

PAPER-1

PART-1 : PHYSICS

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C,D	B,D	A,B	A,D	B,C	B,C	A,D	B,D	A,B	B,D
	Q.	11	12								
SECTION-IV	A.	A,B,D	A,C,D								
	Q.	1	2	3	4	5	6	7	8		
	A.	4	6	2	2	2	4	3	2		

SOLUTION

SECTION-I

1. Ans. (C,D)

Sol. $\frac{R}{L} = \frac{\rho}{\rho_w}$ → same

$$B = \rho V g_{\text{eff}}$$

$$p = p_0 + h \rho g_{\text{eff}}$$

2. Ans. (B,D)

Sol. (A) $\mu mg = m\omega^2 r$
 $= \alpha^2 r t^2$

$$t = \sqrt{\frac{\mu g}{\alpha^2 r}}$$

(C) $N = mr\alpha$

$$\mu N = m\omega^2 r$$

$$\mu m r \alpha = m \omega^2 r$$

$$= \alpha^2 t^2$$

$$t = \sqrt{\frac{\mu}{\alpha}}$$

3. Ans. (A,B)

Sol. $PV^\gamma = C \Rightarrow P \times (R^3)^{\frac{5}{3}} = C$

$$TV^{2/3} = \text{constant}$$

$$T \times R^{3 \times 2/3} = \text{constant}$$

4. Ans. (A,D)

Sol. (A) $\frac{4\pi R_s^2 \sigma T_s^4}{4\pi d^2} \times \pi R_p^4 = 4\pi R_p^2 \sigma T_p^4$

$$T_p = T_s \sqrt{\frac{R_s}{2d}}$$

(C) $\frac{4\pi R_p^2 \times \sigma T_p^4}{4\pi d^2} \times \pi R_s^2 = \frac{dE}{dt}$

$$\frac{\pi R_s^2 \sigma T_s^4}{d^2} \times \frac{R_s^2}{4d^2}$$

5. Ans. (B,C)

Sol. (A) $2\pi a = \frac{\lambda}{4} \Rightarrow \lambda = \frac{340}{85} = 4m$

$$\Rightarrow 2\pi a = 1$$

$$a = \frac{1}{2\pi} m$$

(B) closed organ pipe

(C) Sealed end is PAN

$$2\pi a = \left(2 \frac{1}{2}\right) \frac{\lambda}{2}$$

$$2\pi a = \frac{5\lambda}{4}$$

$$\lambda = \frac{8\pi a}{5}$$

$$\theta = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$$

$$\Rightarrow \theta = \frac{4\pi}{5}, \frac{8\pi}{5}$$

6. Ans. (B,C)

Sol. $Dx = 0.3 = n\lambda$

$$= n \times \frac{330}{f}$$

$$f = \frac{3300n}{0.3}$$

$$f_{\max} = 20000$$

$$n = \frac{20000}{1100}$$

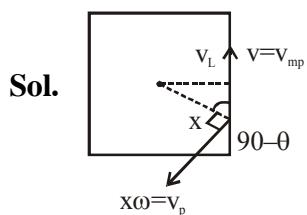
$$\Delta x = 0.3 = \left(n + \frac{1}{2}\right)\lambda$$

$$f = 1100 \left(n + \frac{1}{2}\right)$$

$$n + \frac{1}{2} = 18.22$$

$$\Rightarrow n = 0 \text{ to } 17$$

7. Ans. (A,D)



$$v_m = v_{mp} + v_p$$

$$= \sqrt{v^2 + x^2\omega^2 + 2vx\omega} \cos(90 + \theta)$$

$$= \sqrt{v^2 + x^2\omega^2 - 2vx\omega \sin \theta}$$

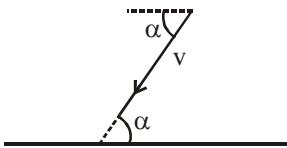
$$L_i = 0 = L_f$$

$$= -\frac{m}{12}(a^2 + a^2)\omega + m(v - x\omega \sin \theta) \frac{\alpha}{2}$$

$$\Rightarrow \frac{a^2}{6}\omega + \frac{a^2}{4}\omega = \frac{va}{2} \Rightarrow \omega = \frac{6v}{5a}$$

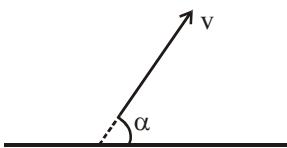
8. Ans. (B,D)

Sol.



Observer moving away from source.

$$v = \frac{C - v \sin \alpha}{C} v_0$$



Observer moving towards source.

$$v_1 = \frac{C - v \sin \alpha}{C} v_0$$

9. Ans. (A,B)

10. Ans. (B,D)

11. Ans. (A,B,D)

Sol. $p_0 + h_1 \rho_1 g = p_0 + h_2 \rho_1 g + h_3 \rho_3 g$

12. Ans. (A,C,D)

Sol. $p_{in} = 10^5 + 5 \times 0.8 \times 10^3 \times 10 + 10 \times 10^3 \times 10$

$$= 10^5 + 4 \times 10^4 + 10^5$$

$$= 2.4 \times 10^5$$

$$p_{out} = 10^5 + 0.8 \times 15 \times 10^3 \times 10 = 2.2 \times 10^5$$

$$p_{out} + \frac{1}{2} \rho v^2 = p_{in}$$

$$\frac{1}{2} \times 10^3 \times v^2 = 0.2 \times 10^5$$

$$v = \sqrt{40} \text{ m/s}$$

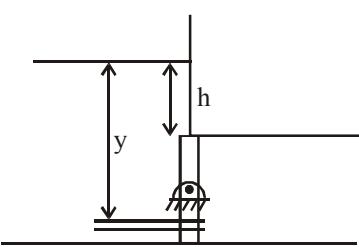
$$\sqrt{40} \times 10^{-6} = 100 \times 10^{-4} v_s$$

$$v_s = \sqrt{40} \times 10^{-4} \text{ m/s}$$

$$= \sqrt{\frac{40}{10}} \text{ mm/s} = \sqrt{\frac{40}{100}} \text{ mm/s} = \sqrt{\frac{2}{5}} \text{ mm/s}$$

SECTION-IV

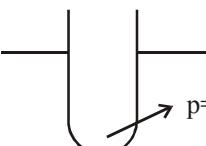
1. Ans. 4



Sol.

$$\begin{aligned} & \int_h^{h+2} \rho g y dy \times 6 \times (y - (h+1)) \\ &= 10^4 \times 6 \left[\frac{y^3}{3} - (h+1) \frac{y^2}{2} \Big|_h^{h+2} \right] \\ &= 6 \times 10^4 \left[\frac{1}{3} ((h+2)^3 - h^3) - \frac{(h+1)}{2} ((h+2)^2 - h^2) \right] \\ &= 6 \times 10^4 \left[\frac{1}{3} \left[6h^2 + \frac{6h}{12} + 8 - \frac{(h+1)}{2} [4^2 h + 4^2] \right] \right] \\ &= 6 \times 10^4 \left[\frac{1}{3} (6h^2 + 12h + 8) - 2(h^2 + 2h + 1) \right] \\ &= 6 \times 10^4 \left[\frac{8}{3} - 2 \right] = 4 \times 10^4 \text{ Nm} \end{aligned}$$

2. Ans. 6



Sol.

$$\Delta p = \frac{25}{r} + h\rho g$$

$$= \frac{2 \times 7 \times 10^{-3}}{0.35 \times 10^{-3}} + 2 \times 10^{-2} \times 10^4 = 240$$

$$\frac{\Delta p}{40} = 6$$

3. Ans. 2

$$\text{Sol. } t_1 = \frac{2u \sin \theta}{g} \Rightarrow u = \frac{10}{2 \sin \theta}$$

$$-h = d \tan \theta - \frac{1}{2} g \frac{d^2}{u^2 \cos^2 \theta}$$

$$-10 = 10 \tan \theta - \frac{1}{2} \times \frac{10 \times 10}{25} \tan^2 \theta$$

$$2\tan^2\theta - \tan \theta - 1 = 0$$

$$\tan \theta = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1+3}{4}, \frac{1-3}{4} = 1, -\frac{1}{2}$$

4. Ans. 2

$$\text{Sol. } T_a = 2\pi \sqrt{\frac{2mR^2}{mgR}}$$

$$T_b = 2\pi \sqrt{\frac{\frac{3}{2}mR^2}{mgR}} = \sqrt{3}$$

$$\Rightarrow 2\pi \sqrt{\frac{mR^2}{2mgR}} = 1$$

$$T_a = 2 \text{ sec}$$

5. Ans. 2

$$\text{Sol. } W = \Delta KE + \Delta H_{\text{friction}}$$

as seen from belt frame

$$\Delta H_{\text{friction}} = \frac{1}{2} m (0^2 - v^2)$$

$$\Rightarrow W = mv^2 \\ = 0.5 \times 2^2 = 2J$$

6. Ans. 4

$$\text{Sol. } mg - N = ma = m \frac{\ell}{2} \alpha$$

$$mg \frac{\ell}{2} = \frac{m\ell^2}{3} \alpha$$

$$\alpha = \frac{3g}{2\ell}$$

$$mg - \frac{m\ell}{2} \times \frac{3g}{2\ell} = N$$

$$N = \frac{mg}{4}$$

7. Ans. 3

Sol. $mv_0 = 2mv_2 + mv_1$

$$e = 1 = \frac{v_2 - v_1}{v_0}$$

$$v_2 - v_1 = v_0$$

$$2v_2 + v_1 = v_0$$

$$3v_2 = 2v_0$$

$$v_2 = \frac{2v_0}{3}$$

$$\frac{1}{2}2mv_2^2 = \frac{1}{2}kd^2$$

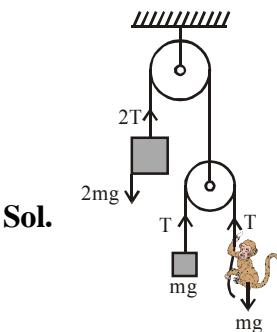
$$v_2 = \sqrt{\frac{k}{2m}}d$$

$$v_0 = \frac{3}{2}d\sqrt{\frac{k}{2m}}$$

$$= \frac{3}{2} \times 0.1 \sqrt{\frac{400}{0.5 \times 2}}$$

$$v_0 = 3$$

8. Ans. 2



$$T - mg = ma_b = ma_m$$

$$2T - 2mg = 2ma_B$$

$$\Rightarrow a_B = a_b = a_m \Rightarrow v_B = v_b = v_m = v$$

$$v_p = -v$$

$$v_R - v_b = 2v_p$$

$$v_R = -2v - v = -3v$$

$$v_{mR} = v - (-3v) = 4v = 8$$

PART-2: CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	A,B,D	A,D	A	A,C,D	B	A,D	C	D
	Q.	11	12								
	A.	C	C								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	6	4	9	2	1	3	5		

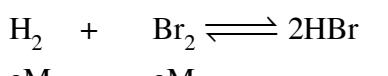
SOLUTION

SECTION - I

1. Ans. (C)

$$282 \times \frac{33.33}{100} = 96 \Rightarrow 3 'S' atoms per molecule.$$

2. Ans. (B)



$$= \frac{25.92/81}{20} = \frac{0.32}{20} = 0.016 M$$

$$64 = \frac{(2x)^2}{(a-x)^2}$$

$$8a - 0.064 = 0.016$$

$$8a = 0.08$$

$$a = 0.01 M$$

$$n_{H_2} = 0.01 \times 20 = 0.2 \text{ mol} \equiv 0.4 \text{ gm}$$

$$n_{Br_2} = 0.01 \times 20 = 0.2 \text{ mol} \equiv 32 \text{ gm}$$

3. Ans. (A, B, D)

$$(A) P \times 1 = 5 \times 0.08 \times 320 \Rightarrow P = 128 \text{ atm}$$

$$(B) \left(P + \frac{4 \times 5^2}{1^2} \right) (1 - 0) = 5 \times 0.08 \times 320$$

$$\Rightarrow P = 28 \text{ atm}$$

$$(C) \left(P + \frac{4 \times 5^2}{12} \right) (1 - 0.04 \times 5) = 128 \Rightarrow P = 60 \text{ atm}$$

$$(D) P (1 - 0.04 \times 5) = 128 \Rightarrow P = 160 \text{ atm}$$

PART-3 : MATHEMATICS

ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B,C	A,B,C	A,B,C	A,B,C	A,B,C,D	A,B,C,D	A,B,D	A,B	B	A,B
	Q.	11	12								
	A.	A,C	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	7	0	7	2	2	2	3		

SOLUTION

SECTION-I

1. **Ans. (B,C)**

$$\cos^2 x = \sin x$$

$$\cos^4 x = 1 - \cos^2 x$$

$$\cos^4 x + \cos^2 x - 1 = 0$$

$$\cos^2 x = \frac{-1 + \sqrt{5}}{2}$$

$$\frac{1}{\sin x} + \cos^4 x = \frac{1}{\cos^2 x} + \cos^4 x$$

$$= \frac{2}{\sqrt{5}-1} + \frac{3-\sqrt{5}}{2} = \frac{\sqrt{5}+1}{2} + \frac{3-\sqrt{5}}{2} = 2$$

(B) & (C) are correct.

2. **Ans. (A,B,C)**

$$f(n) = \sum_{n=1}^n \tan^{-1} \left(\frac{2 \cdot 3^{n-1}}{1 + 3^n \cdot 3^{n-1}} \right)$$

$$= \sum_{n=1}^n \tan^{-1} (3^n) - \tan^{-1} (3^{n-1})$$

$$f(n) = \tan^{-1} 3^n - \frac{\pi}{4}$$

Now check option.

3. **Ans. (A,B,C)**

$$f(x) = \cos^{-1} \left(\frac{1 - (x+1)^2}{1 + (x+1)^2} \right)$$

$$x+1 = \tan \theta \quad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right)$$

$$y = \cos^{-1} \cos 2\theta$$

$$= \begin{cases} -2 \tan^{-1}(x+1), & x < -1 \\ 2 \tan^{-1}(x+1), & x \geq -1 \end{cases}$$

4. **Ans. (A,B,C)**

5. **Ans. (A,B,C,D)**

$$r_n = \frac{n}{r^{n-1}}$$

$$f(r) = 1 + \frac{2}{r} + \frac{3}{r^2} + \frac{4}{r^3} + \dots \infty \quad \dots(1)$$

$$\frac{f(r)}{r} = \frac{1}{r} + \frac{2}{r^2} + \frac{3}{r^3} + \dots \infty \quad \dots(2)$$

$$\Rightarrow f(r) = \left(\frac{r}{1-r} \right)^2$$

Now check options.

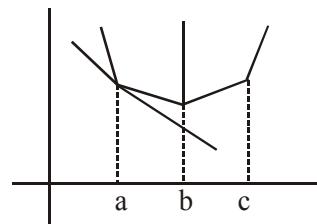
6. **Ans. (A,B,C,D)**

$$f(x) = \frac{(x^2 - x + 1)(x^2 + x + 1)}{(x^2 + x + 1)^2}$$

$$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$$

Now check options.

7. **Ans. (A,B,D)**



By comparing the slopes, condition in problem is satisfied for $(a, b+c-2a)$ to lie on the line. Putting in line we get

$$b+c=2003$$

$$b_{\max} = 1004$$

$$c_{\min} = 1002$$

8. **Ans. (A,B)**

Let P denotes total number of ways of selecting even number of red balls and Q denotes total number of ways of selecting odd number of red balls.

$$(2-1)(4-1)(6-1)(8-1)(10-1) = P - Q$$

$$(2+1)(4+1)(6+1)(8+1)(10+1) = P + Q$$

Now solve to get P & Q.

Paragraph for Question 9 to 10

$$N = 2^a 3^b 5^c 7^d$$

$$a = \sum_{m=2}^{10} \sum_{n=1}^{\infty} \left\lceil \frac{m}{2^n} \right\rceil = 38$$

similarly b = 17, c = 7, d = 4

$$N = 2^{38} 3^{17} 5^7 7^4$$

obviously P = 7

$$\begin{aligned} Q &= 13 \times 6 \times 3 \times 2 \\ &= 2^2 \cdot 3^2 \times 13^1 \end{aligned}$$

9. Ans. (B)

10. Ans. (A,B)

Paragraph for Question 11 to 12

$$y = \sqrt[3]{x + \sqrt{x^2 + 1}} + \sqrt[3]{x - \sqrt{x^2 + 1}}$$

Image in $y = x$ & then cubing

$$x^3 = \left(\sqrt[3]{y + \sqrt{y^2 + 1}} + \sqrt[3]{y - \sqrt{y^2 + 1}} \right)^3$$

$$x^3 = 2y + 3(-1)x$$

$$f^{-1}(x) = \frac{x^3 + 3x}{2}$$

11. Ans. (A,C)

12. Ans. (B)

SECTION-IV

1. Ans. 5

Replace x by $1-x$ & solve to get $f(x)$

2. Ans. 7

$$H_1 - H_2 - H_3 - H_4 = 4 = 24$$

$$\boxed{H_1 H_2} - H_3 - H_4 = {}^4C_2 | 3 = 36$$

$$\boxed{H_1 H_2} - \boxed{H_3 H_4} = \frac{{}^4C_2}{2} | 2 = 6$$

$$\boxed{H_1 H_2 H_3} - H_4 = {}^4C_3 | 2 = 8$$

$$H_1 H_2 H_3 H_4 = 1 \Rightarrow 75$$

3. Ans. 0

$$s = \sum_{r=1}^3 \frac{\cos(2 \cdot 3^{r-1} \theta)}{\sin 3^r \theta} \quad \left(\theta = \frac{\pi}{28} \right)$$

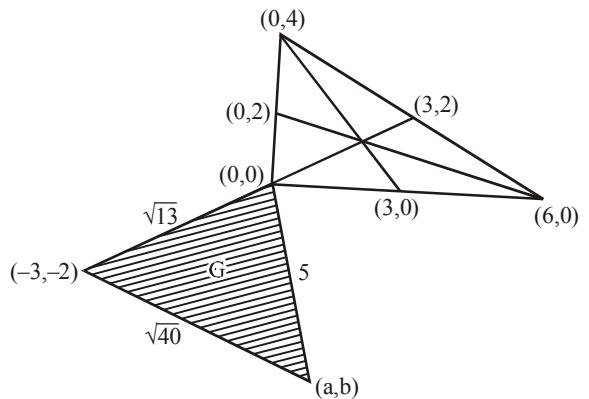
$$\Rightarrow s = \sum_{r=1}^3 \frac{\sin(3^{r-1} \theta) \cos(2 \cdot 3^{r-1} \theta)}{\sin(3^r \theta) \sin(3^{r-1} \theta)}$$

$$= \frac{1}{2} \sum_{r=1}^3 \frac{\sin 3^r \theta - \sin 3^{r-1} \theta}{\sin 3^r \theta \sin 3^{r-1} \theta}$$

$$S = \sum_{r=1}^3 \operatorname{cosec}(3^{r-1} \theta) - \operatorname{cosec}(3^r \theta)$$

$$= \operatorname{cosec} \theta - \operatorname{cosec} 27\theta = 0 \quad \{ \because \theta + 27\theta = \pi \}$$

4. Ans. 7



$$a^2 + b^2 = 25 \quad \dots \dots (1)$$

$$(a+3)^2 + (b+2)^2 = 40$$

$$6a + 4b = 2 \quad \dots \dots (2)$$

Solving (1) & (2)

possible value of a is 3

$$\Rightarrow b = 4$$

$$\frac{G}{S} = \frac{3}{4}$$

5. Ans. 2

$$\left. \begin{aligned} 2f(x) + f\left(\frac{x+1}{x-1}\right) &= x \\ x \rightarrow \frac{x+1}{x-1} & \\ \Rightarrow f(x) + 2f\left(\frac{x+1}{x-1}\right) &= \frac{x+1}{x-1} \end{aligned} \right\} \begin{aligned} &\text{Solving} \\ &3f(x) = 2x - \frac{x+1}{x-1} \\ &3f(x) + \frac{x+1}{x-1} = 2x \\ &\Rightarrow \lambda = 2 \end{aligned}$$

6. Ans. 2

$$y = \sin^2 \alpha (1 - \sin^2 \alpha)^3$$

$$y = t(1-t)^3 \quad 0 < t \leq 1$$

$$\frac{t + \frac{3(1-t)}{3}}{4} \geq \sqrt[4]{\frac{t(1-t)^3}{27}}$$

$$\frac{t(1-t)^3}{27} \leq \frac{1}{4^4}$$

$$t(1-t)^3 \leq \frac{3^3}{256}$$

7. Ans. 2

$$\frac{5}{2 \sin \theta} = \frac{5}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$\Rightarrow \angle BOA = 90^\circ$$

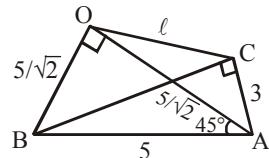
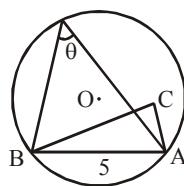
$$\angle ACO = 135^\circ$$

$$-\frac{1}{\sqrt{2}} = \frac{9 + \ell^2 - \frac{25}{2}}{6\ell}$$

$$-3\sqrt{2}\ell = \ell^2 - \frac{7}{2}$$

$$\ell^2 + 3\sqrt{2}\ell - \frac{7}{2} = 0$$

$$\ell = \frac{-3\sqrt{2} + \sqrt{18+14}}{2} = \frac{1}{\sqrt{2}}$$



8. Ans. 3

$$s = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1001x^{1000}(1+x)^0$$

$$\frac{xs}{(1+x)} = x(1+x)^{999} + 2x^2(1+x)^{998} + \dots$$

$$\dots + 1001x^{1000} + \frac{1001+x^{1001}}{(1+x)}$$

$$\frac{s \cdot 1}{(1+x)} = (1+x)^{1000} + x(1+x)^{999} + \dots$$

$$\dots + x^{1000}(1+x)^0 - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = (1+x)^{1000} \left\{ \frac{\left(\frac{x}{1+x} \right)^{1001} - 1}{\frac{x}{1+x} - 1} \right\} - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = \frac{x^{1001} - (1+x)^{1001}}{-1} - \frac{1001x^{1001}}{(1+x)}$$

$$s = (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$$

$$\text{coeff. of } x^{50} = {}^{1002}C_{50}$$