ANSWERKEY

Q.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A.	**3**	**3**	**1**	**1**	**4**	**4**	**2**	**2**	**3**	**3**	**4**	**3**	**3**	**2**	**1**	**3**	**2**	**3**	**3**	**1**
Q.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A.	**2**	**4**	**3**	**3**	**3**	**4**	**2**	**2**	**3**	**4**	**4**	**1**	**3**	**3**	**4**	**4**	**4**	**2**	**4**	**3**
Q.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A.	**3**	**1**	**3**	**2**	**1**	**3**	**3**	**2**	**3**	**3**	**2**	**4**	**2**	**3**	**1**	**2**	**4**	**1**	**1**	**3**
Q.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
A.	**2**	**3**	**4**	**3**	**1**	**4**	**2**	**1**	**4**	**3**	**2**	**2**	**1**	**4**	**4**	**4**	**3**	**1**	**4**	**2**
Q.	81	82	83	84	85	86	87	88	89	90										
A.	**4**	**2**	**3**	**3**	**2**	**1**	**4**	**3**	**1**	**3**										

SOLUTION

1. Ans. (3)

Period of $|\sin 4x| + |\cos 4x|$ is $\frac{\pi}{8}$

& period of

$|\sin 4x - \cos 4x| + |\sin 4x + \cos 4x| = \frac{\pi}{8}$

$\therefore$ period of given function $= \frac{\pi}{8}$

2. Ans. (3)

If lines intersect on x-axis then y = 0

$\Rightarrow 4x^2 - 16x + \beta = 0$

should be a perfect square $\Rightarrow \beta = 16$

3. Ans. (1)

The family of cirlces (concentric) will be given by $x^2 + y^2 - 2x + 4y + c = 0$ and for required circle $2((-1)x(-2) + 2x(-2)) = -1 + c \Rightarrow c = -3$.

4. Ans. (1)

If ℓ,m,n are dc's of normal then

$\ell + m + n = 0$

$\ell + m + nd = 0 \Rightarrow n(1 - d) = 0 \Rightarrow n = 0$

$\Rightarrow \ell = -m$ & $\ell^2 + m^2 = 1$

$\therefore$ dc's will be $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ or

$\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

5. Ans. (4)

$T_{r+1} = {}^{38}C_r 2^{38-r}(ix)^r$ will be negative real number for r = 2,6,10......38 (10 terms)

6. Ans. (4)

$S_n = 0 \ \forall \ n \geq 2$

$$P_n = \begin{cases} 1 : \text{if } n \text{ is odd} \\ -1 : \text{if } n \text{ is even} \end{cases}$$

$$\therefore \sum_{r=2}^{15}\left(S_r + P_r + S_r P_r\right) = 0$$

7. Ans. (2)

$4\vec{a} + 5\vec{b} + 9\vec{c} = \vec{0} \Rightarrow \vec{a}, \vec{b}$ & $\vec{c}$ are coplanar and

$(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ are collinear

$\Rightarrow (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = \vec{0}$

8. Ans. (2)

$$\sin\alpha + \int_{\alpha}^{2\alpha} \cos 2x\,dx = 0$$

$$\Rightarrow \sin\alpha + \left.\frac{\sin 2x}{2}\right|_{\alpha}^{2\alpha} = 0$$

$\Rightarrow \sin\alpha(1 + \cos 3\alpha) = 0$

$$\Rightarrow \alpha = -\pi, 0, -\frac{\pi}{3}$$

9. **Ans. (3)**

$$\bar{x} = \frac{2+4+6+8+10}{5} = 6$$

$$\therefore \text{ variance } = \frac{1}{n}\sum\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{5}\left\{(2-6)^2 + (4-6)^2 + (6-6)^2 + (8-6)^2 + (10-6)^2\right\} = 8$$

10. **Ans. (3)**

$$AA^T = I \Rightarrow \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}\begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+4+y^2 \end{bmatrix}$$

$\Rightarrow x = -2;\ y = -1$

11. **Ans. (4)**

$$\int \operatorname{cosec}^2 x.\cos^{-2014} x.dx - 2014\int \cos^{-2014} x\,dx$$

$= -\cot x \cos^{-2014} x\ +\ 2014$

$$\int \cot x.\cos^{-2015} x.\sin dx - 2014\int \cos^{-2014} x dx$$

$= -\ \cot x \cos^{-2014} x\ +\ 2014$

$$\int \cos^{-2014} x dx - 2014\int \cos^{-2014} x dx + c$$

$$= -\cot x.\cos^{-2014} x + c = -\frac{\cos ec x}{\cos^{2013}} + c$$

12. **Ans. (3)**

Differentiating the given equation w.r.t 'x'

$f(x) = 2x + 2 - xf(x) \Rightarrow f(x) = 2$ (constant function)

$f(x)$ is periodic many-one. Differentiable but not invertiable due to not a bijective function.

13. **Ans. (3)**

Equation of tangent is

$$y = mx + \sqrt{9m^2 - 16} \Rightarrow \sqrt{9m^2 - 16} = 2\sqrt{5}$$

$\Rightarrow m = \pm 2$

14. **Ans. (2)**

$$\text{Lines are coplanar if } \begin{vmatrix} 1 & 1 & 1 \\ 2 & \alpha & \beta \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$\Rightarrow 2\alpha - \beta = 2$

$$\text{sum of the roots } = \frac{-(\beta+2)}{\alpha} = -2$$

15. **Ans. (1)**

$\tan\alpha \tan 2\alpha \tan 3\alpha ...... \tan(2n-1)\alpha$

$= (\tan\alpha.\ \tan(2n-1)\alpha)$

$(\tan 2\alpha.\ \tan(2n-2)\alpha)....\tan(n\alpha)$

$$= \left(\tan\alpha.\tan\left(\frac{\pi}{2} - \alpha\right)\right)$$

$$\left(\tan 2\alpha.\tan\left(\frac{\pi}{2} - 2\alpha\right)\right) ..... \tan\frac{\pi}{4} = 1$$

16. **Ans. (3)**

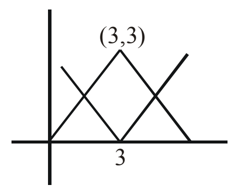
$$\frac{2a^2 + 5a + 2}{a} = 2\left(a + \frac{1}{a}\right) + 5 \geq 9$$

$$\therefore\ \frac{\left(2a^2 + 5a + 2\right)\left(2b^2 + 5b + 2\right)\left(2c^2 + 5c + 2\right)}{abc}$$

$\geq 9^3 = 729$

17. **Ans. (2)**

Required Area



$$\text{area } = \left(\frac{3}{\sqrt{2}}\right)^2 = \frac{9}{4}$$

18. **Ans. (3)**

$$\left(\frac{f'(x)}{f(x)}\right)^2 + 4\frac{f'(x)}{f(x)} + 1 = 0$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$$

$$\Rightarrow \ell n\left(f(x)\right) = \left(-2 \pm \sqrt{3}\right)x + c$$

But $c = 0 \because f(0) = 1$

$$\Rightarrow f(x) = e^{\left(-2\pm\sqrt{3}\right)x} \Rightarrow f_1(x) = e^{\left(-2+\sqrt{3}\right)x}$$

$$\& \Rightarrow f_2(x) = e^{\left(-2-\sqrt{3}\right)x}$$

19. **Ans. (3)**

since $\sin^{-1}(\sin x) + \cos^{-1}(\sin x)$ is a periodic function with period 2π

$$\Rightarrow \int_{t+4\pi}^{t+\frac{9\pi}{2}} \left(\sin^{-1}(\sin x) + \cos^{-1}(\sin x)\right)dx$$

$$= \frac{\pi}{2}\int_{4\pi}^{\frac{9\pi}{2}} dx = \frac{\pi^2}{4}$$

20. Ans. (1)

Eqaution of tangent is $y = 2x \pm \sqrt{4a^2 + b^2}$

If this is normal to circle $x^2 + y^2 + 4x + 1 = 0$ then it should be passes through centre (–2,0)

$\Rightarrow -4 \pm \sqrt{4a^2 + b^2} = 0 \Rightarrow 4a^2 + b^2 = 16$

$\Rightarrow \because$ AM $\geq$ GM

$\Rightarrow \frac{4a^2 + b^2}{2} \geq \sqrt{4a^2 b^2} \Rightarrow ab \leq 4$

21. Ans. (2)

$\because f(x)$ is non monotonic

$\therefore$ unity will be the double repeated root of $f(x) = 0$.

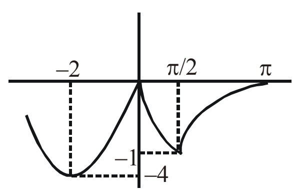
$\therefore f(1) = f'(1) = 0$

$\Rightarrow p + 2q + r = 0$

& $3p + 3q = 0 \Rightarrow p = -q = r$

22. Ans. (4)

The graph of given function is



23. Ans. (3)

$f(x)$ is monotonic $\Rightarrow f'(x) < 0$ or $f'(x) > 0$ $\forall x \in R$

$\Rightarrow f'(mx) < 0$ or $f'(mx) > 0$ $\forall x \in R, m \in R$

$\Rightarrow f(mx)$ is monotonic

$\Rightarrow f(x) + f(3x) + f(5x) + ... + f((2k-1)x)$ is also a monotonic polynomial of degree $(2k - 1)$

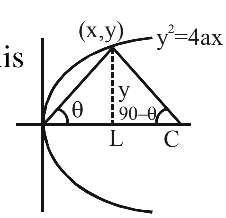
$\therefore$ It will attains all real values only once.

24. Ans. (3)

$\tan\theta = \frac{y}{x}$

Projection of BC on the x-axis

$\Rightarrow LC = \frac{y}{\tan(90-\theta)} = y\tan\theta$



$\Rightarrow \frac{y^2}{x} = 4a$

25. Ans. (3)

$\because b = \frac{7}{9}a$

$\therefore (a,b) \equiv (9,7); (18,14); (27,21); (36,28)$

$\therefore$ Required probability $= \frac{4}{^{39}C_2} = \frac{4}{741}$

26. Ans. (4)

$$a_1 = f'(0) \ \& \ f'(x) = \begin{vmatrix} 2x & 1 & x+1 \\ 4x & 1 & x+2 \\ 6x & 1 & x+3 \end{vmatrix} + 0 + 0$$

$\therefore f'(0) = 0$

27. Ans. (2)

1^{st} key will be tried for at the most

= (n – 1) locks

2^{nd} key will be tried for at the most

= (n – 2) locks

$(n-1)^{th}$ key will be tried for at the most

= 1 locks

$\therefore$ Maximum number of trials needed

$= (n-1) + (n-2) + ... + 1 = \frac{n(n-1)}{2}$

also $\sum_{k=2}^{n}(k-1) = \frac{n(n-1)}{2}$

28. Ans. (2)

$f(1^{-1}) > f(1) < f(1^{+})$

$\Rightarrow$ Statement 1 is correct.

But $f'(\alpha) = 0 \not\Rightarrow$ that at $x = \alpha$, $f(x)$ has an extremum.

29. Ans. (3)

If chord PQ subtends right angle at vertex then

$t_1t_2 = -4 \Rightarrow P(at^2, 2at)$: $Q\left(\frac{16a}{t^2}, \frac{-8a}{t}\right)$ mid point of PQ will be $\left(a\left(t^2 + \frac{16}{t^2}\right), 2a\left(t - \frac{4}{t}\right)\right)$ and its locus will be $y^2 = 2a(x - 4a)$

30. Ans. (4)

$\because$ S(1) is not true.

Mathematical induction can't be apply.

But if S(k) if true means $S(k) = 1 + k^3$

$S(k + 1) = 1 + k^3 + 3k^2 + 3k + 1$

$= 1 + (k + 1)^3$ also true.

31. Ans. (4)

32. Ans. (1)

33. Ans. (3)

34. Ans. (3)

35. Ans. (4)

36. Ans. (4)

Sol. $Q_2 = \frac{(2\omega_1)L}{R}$ (as frequency is doubled) = 2Q,

where $Q_1 = \frac{L\omega_1}{R} \Rightarrow$ Q is doubled

$I_1 = \frac{V}{\sqrt{(L\omega_1)^2 + R^2}} \simeq \frac{V}{L\omega_1}$ (as $L\omega >> R$ for high Q coils) Similarly, $I_2 \simeq \frac{V}{L(2\omega_1)}$

$\Rightarrow I_2 = \frac{I_1}{2}$

$P_2 = I_2R^2 = \left(\frac{I_1}{2}\right)R^2 = \frac{I_2R^2}{4} = \frac{P_1}{4}$

$\Rightarrow$ P decreases 4 times.

37. Ans. (4)

Sol. x + 2f + x' + 2f > 4f; x + x' > 2f

38. Ans. (2)

Sol. $f_b = \frac{v}{4\ell} - \frac{v}{4(\ell + x)}$

$= = \frac{v}{4}\left(\frac{1}{\ell} - \frac{1}{(\ell + x)}\right) = \frac{v}{4}\left(\frac{\ell + x - \ell}{\ell(\ell + x)}\right)$

$\simeq \frac{vx}{4\ell^2}$ $\quad (\because x << \ell)$

39. Ans. (4)

40. Ans. (3)

Sol. $a = \frac{2Kqq}{m} \cdot \frac{x}{(a^2 + x^2)^{3/2}}$

when x = 0, then a = 0 and when $x = \pm\frac{a}{\sqrt{2}}$ then the acceleration is maximum.

41. Ans. (3)

42. Ans. (1)

43. Ans. (3)

44. Ans. (2)

45. Ans. (1)

Sol. 2nd fragment will move vertically downward with velocity u

So Δt = time of returning of 1st fragment to height $H = \frac{2u}{g}$

46. Ans. (3)

Sol. Width of central maxima

$= \frac{2\lambda D}{a} = \frac{2(500 \times 10^{-9})(1)}{(0.1 \times 10^{-3})}m = 10^{-2}m = 10mm$

47. Ans. (3)

Sol. Resistance of galvanometer

$G = 50\ \Omega$

Full scale current $i_g = 0.05$

$A = 2.97 \times 10^{-2}\ cm^2$

$= 2.97 \times 10^{-2} \times 10^{-4}\ m^2$

$= 2.97 \times 10^{-6}\ m^2$

$i = 5A$

$\rho = 5 \times 10^{-7}\ \Omega m$

Required resistance to convert the galvanometer into ammeter.

$R = \frac{i_gG}{i - i_g} = \frac{0.05 \times 50}{5 - 0.05} = \frac{2.5}{4.95}$

$\rho\frac{\ell}{A} = \frac{50}{99}$

$\ell = \frac{50}{99} \times \frac{A}{\rho} = \frac{50}{99} \times \frac{2.97 \times 10^{-6}}{5 \times 10^{-7}}$

$= \frac{50}{99} \times \frac{29.7}{5} = 10 \times 0.3 = 3m$

48. Ans. (2)

49. Ans. (3)

50. Ans. (3)

51. **Ans. (2)**

Sol. $I' = I\left(1-\dfrac{30}{360}\right)$

52. **Ans. (4)**

Sol. Here, the state of maximum amplitude of the oscillation is a measure of resonance.

53. **Ans. (2)**

54. **Ans. (3)**

55. **Ans. (1)**

56. **Ans. (2)**

57. **Ans. (4)**

58. **Ans. (1)**

59. **Ans. (1)**

60. **Ans. (3)**

61. **Ans. (2)**

62. **Ans. (3)**

63. **Ans. (4)**

64. **Ans. (3)**

65. **Ans. (1)**

66. **Ans. (4)**

67. **Ans. (2)**

68. **Ans. (1)**

69. **Ans. (4)**

70. **Ans. (3)**

71. **Ans. (2)**

72. **Ans. (2)**

73. **Ans. (1)**

74. **Ans. (4)**

75. **Ans. (4)**

76. **Ans. (4)**

77. **Ans. (3)**

78. **Ans. (1)**

79. **Ans. (4)**

80. **Ans. (2)**

81. **Ans. (4)**

82. **Ans. (2)**

83. **Ans. (3)**

84. **Ans. (3)**

85. **Ans. (2)**

86. **Ans. (1)**

87. **Ans. (4)**

88. **Ans. (3)**

89. **Ans. (1)**

90. **Ans. (3)**