RITS-30
JEE MAINS-2019
ANSWER KEY
Code: 119619

PHYSICS		CHEMISTRY		MATHEMATICS	
1	2	1	2	1	2
2	3	2	2	2	3
3	4	3	2	3	1
4	4	4	1	4	2
5	2	5	2	5	2
6	1	6	4	6	4
7	2	7	4	7	3
8	2	8	4	8	2
9	1	9	1	9	2
10	3	10	2	10	2
11	2	11	2	11	3
12	2	12	3	12	4
13	3	13	2	13	2
14	2	14	3	14	1
15	1	15	3	15	3
16	2	16	2	16	4
17	2	17	4	17	3
18	3	18	4	18	1
19	2	19	2	19	4
20	1	20	2	20	2
21	3	21	1	21	4
22	3	22	4	22	2
23	3	23	2	23	3
24	2	24	4	24	3
25	2	25	3	25	4
26	3	26	2	26	1
27	1	27	1	27	2
28	4	28	3	28	2
29	4	29	3	29	3
30	1	30	2	30	4

SOLUTION

1. Ans. (2)

Sol. Optical source frequency

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{1.3\times10^{-6}} = 2.3\times10^{14}\,Hz$$

$\therefore$ Number of channels or subscribers $= \dfrac{2.3\times10^{14}}{20\times10^{3}}$

$= 1.15 \times 10^{10}$

2. Ans. (3)

Sol. $V = -mS + V_0$

$$\left(\frac{dV}{dS}\right) = -m$$

$$a = V\left(\frac{dV}{dS}\right) = [-mS + V_0][-m]$$

$a = m^2S - mV_0$

This is eqn. of straight line.

3. Ans. (4)

Sol. $\vec{V} = 2t\,\hat{i} + t^2\hat{j}$

$\vec{a} = 2\hat{i} + 2t\,\hat{j}$

$|\vec{V}| = V = \sqrt{4t^2 + t^4}$

$$a_t = \left(\frac{dV}{dt}\right) = \frac{1}{2\sqrt{4t^2+t^4}} \times (8t + 4t^3)$$

at t = 1 $\quad a_t = \left(\dfrac{6}{\sqrt{5}}\right) m/s^2$

$a = \sqrt{2^2 + (2t)^2} = \sqrt{8}\, m/s^2$

$r = \sqrt{5}$

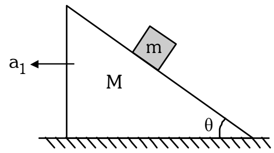
$$a_c = \sqrt{a^2 - a_t^2} = \sqrt{8 - \frac{36}{5}} = \left(\frac{2}{\sqrt{5}}\right)$$

$$R = \left(\frac{V^2}{a_c}\right) = \frac{5}{\left(\frac{2}{\sqrt{5}}\right)} = \frac{5\sqrt{5}}{2}$$

$$R = \left(\frac{5\sqrt{5}}{2}\right) m$$

4. Ans. (4)

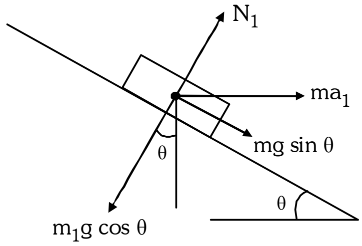
Sol.



For entire system, $\vec{F}_y = (M+m)a_{c\,my}$

$(M + m)g - N = (M + m)\, a_{c\,my}$

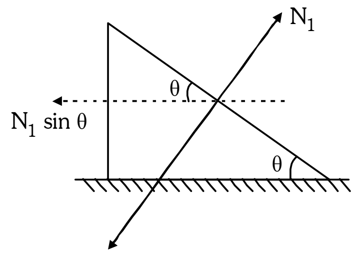
$N = (m + M)\,(g - a_{c\,my})$ ....(1)



$N_1 + ma_1 \sin\theta = mg\cos\theta$

$N_1 = m[g\cos\theta - a_1\sin\theta]$ ....(2)

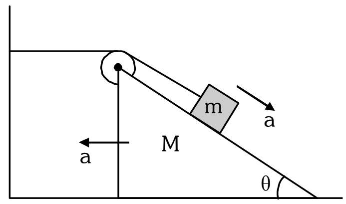
For M :



$N_1 \sin\theta = Ma_1$

5. Ans. (2)

Sol.



$$a_0 = \sqrt{a^2 + a^2 + 2a^2\cos(\pi - \alpha)}$$

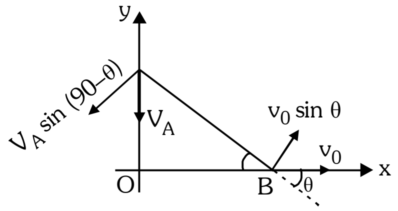
$$a_0 = 2a\sin\left(\frac{\alpha}{2}\right)$$

6. Ans. (1)

Sol. The acceleration of point of contact is $\left(\frac{V^2}{R}\right)$ towards centre. The a_{cm} and friction may or may not be zero.

7. Ans. (2)

Sol.



$V_A \cos(90-\theta) = V_0 \cos\theta$

$V_A \sin\theta = V_0 \cos\theta$

$$V_A = \frac{V_0}{\tan\theta}$$

$$V_A = \frac{4}{3}V_0$$

$$\omega = \frac{[V_A \sin(90-\theta) + V_0 \sin\theta]}{\ell}$$

8. Ans. (2)

Sol. $F(2S) = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}w^2$

9. Ans. (1)

Sol. $\vec{L} = \vec{r} \times \vec{P}_{cm} + I_{cm}\vec{\omega}$

10. Ans. (3)

Sol. $VT = C_0$

$PV^2 = C_0, \quad x = 2$

$$C = C_v + \frac{R}{1-r} = \left(\frac{3R}{2}\right)$$

$$W = \left[\frac{nR\Delta T}{(-x)}\right] = \frac{2R \times 200}{(1-2)} = -400R$$

11. Ans. (2)

Sol.

	Q	ΔU	W
AB	–70	0	–70
BC	150	150	0
CA	60	-150	210

$$\eta = \frac{140}{210} \times 100\% = 60\%$$

12. Ans. (2)

Sol. $x = \frac{y_1}{2} + 2y_2 + 2y_3$ ....(1)

$\Delta T = ma$ ....(2)

$2\Delta T = Ky_2$ ....(3)

$2\Delta T = Ky_3$ ....(4)

$\frac{\Delta T}{2} = Ky_1$ ....(5)

Solving eqn.

$$x = \Delta T\left(\frac{33}{4K}\right)$$

$$w^2 = \left(\frac{4K}{33}\right)$$

13. Ans. (3)

Sol. Fg = 5N, Fe = 11N

The minimum possible velocity at lowest point can be $\sqrt{7.2}$ m/s. The corresponding velocity at the highest point will also be minimum

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mg(2r) + qE_0(2r)$$

$v_s = 6$ m/s

14. Ans. (2)

Sol. $\frac{1}{2}mV_1^2 - \frac{KQq}{r_1} = \frac{1}{2}mV_2^2 - \frac{KQq}{r_2}$

$mV_1r_1 = L_0$

$mV_2r_2 = L_0$

Solving eqns :

$$L = \sqrt{\frac{mr_1r_2Qq}{2\pi\varepsilon_0(r_1+r_2)}}$$

15. Ans. (1)

Sol. $i = \frac{E}{(R_1+R_2)} \cdot e^{\frac{-t}{(R_1+R_2)C}}$

$$P_1 = i^2R_1 = \frac{E^2R_1}{(R_1+R_2)^2} \cdot e^{\frac{-2t}{(R_1+R_2)C}}$$

16. Ans. (2)

Sol. $i_0 = \frac{10V}{10\Omega} = 1A$

$$i_0 = \frac{9}{100} \times \ell_1 = 5V$$

$$\ell_1 = \frac{500}{9} = 55.55$$

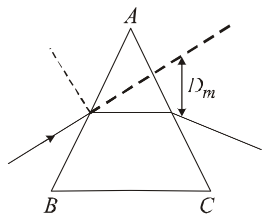
17. Ans. (2)

Sol. $v_0 \cos 45 = 20\sqrt{2}$

$v_0 = 40$ cm/s

18. Ans. (3)

Sol . $\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$



$$\sqrt{2}\sin\left(\frac{A}{2}\right) = \sin\left(\frac{A+D_m}{2}\right)$$

Only for $A = 60°$ & $D_m = 30°$ satisfy

19. Ans. (2)

Sol. $f = \frac{2\pi}{\lambda}\Delta x$,

$I_{res} = 2I(1 + \cos\theta)$

$\Delta x = \frac{\lambda}{4} = \frac{K}{2}(1 + \cos\theta)$

$\theta = \frac{\pi}{2}$, So $I_{res} = \left(\frac{K}{2}\right)$

20. Ans. (1)

Sol. $(\mu - 1)t - \frac{dy}{D} = n\lambda$

$t = \frac{dy}{D(\mu-1)} + \left(\frac{n\lambda}{\mu-1}\right)$

$t_{min.} = \frac{\lambda}{\mu-1} = 2\lambda$

21. Ans. (3)

Sol. $\frac{I_{max}}{2I} = \frac{4I}{2I} = \frac{2}{1}$

22. Ans. (3)

Sol. $\frac{dN}{dt} = -\lambda_1 N - \lambda_2 N - \lambda_3 N = -(\lambda_1 + \lambda_2 + \lambda_3)N$

$\lambda_{eff.} = \lambda_1 + \lambda_2 + \lambda_3$

23. Ans. (3)

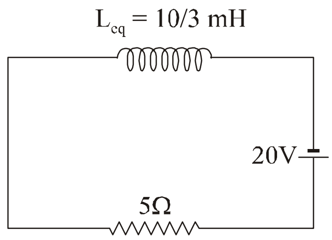
Sol. $\frac{1}{\lambda} \propto (Z - b)2$

$\frac{\lambda_2}{\lambda_1} = \frac{(Z_1-b)^2}{(Z_2-b)^2} = \left(\frac{28}{14}\right)^2 = 4$

$\boxed{\lambda_2 = 4\lambda_1}$

24. Ans. (2)

Sol.



Leq. $= \frac{5\times10}{5\times10} = \frac{10}{3}$ mH

Current in steady start,

$I = \frac{20}{5} = 4A$

As L1& L2 are in parallel

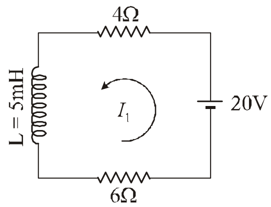
$I_1 = I\left(\frac{I_2}{L_1 + L_2}\right) = 4\times\frac{10}{10+5}$

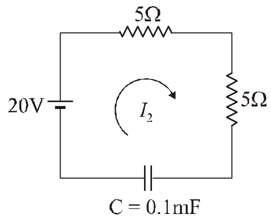
$I_1 = \frac{4\times10}{15} = \frac{8}{3}A$

25. Ans. (2)

Sol. $I_1 = \frac{20}{10}\left(1 - e^{\frac{-t}{5\times10^{-4}}}\right) = 2\left(1-\frac{1}{4}\right) = \frac{3}{2} = 1.54$

$I_2 = \frac{20}{10}e^{\frac{-t}{10^{-3}}} = 1A$





From superposition

$I = I_1 + I_2 = 2.5$ A

26. Ans. (3)

Sol. $F - i\ell B = ma$ ....(1)

$B\ell v = i(R + 2\lambda x)$ ....(2)

$\left(\frac{dV}{dx}\right) = \left(\frac{2\lambda i}{B\ell}\right)$ ....(3)

$a = V\left(\frac{dv}{dx}\right) = \left(\frac{2\lambda i}{B\ell}\right)\left(\frac{i(R+2\lambda x)}{B\ell}\right)$

$a = \frac{2\lambda i^2}{B^2\ell^2}(R + 2\lambda x)$ ....(4)

27. Ans. (1)

Sol. $B = \frac{\mu_0 i}{2x}$

$\phi = \frac{\mu_0 i}{2x}(\pi a^2)$ ....(1)

$\varepsilon = \frac{d\phi}{dt} = \frac{\mu_0 i(\pi a^2)}{2}\left(\frac{-1}{x^2}\right)(v)$ ....(2)

$i = \frac{\mu_0 i_0 \pi a^2 v}{rx^2}$ ....(3)

$P = i^2 r$ ....(4)

28. Ans. (4)

Sol. The mutual inductance $M = \frac{\mu_0 N_1 \times N_2 \times A}{\ell}$

where, N_1 = 300 turns, N_2 = 400 turns,

A = 10 cm^2 and ℓ = 20 cm

Substituting the values in the given formula, we get,

$M = 2.4\,\pi \times 10^{-4}$ H

29. Ans. (4)

Sol. If we give the following inputs to A and B, then corresponding output is shown in table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

The above table is similar to OR gate.

30. Ans. (1)

Sol. The reverse biasing results in very high resistance across the p-n junction.
If the value of the resistance R is increased, the current in the forward biased input circuit decreases. The emitter current I_E decreases and hence the collector current ($I_C = I_E - I_B$) also decreases. The glowness of the bulb decreases. Due to decrease in I_C, the potential drop across the bulb B decreases and hence the voltmeter shows a lower voltage.

31. Ans. (2)

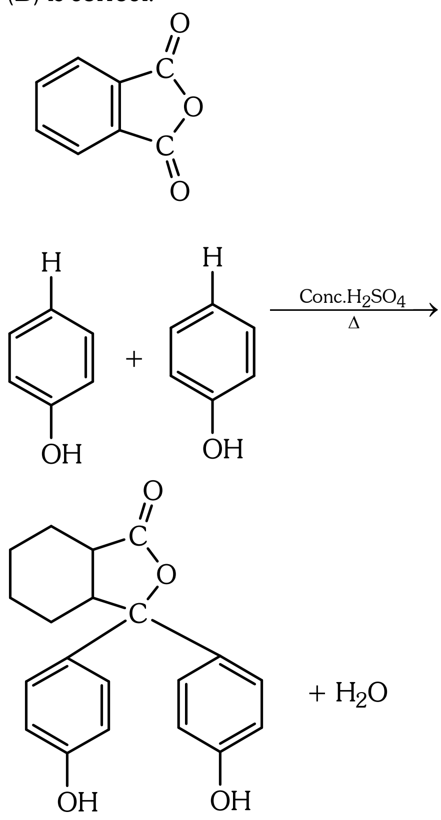
Sol. $1 \text{ kJ mol}^{-1} = 83.7 \text{ cm}^{-1}$

$$\Delta_0 = \frac{20300}{83.7} = 243 \text{ kJ mol}^{-1}$$

∴ (B) is correct answer.

32. Ans. (2)

Sol. (B) is correct.



Phenolphthalein

(A), (C) and (D) are not possible.

33. Ans. (2)

Sol. $$h = \frac{\lambda_c - \lambda_{c'}}{\lambda_a - \lambda_c} = \frac{122.5 - 106.5}{426 - 106.5}$$

$$h = \frac{16}{319.5} = 0.05$$

$$K_h = \frac{ch^2}{(1-h)} = \frac{0.01 \times 0.05 \times 0.05}{(1 - 0.05)}$$

$$K_h = \frac{25 \times 10^{-6}}{0.95} = 2.63 \times 10^{-5}$$

34. Ans. (1)

Sol. (A) is correct answer, it is non–magnetic ore.
(B), (C) and (D) are magnetic ores, therefore, cocentrated by electromagnetic separation.

35 Ans. (2)

Sol. $2A + B \rightleftharpoons 3C + 2D$

4P	P	0	0	Initially
2P	0.5 P	1.5 P	2P	At equilibrium

$$K_p = \frac{(1.5P)^3(2P)^2}{(2P)^2(0.5P)} = 6.75P^2$$

$$P = \sqrt{\frac{K_P}{6.75}} = 0.4\sqrt{K_P}$$

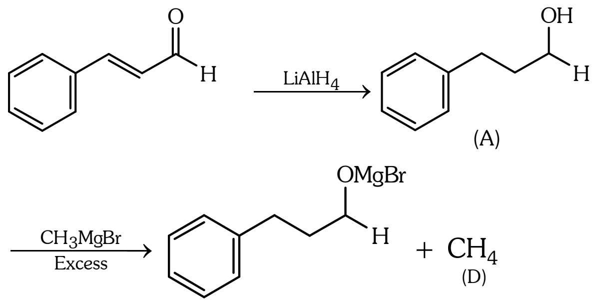
∴ (B) is correct answer.

36. Ans. (4)

Sol. Acidic nature of C, Si, Ge and Pb decreases down the group hence reactivity decreases.
Greater the number of ionsa, greater will be the electrical conductivity. III gives 4 ions, II gives 3 ions, IV gives 2 ions; I does not give ions.

37. Ans. (4)

Sol.



38. Ans. (4)

Sol. Energy of photon = 10 eV
Work function = 4e V
$K.E._{max} = 6$ eV

$$\lambda = \left(\frac{150}{V}\right)^{1/2} = \left(\frac{150}{6}\right)^{1/2} \text{Å}$$

$= (25)^{1/2}$ Å = 5 Å = 0.5 nm.

39. Ans. (1)

Sol. In $O_2[AsF_6]$, O_2^+ ion have one unpaired electron.
Al_2O_3 is oxide in which Al^{3+} and O^{2-} ion do not have any unpaired electrons.

$$C_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2, \pi 2p_y^2$$

Number unpaired electrons.

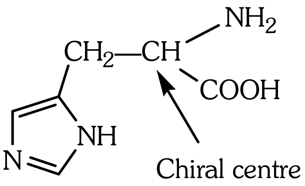
$$Be_2 = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$$

No unpaired electrons.

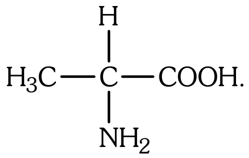
40. Ans. (2)

Sol. (B) is correct. Glycine is $H_2N-\underset{H}{\overset{H}{\underset{|}{\overset{|}{C}}}}-COOH$

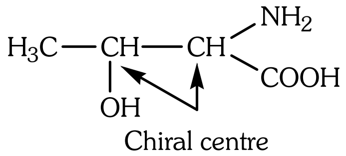
(No chiral centre)

Histidine is 

Hence (A) is wrong.

(C) α–Alanine is 

Hence (C) is wrong.

(D) Threonine is 

41. Ans. (2)

Sol. AgCl is not soluble in H_2O because lattice energy is more than hydration energy.

$$AgCl + 2KCN \longrightarrow \underset{\text{soluble}}{K[Ag(CN)_2]} + KCl$$

$$Zn + 10HNO_3(\text{v.v dil}) \longrightarrow 4Zn(NO_3)_2 + NH_4NO_3 + 3H_2O$$

On heating,

$$4Zn + 10HNO_3(\text{v.v.dil}) \xrightarrow{\text{heat}} 4Zn(NO_3)_2 + N_2O + 5H_2O$$

$$2CuSO_4 + 4KI \longrightarrow Cu_2I_2 + I_2 + 2K_2SO_4$$

$$\underset{\text{blue}}{CuSO_4.5H_2O} \xrightleftharpoons{\text{exposes}} \underset{\text{pale blue}}{CuSO_4.3H_2O} \xrightleftharpoons{100^\circ C} \underset{\text{bluish white}}{CuSO_4.H_2O}$$

$$\xrightleftharpoons{230^\circ C} \underset{\text{white}}{CuSO_4} \xrightarrow[\Delta]{720^\circ C} CuO + SO_2 + \frac{1}{2}O_2$$

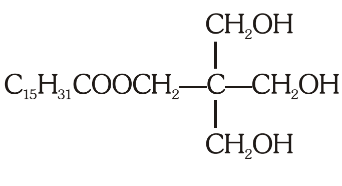
42. Ans. (3)

Sol. $As_2O_3 + 3H_2S \longrightarrow As_2S_3 + 3H_2O$

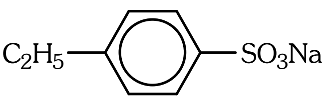
It is double decomposition reaction.

Sulphur sol is likely to be negatively charged due to adsorption of S^{2-} ion.

$\therefore$ Al^{3+} will be most effective.



can not form ion.



$\longrightarrow$ Shows miccelle formation on higher concentration.

43. Ans. (2)

Sol. In benzene acetic acid undergoes dimerization while in water, acetic acid remain in monomeric state so

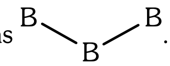
$K_{Water} < K_{Benzene}$

$\therefore$ (B) is correct answer.

44. Ans. (3)

Sol. (A) $2Hg + O_3 \longrightarrow Hg_2O + O_2$

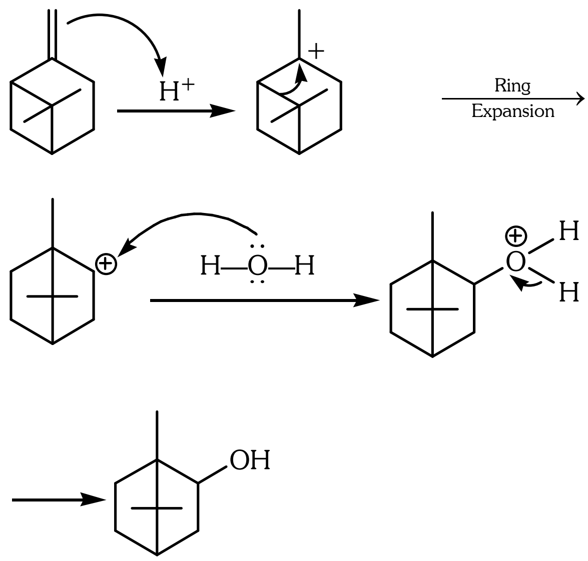
(B) C_3O_2 (carbon suboxide) is amphoteric.

(C) Ni_3B having isolated boron atoms and FeB having zig zag chains of boron atoms .

(D) V_3B_2 having pairs of boron elements e.g. B–B and NaB_{15} having icosahedron arrangement of boron atoms.

45. Ans. (3)

Sol.



46. Ans. (2)

Sol. (B) is incorrect statement because heat capacity of liquid is more than gaseous state.

(A), (C) and (D) are correct statements.

47. Ans. (4)

Sol. Only Au is present in anode mud as it is less reactive than silver.

$\therefore$ (D) is correct.

48. Ans. (4)

Sol. In Mn, there is broken crystal lattice due to which its melting point is low. In vanadium, there are maximum number of unpaired electrons.

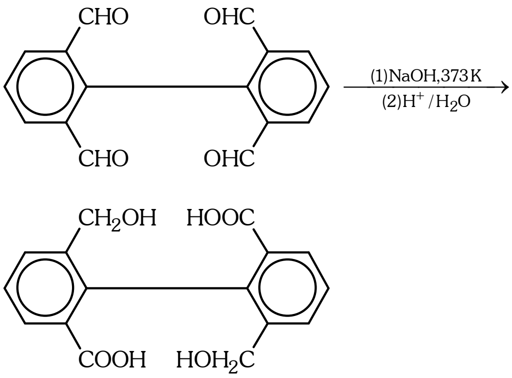
Co and Ni have lesser radii than Cu and Zn. Also in Co the number of unpaired electrons are more.

Na_2S is ionic, soluble in water, has highest solubility.

K_{sp} of ZnS is higher than CuS, therefore, solubility is higher.

49. Ans. (2)

Sol.



50. Ans. (2)

Sol. $[A]_t = [A_0] - Kt$

$= 5 \times 10^{-3} - 20 \times 10^{-x}$

Now $\Delta T_f = K_f m$

$\Rightarrow 0.186 = 1.86m \Rightarrow m = 0.1$

$\therefore 0.1 = \dfrac{5\times10^{-3} - 20\times10^{-x} + 6\times10^{-3}}{90\times10^{-3}}$

$\Rightarrow 0.1 = \dfrac{11\times10^{-3} - 20\times10^{-x}}{90\times10^{-3}}$

$\Rightarrow 0.1 = \dfrac{11 - 20\times10^{-x+3}}{90}$

$\Rightarrow 9 = 11 - 20 \times 10^{-x+3}$

$\Rightarrow 2 = 20 \times 10^{-x+3} \Rightarrow 10^{-x+4} = 1 \Rightarrow x = 4$

51. Ans. (1)

Sol. Packing fracton $= \dfrac{\text{density of unit cell}}{\text{density of atom}}$

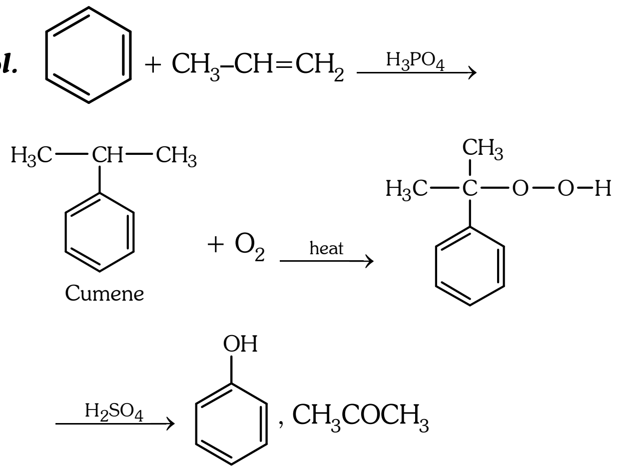
$\Rightarrow \dfrac{10.6}{14.32} = 0.74$

52. Ans. (4)

Sol. Because it has maximum branching $\therefore$ it is not attacked by micro–organisms easily hence least biodegradable. Bacteria cannot degrade benzene ring easily.

53. Ans. (2)

Sol.



(A) is not possible because $CH_3 - CH_2 - \overset{\oplus}{C}H_2$ is less stable than $CH_3 - \overset{\oplus}{C}H - CH_3$

(C) is not possible because acetophenone and CH_3OH cannot be formed.

(D) is not possible because $CH_3—\underset{}{\overset{OH}{CH}}—CH_3$ cannot be formed.

54. Ans. (4)

Sol. $KMnO_4 + 8HCl \rightarrow MnCl_2 + \frac{5}{2}Cl_2 + KCl + 4H_2O$

nf = 5

	Cl_2	H_2O
moles	5/2	4
	1	$\frac{4}{5}\times 2 = \frac{8}{5}$

55. Ans. (3)

Sol. By Charle's law

$\dfrac{V_1}{V_2} = \dfrac{T_1}{T_2}$ (T_1 and T_2 are in Kelvin)

$\dfrac{d_2}{d_1} = \dfrac{T_1}{T_2}$

56. Ans. (2)

Sol. The negative value of ΔH_f of fluoride is higher due to small size.
The enthalpy of formation of chloride becomes more electronegative than on descending the group.
For fluorides enthalpy of formation decreases down the group.

57. Ans. (1)

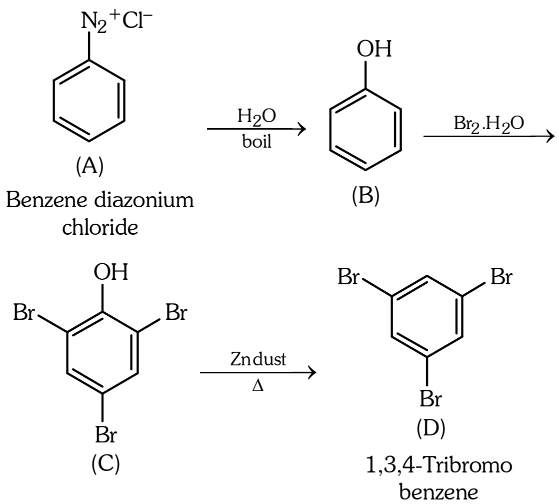
Sol. Moles of $N_2 = \dfrac{168}{28} = 6$ Moles of $H_2 = \dfrac{30}{2} = 15$

$\Delta U = \Delta H - \Delta n_g RT$ Limiting reagent is H_2

$= (-24) \times 5 - (2) \times \dfrac{2}{1000} \times 700 \times 5 = -106\,\text{Kcal}$

58. Ans. (3)

Sol. (C) is correct.



59. Ans. (3)

Sol. $\frac{P^0 - P_s}{P^0} = i \times X_{solute}$

$\frac{760-750}{760} = i \times X_{solute}$

$\frac{10}{760} = 4 \times X_{solute}$

$X_{solute} = \frac{1}{304}$

$X_{solute} = \frac{n_{solute}}{n_{solute} + n_{sovent}} \Rightarrow \frac{1}{304} = \frac{\frac{1}{300}}{\frac{1}{304} + \left(1 - \frac{1}{304}\right)}$

$n_{sovent} = 1 - \frac{1}{304}$

$n_{solvent} H_2O = \frac{303}{304}$

$\simeq 1$

Mass of H_2O = 18 gm

Volume = 18 ml

= 0.018 L

$\text{Molarity} = \frac{1}{304 \times 0.018}$ *= 0.1827* $= \frac{10}{54}$

60. Ans. (2)

Sol. For every 10° rise in temperature, the rate of reaction doubles.

$u \propto \sqrt{T}$,

∴ will not be doubled for every 10° rise in temperature. Hence, choice (A) is incorrect.

Also on increasing temperature, number of molecules forming activated complex doubles.

Hence (B) is correct.

On increasing temperature, the concentration does not double and the collision frequency increases but not doubles.

61. Ans. (2)

Sol. For concurrent lines, $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

Applying $c_2 - c_1$ and $c_3 - c_1$, we get

$\Delta = \begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$

$a(b-1)(c-1) - (1-a)(c-1) - (1-a)(b-1) = 0$

Dividing $(1-a)(1-b)(1-c)$, we get

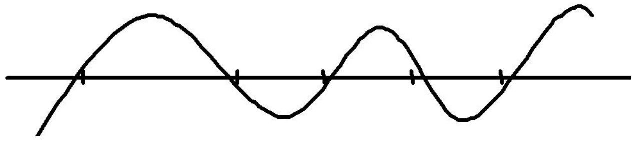
$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \frac{1-a}{1-a} = 1$

62. Ans. (3)

Sol. $(x+1)(e^x - 1)(x-4)(x+5)(x-6) > 0$

CP's are {–1, 0, 4, –5, 6}



$x \in (-5, -1) \cup (0, 4) \cup (6, \infty)$

63. Ans. (1)

Sol. $\int_0^1 (1 + |\sin x|)(ax^2 + bx + c)dx$

$= \int_0^1 (1 + |\sin x|)(ax^2 + bx + c)dx + \int_1^2 (1 + |\sin x|)(ax^2 + bx + c)dx$

$\int_1^2 (1 + |\sin x|)(ax^2 + bx + c)dx = 0$

$\because 1 + |\sin x| > 0$

So, quadratic equation $ax^2 + bx + c$ must have at least one real root between (1, 2).

64. Ans. (2)

Sol. $S_n = cn(n-1)$

$S_{n-1} = c(n-1)(n-2)$

$t_n = S_n - S_n = c(n-1)(n-(n-2)) = 2c(n-1)$

So, for the new series

$T_n = 4c^2(n-1)^2 = 4c^2\left[n^2 - 2n + 1\right]$

So, S_n for the new series will give

$S_n = 4c^2\left[\sum n^2 - 2\sum n + n\right] = \frac{2}{3}c^2 n(n-1)(2n-1)$

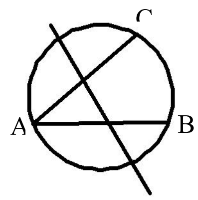
65. Ans. (2)

Sol. Cos (tan x) varies from [–1, 1]

So, sine function is increasing in between [–1, 1], so maximum value of $\sin(\cos(\tan x))$ is sin1.

66. Ans. (4)

Sol.



$\because$ $A\left[\left(\frac{1+\sqrt{2}a}{2}\right), \left(\frac{1-\sqrt{2}a}{2}\right)\right]$ lies on the circle and

$4 + x = 0$ line bisects the chords AB and AC so that M (h, –h)

So, coordinates of B is $x = 2h - \frac{1+\sqrt{2}a}{2}$,

$y = -2h - \frac{1-\sqrt{2}a}{2}$

$\because$ this point lies on circle, so by satisfying value of (x, y) on the circle, we will get equation

$8h^2 - 6\sqrt{2}ah + 1 + 2a^2 = 0$

So, for $\Delta > 0$

$a^2 - 4 > 0$

$a \in (-\infty, -2) \cup (2, \infty)$

67. Ans. (3)

Sol. n = normal to the plane of given vectors is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 5(\hat{i} - \hat{j} - \hat{k})$$

If θ be the angle between vector, x, y, z and plane, then $(90^\circ - \theta)$ is the angle between x, y, z and normal to the plane.

$$\therefore \cos(90^\circ - \theta) = \frac{5(1, -1, -1).(x, y, z)}{5\sqrt{3}.\sqrt{x^2 + y^2 + z^2}}$$

$\sin^2\theta \times 3(x^2 + y^2 + z^2) = (x - y - z)^2$

$\left(\because \cot\theta = \sqrt{2},\ \sin\theta = \frac{1}{\sqrt{3}}\right)$

So, $\frac{1}{3}.3.(x^2 + y^2 + z^2) = (x - y - z)^2$

$-2xy + 2yz - 2zx = 0$

So, $yz = x(y + z)$

68. Ans. (2)

Sol. Let A (o, b, c) in yz plane & B (a, o, c) in zx plane through O is px + qy + rz = 0. It passes through A & B so Op + qb + rc = 0 & pa + ob + rc = 0

$$\therefore \frac{p}{bc} = \frac{q}{ca} = \frac{r}{-ab}$$

So equation of plane is

bcx + cay – abz = 0

69. Ans. (2)

Sol. Let $a = \cos\theta + i\sin\theta, b = \cos\phi + i\sin\phi, c = \cos\psi + i\sin\psi$

$a + 2b + 3c = 0 + i.0$

so,

$a + 2b + 3c = 0$

$a^3 + 8b^3 + 27c^3 = 3.(a.2b.3c)$

$= 18abc$

so,

$\cos 3\theta + 8.\cos 3\phi + 27\cos\psi = 18\cos(\theta + \phi + \psi)$

70. Ans. (2)

Sol. $\alpha + \beta = 2$

$\alpha.\beta = 4$

by solving,

$\alpha = 2\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$\beta = 2\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]$

so,

$$2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right] + 2^n\left[\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right]$$

$$= 2^n\left[\cos\frac{n\pi}{3}\right]$$

$$= 2^{n+1}.\cos\frac{n\pi}{3}$$

71. Ans. (3)

Sol. $S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \dots\dots\dots\dots + \frac{1}{2^{n-1}}$

$$S_n = \frac{1\left(1 - \left(\frac{1}{2}\right)^n\right)}{\left(1 - \frac{1}{2}\right)} = 2\left[1 - \frac{1}{2^n}\right]$$

$2 - Sn < \frac{1}{100}$

$\frac{2}{2^n} < \frac{1}{100}$

$n \geq 8$

72. Ans. (4)

Sol. $f(x) = \frac{1}{\sqrt{1 - x^2}} + \left[\frac{x^2 + x + 1}{4}\right]$

domain of $f(x)$ is $x \in (-1, 1)$

within that domain, $\left[\frac{x^2 + x + 1}{4}\right] = 0$

so,

$f(x) = \frac{1}{\sqrt{1 - x^2}}$ i.e, an even function.

73. Ans. (2)

Sol. $y = mx \pm \sqrt{a^2m^2 + b^2}$

passing through a point (h, k)

$k = mh \pm \sqrt{a^2m^2 + b^2}$

$m^2(a^2 - h^2) + 2mhk - (k^2 - b^2) = 0$

$\because$ tangents are $\perp$

so, $m_1.m_2 = -1$

$m_1.m_2 = \frac{c}{a} = \frac{-k^2 - b^2}{a^2 - h^2} = -1$

so, $x^2 + y^2 = a^2 + b^2$

74. Ans. (1)

Sol. $ax + by = 0$

$cx + dy = 0$

so, $(0,0)$ is always the solution for a,b,c,d $\in \langle 0,1 \rangle$

for unique solution

$\frac{a}{c} \neq b/d$

$ad \neq bc$

for $ad \neq bc$

two cases are there

when ad=0, then bc= 1

& ad=1, then bc=0

favourable case= 6

so, probability= $\frac{6}{2^4} = \frac{3}{8}$

75. Ans. (3)

Sol. $\cos B = \frac{a^2 + b^2 + c^2}{2ac}$

$a^2 - (2c\cos B)a + (c^2 - b^2) = 0$

$a = c\cos B \pm \sqrt{(c\cos B)^2 - (c^2 - b^2)}$

$a = c\cos B \pm \sqrt{b^2 - (c\text{Sin}B^2)}$

$\because b > c\text{Sin}B$

there will be 2 values of c

but here b=5 cm & c=3 cm

c<b

and b>cSinB

so, only 1 triangle is possible.

76. Ans. (4)

Sol. S = 1-2+3-4+..................n terms

for n to be even, let n=2m

S= 1-2+3-4+................2m terms

$S = (1-2)+(3-4)+(5-6)+$................m terms

$S = (-1)+(-1)+(-1)+$................m terms

$S = -m = \frac{-n}{2}$

for n to be odd let it be 2m+1 so,

S= 1-2+3-4+................(2m+1) terms

$S = (1-2)+(3-4)+(5-6)+$................$[(2m-1)-2m](2m+1)$

$-m + 2m + 1$

$= (m+1) \quad \left\langle \begin{matrix} n = 2m+1 \\ m = \frac{n-1}{2} \end{matrix} \right\rangle$

$\frac{n-1}{2} + 1$

$= \frac{n+1}{2}$

77. Ans. (3)

Sol. $\lim_{n\to\infty} \sum_{r=1}^{r=n} \frac{n+r}{n^2 + r^2}$

$= \lim_{n\to\infty} \sum \frac{1 + \frac{r}{n}}{1 + \left(\frac{r}{n}\right)^2} \times \frac{1}{n}$

$= \int_0^1 \frac{1+x}{1+x^2} dx = \pi/4 + \frac{1}{2}\ln 2$

78. Ans. (1)

Sol. $|y| + \frac{1}{2} = e^{-|x|}$

$|y| = e^{-|x|} - \frac{1}{2}$

$e^{-x} - \frac{1}{2} = 0 \Rightarrow x = \ln 2$

Required area $= 4\int_0^{\ln 2} \left(e^{-x} - \frac{1}{2}\right) dx$

Solving, we get the required area $= 2[1 - \ln 2]$

79. Ans. (4)

Sol. $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

$\alpha^2 = 1 \Rightarrow \alpha = \pm 1$

$\alpha + 1 = 5 \Rightarrow \alpha = 4$

$\langle$no common value of $\alpha\rangle$

so no real α.

80. Ans. (2)

Sol. $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots.C_nx^n$

$x \to x^2$

$(1+x^2)^n = C_0 + C_1x^2 + C_2x^4 + \ldots.C_nx^{2n}$

Multiplying x,

$x(1+x^2)^n = C_0x + C_1x^3 + C_2x^5 + \ldots.C_nx^{2n+1}$

Differentiating both sides w.r.t. x, then

$x.n.(1+x^2)^{n-1}.2x + (1+x^2)^n = C_0 + 3C_1x^2 + 5C_2x^4 + \ldots\ldots + (2n+1)C_nx^{2n}$

Put $x = i$ in both sides

$0 + 0 = C_0 - 3C_1 + 5C_2 - \ldots\ldots.(-1)^n(2n+1)C_n = 0$

81. ***Ans. (4)***

Sol. $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$

$1(-4k+6) - k(-12+4) + 3(-9-2k) = 0$

$-4k + 6 + 12k - 4k + 27 - 6k = 0$

$2k = 33$

$k = 33/2$

82. ***Ans. (2)***

Sol. $nP = 2$

variance $nPq = 1$

$q = 1/2$

$P = 1 - q = 1/2$

$n = 4$

so, binomial is $\left(\frac{1}{2} + \frac{1}{2}\right)^4$

so, $P(X > 1) = P(X = 2) + P(X = 3) + P(X = 4)$

$= \frac{4}{2}\left(\frac{1}{2}\right)^2 . \left(\frac{1}{2}\right)^2 + \frac{4}{3}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + \frac{4}{4}\left(\frac{1}{2}\right)^4$

$= \frac{11}{16}$

83. ***Ans. (3)***

Sol. $Y - y = \frac{dy}{dx}(X - x)$

when $X = 0$

$Y = y - x\frac{dy}{dx}$

$\therefore Y = ky^2 \langle \text{as } Y \propto y^2 \rangle$

$y - x\frac{dy}{dx} = ky^2 \text{ i.e. } \frac{dy}{dx} - \frac{1}{x}y = -k\frac{y^2}{x}$

$-y^{-2}\frac{dy}{dx} + \frac{1}{x}.\frac{1}{y} = \frac{k}{x}$

$\frac{x}{y} = kx + c$

$\frac{c_1}{x} + \frac{c_2}{y} = 1$

84. ***Ans. (3)***

Sol. Let $I = \int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta - x}}\, dx$

$x = \alpha\cos^2 t + \beta\sin^2 t$

$x - \alpha = (\beta - \alpha)\sin^2 t$

$\beta - x = (\beta - \alpha)\cos^2 t$

$(\beta - \alpha)\int_0^{\pi/2}(1 - \cos 2t)\,dt = (\beta - \alpha)\left[t - \frac{\sin 2t}{2}\right]_0^{\pi/2} = \frac{\pi}{2}(\beta - \alpha)$

$I = \frac{\pi}{2}(\beta - \alpha)$

85. ***Ans. (4)***

Sol. $\frac{y - 3x}{2} = 1$

By putting values in the equation of curve,

$119x^2 - 34xy - 17y^2 = 0$

$7x^2 - 2xy - y^2 = 0$

$\tan\theta = \left|\frac{2\sqrt{(-1^2) - 7x(-1)}}{7 + (-1)}\right|$

$\tan\theta = \frac{2\sqrt{2}}{3}$

$\theta = \tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

86. ***Ans. (1)***

Sol. $y = mx + a/m$

let it be passed through (h, k)

$k = mh + a/m$

$m^2h - mk + a = 0$

$m_1 + m_2 = k/h$

$m_1.m_2 = a/h$

$\tan\theta_1 + \tan\theta_2 = k/h$

$\tan\theta_1.\tan\theta_2 = a/h$

$\therefore \cot\theta_1 + \cot\theta_2 = c$

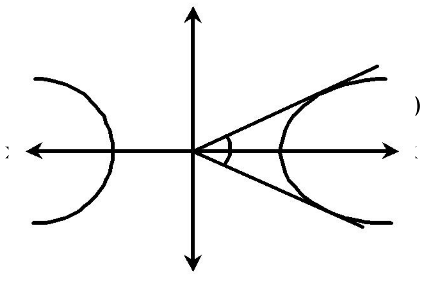
$\frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1\tan\theta_2} = c$

$\Rightarrow k = ac$

$\Rightarrow y = ac$

87. ***Ans. (2)***

Sol.



Let equation of CP is $y = mx$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} - \frac{m^2x^2}{b^2} = 1$

After solving, $x^2 = \frac{a^2b^2}{b^2 - a^2m^2}$ and

$y^2 = \frac{a^2m^2b^2}{b^2 - a^2m^2}$

For CQ, equation of line is $y = \frac{-1}{m}x$

Again by solving $CQ^2 = \frac{a^2b^2(1+m^2)}{b^2m^2 - a^2}$

So, $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{b^2(1+m^2) - a^2(1+m^2)}{a^2b^2(1+m^2)} = \frac{1}{a^2} - \frac{1}{b^2}$

88. Ans. (2)

Sol. $a = 3\sqrt{3}, b = 1$

eqn. of tangent is

$\frac{x}{3\sqrt{3}}\cos\theta + y\sin\theta = 1$

$x - \text{intercept} = 3\sqrt{3}\sec\theta$

$y - \text{intercept} = \text{cosec}\,\theta$

so let $P = 3\sqrt{3}\sec\theta + \text{cosec}\,\theta$

so, $\frac{dP}{d\theta} = 3\sqrt{3}\sec\theta + \tan\theta - \text{cosec}\,\theta.\cot\theta$

so, $\frac{dP}{d\theta} = 0 \Rightarrow \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} = \frac{\cos\theta}{\sin^2\theta}$

$\tan^3\theta = \left(\frac{1}{\sqrt{3}}\right)^3 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$ $\qquad \because \left(\frac{d^2P}{d\theta^2}\right)_{\theta=\frac{\pi}{6}} = +ve$

$\theta = \frac{\pi}{6}$

89. Ans. (3)

Sol. Let $\sqrt{x-1} = t$

$x - 1 = t^2 \Rightarrow x = 1 + t^2$

$\sqrt{1+t^2+3-4t} \Rightarrow \sqrt{1+t^2+8-6t} = 1$

$|t-2| + |t-3| = 1$

$\therefore 2 \le t \le 3$

$4 \le t^2 \le 9$

$4 \le x - 1 \le 9$

$5 \le x \le 10$

$x \in [5, 10]$

90. Ans. (4)

Sol. $2\cos^2\theta + \sin\theta \le 2$

$\sin\theta(2\sin\theta - 1) \ge 0$

$\sin\theta \le 0, \sin\theta \ge 1/2$

for $\sin\theta \le 0$

$\theta \in \left[\pi, 3\pi/2\right]$

for $\sin\theta \ge 1/2$

$\theta \in \left[\pi/6, 5\pi/6\right]$

so after taking common

we will get $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$