ we get

$$\Rightarrow n(1 - 1 + 1)^{n-1} (1 - 2) = a_1 - 2a_2 + 3a_3 - \dots - 2na_{2n}$$

$$\Rightarrow a_1 - 2a_2 + 3a_3 - \dots - 2na_{2n} = -n$$

49. If $|x| < 1$, then.....

$$\text{Sol. (A)} \quad (1 + x + x^2)^{-3} = \left[\frac{1}{(1 + x + x^2)} \right]^3$$

$$= \left[\frac{1-x}{1-x^3} \right]^3$$

$$= (1-x)^3 (1-x^3)^{-3}$$

$$= (1-x^3 + 3x^2 - 3x) (1 + 3x^3 + 6x^6 + \dots)$$

\therefore Coefficient of x^6 in $(1 + x + x^2)^{-3}$

$$= 6 - 3 = 3$$

50. The digit at the unit.....

Sol. (B)

$$(19)^{2005} + (11)^{2005} - (9)^{2005}$$

$$= (10 + 9)^{2005} + (10 + 1)^{2005} - (9)^{2005}$$

$$= (9^{2005} + {}^{2005} C_1 (9)^{2004} \times 10 + \dots) + ({}^{2005} C_0 + {}^{2005} C_1 10 + \dots) - (9)^{2005}$$

$$\therefore \text{Unit digit} = 1$$

51. If the sum of the coefficients

Sol. (D)

$$(a^2 - 6a + 11)^{10} = 1024$$

$$\Rightarrow (a^2 - 6a + 11)^{10} = 2^{10}$$

$$\Rightarrow a^2 - 6a + 11 = 2$$

$$\Rightarrow a^2 - 6a + 9 = 0$$

$$\Rightarrow (a - 3)^2 = 0$$

$$\Rightarrow a = 3$$

52. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors.....

Sol. (C)

Condition for coplanarity

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$$

$$\Rightarrow (2\lambda - 1)(\lambda) = 0 \Rightarrow \lambda = 0, \frac{1}{2}$$

53. Kanchan has 10 friend among.....

Sol. (B, D)

The number of ways of inviting, with the couple not included = 8C_5 .

The number of ways of inviting, with the couple included = 8C_3 .

∴ The required number of ways

$$= {}^8C_5 + {}^8C_3 = {}^8C_3 + {}^8C_3 (\because {}^8C_5 = {}^8C_3)$$

$$\text{Also } {}^{10}C_5 - 2 \times {}^8C_4 = \frac{10!}{5!5!} - 2 \times \frac{8!}{4!4!}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{120} - 2 \times \frac{8 \cdot 7 \cdot 6 \cdot 5}{24}$$

$$= 9 \times 4 \times 7 - 140 = 112 = 2 \times \frac{8!}{3!5!}$$

54. The term independent of x in the expansion

Sol. (A,C)

$$\text{Expression} = (-1)^n \cdot \frac{(1-x^2)^n}{x^n}$$

∴ the term independent of x

= the coefficient of x^n in $(-1)^n \cdot (1-x^2)^n$

= the coefficient in

$$(-1)^n \{ {}^nC_0 + {}^nC_1(-x^2) + {}^nC_2(-x^2)^2 + \dots + {}^nC_n(-x^2)^n \}$$

and the expansion contains only even powers of x.

55. Let $R = (8 + 3\sqrt{7})^{20}$

Sol. (B, C)

$R = [R] + g = (8+3\sqrt{7})^{20} = {}^{20}C_0 8^{20} + {}^{20}C_1 8^{19}(3\sqrt{7}) + \dots$, where $0 < g < 1$.

$f = (8 - 3\sqrt{7})^{20} = {}^{20}C_0 \cdot 8^{20} - {}^{20}C_1 8^{19}(3\sqrt{7}) + \dots$, where $0 < f < 1$.

∴ $[R] + g + f = 2[{}^{20}C_0 8^{20} + {}^{20}C_2 8^{18}(3\sqrt{7})^2 + \dots]$ = an even integer

∴ $g + f = \text{an integer} = 1$ as $0 < g < 1, 0 < f < 1$.

∴ $[R] = \text{an even integer} - 1 = \text{an odd integer}$

$$\text{Also, } R - [R] = g = 1 - f = 1 - (8 - 3\sqrt{7})^{20} = 1 - \frac{1}{(8 + 3\sqrt{7})^{20}}$$

56. If $\vec{a}, \vec{b}, \vec{c}$ are noncoplanar

Sol. (A, B, C)

$$\vec{b} \times \vec{c} = \vec{a} \Rightarrow [\vec{a} \vec{b} \vec{c}] = (\vec{a} \cdot \vec{a}) = |\vec{a}|^2$$

$$\text{Similarly } [\vec{a} \vec{b} \vec{c}] = |\vec{b}|^2 = |\vec{c}|^2$$

$$\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{[\vec{a} \vec{b} \vec{c}]}$$

$$\text{Now, } [\vec{b} \times \vec{c} \vec{c} \times \vec{a} \vec{a} \times \vec{b}] = [\vec{a} \vec{b} \vec{c}]^2$$

$$\therefore [\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{b} \vec{c}]^2 \text{ (from the question)}$$

$$\text{But } [\vec{a} \vec{b} \vec{c}] \neq 0. \text{ So, } [\vec{a} \vec{b} \vec{c}] = 1.$$

$$\text{Hence, } |\vec{a}| = |\vec{b}| = |\vec{c}| = 1.$$

57. The equation of a

Sol. (BC) For A(1, 1, 1), $2x - y - 3z - 5 = 2 - 1 - 3 - 5 < 0$

For B(2, 1, -3), $2x - y - 3z - 5 = 4 - 1 + 9 - 5 > 0$

For C(1, -2, -2), $2x - y - 3z - 5 = 2 + 2 + 6 - 5 > 0$.

For D(-3, 1, 2), $2x - y - 3z - 5 = -6 - 1 - 6 - 5 < 0$

∴ A, D are on one side of the plane and B, C are on the other side.

∴ the line segments AB, AC, BD, CD intersect the plane.

58. If p is a prime.....

Sol. (B) $(p-1)\sigma(p^n)$

$$= (p-1)(1 + p + p^2 + \dots + p^n)$$

$$= p^{n+1} - 1$$

59. If p, q are two

Sol. (B) $\sigma(pq) = 1 + p + q + pq = (1+p)(1+q)$

60. In a certain test there are.....

Ans. 6

Sol. Since the number of students giving wrong answers to at least i question ($i = 1, 2, \dots, n$) = 2^{n-i}

The number of student answering exactly i ($1 \leq i \leq n-1$) questions wrongly = {the number of students answering at least i questions wrong, $i = 1, 2, \dots, n$ } - {the number of students answering at least (i+1) question wrong ($2 \leq i+1 \leq n$)} = $2^{n-i} - 2^{n-(i+1)}$ ($1 \leq i \leq n-1$).

Now, the number of students answering all the question wrongly = $2^{n-n} = 2^0$.

Thus the total number of wrong answers

$$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4}) + \dots + (n-1)(2^1 - 2^0) + n(2^0)$$

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \dots + 2^0 = 2^n - 1 \text{ [}\because \text{ its a G.P.]}$$

$$\therefore \text{ As given } 2^n - 1 = 2047 \Rightarrow 2^n = 2048 = 2^{11}$$

$$\Rightarrow n = 11 \quad \Rightarrow n - 5 = 6$$

61. If the sum of the

Ans. 1

Sol. Accordingly, $(\alpha - 2 + 1)^{35} = (1 - \alpha)^{35}$

$$\Rightarrow (\alpha - 1)^{35} = (1 - \alpha)^{35} \Rightarrow \alpha = 1.$$

62. If the planes $x = cy + bz, \dots$

Ans. 1

Sol. Let l, m, n direction ratios of the line lying in the three planes, then the line is perpendicular to the normals to these planes, so that we have

$$l - cm - bn = 0$$

$$-cl + m - an = 0$$

$$-bl - am + n = 0$$

Elimination l, m, n from these questions we get

$$\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1 - a^2 + c(-c - ab) - b(ac + b) = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

63. If $[\bar{b} \bar{c} \bar{d}] = 4$ and

Ans. 8

Sol. We know that $(a \times b) \times c = (a \cdot c)b - (b \cdot c)a$. Putting

$$c \times d = e,$$

we get

$$(a \times b) \times (c \times d) = (a \times b) \times e = (a \cdot e)b - (b \cdot e)a$$

$$= \{a \cdot (c \times d)\}b - \{b \cdot ((c \times d))\}a$$

$$= [a \cdot c]b - [b \cdot c]a \quad \dots(1)$$

Similarly,

$$(a \times c) \times (d \times b) = [a \cdot d]b - [c \cdot d]a$$

$$= [a \cdot d]b - [b \cdot c]a \quad \dots(2)$$

Also

$$(a \times d) \times (b \times c) = - (b \times c) \times (a \times d)$$

$$= (b \times c) \times (d \times a)$$

$$= [b \cdot d]a - [c \cdot d]b$$

$$= - [a \cdot d]b - [a \cdot c]d \quad \dots(3)$$

from (1), (2) and (3), we get -

$$a = (a \times b) \times (c \times d) + (a \times c) \times (d \times b) + (a \times d) \times (b \times c)$$

$$= [a \cdot c]b - [b \cdot c]a + [a \cdot d]b - [b \cdot c]a - [a \cdot d]b - [a \cdot c]d$$

$$= -2[b \cdot c]a = -8a.$$