PAPER-2

PART-1 : PHYSICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	A,B,C,D	A,B,C,D	A,B,C	A,C	A,B,C	A,B,C,D	A,C	A,B,C,D	A,B,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		Q,T	Q,R	P,S	P		P,R,S,T	P,R,S,T	Q,R,T	Q,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	5	4	3	7	1	5	4		

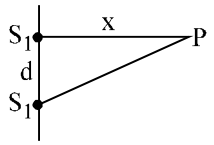
SOLUTION

SECTION-I

1. Ans. (A,B,C)

Sol. Path difference

$$\Delta x = S_2P - S_1P = \sqrt{x^2 + d^2} - x$$



(A) when $x \approx \infty$

$\Delta x = 0$, so phase difference is zero

(B) In some phase so constructive

(C) when path difference $\Delta x = n\lambda$, then constructive interference

when $\Delta x = (2n + 1)\lambda/2$ where n = 0, 1, 2, … destructive

2. Ans. (A,B,C,D)

Sol. We know that $I = \frac{2\pi^2 B}{v} s_0^2 \nu^2$ and $P = I \times$ area

also $I = \frac{p_0^2}{2\rho v}$ and here $s_0 = \Delta R$, $v = \sqrt{\frac{B}{\rho}}$ and $\nu = f$

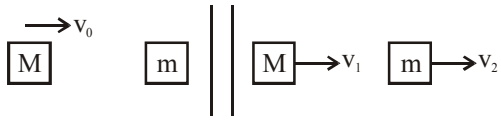
3. Ans. (A,B,C,D)

Sol. Block B & C will perform SHM about COM

$$\therefore \quad \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{m/2}} = \sqrt{\frac{2k}{m}}$$

For collision between M & m

$$Mv_0 = Mv_1 + mv_2 \ \&$$



$$v_2 - v_1 = v_0$$

$$\therefore \quad v_0 = v_1 + \gamma v_2$$

$$v_0 = v_2 - v_1$$

$$2v_0 = v_2(1 + \gamma)$$

$$\therefore \quad v_2 = \frac{2v_0}{1+\gamma}$$

$$\therefore \quad v_{CM} = \frac{mv_2 + m \times 0}{m + m} = \frac{v_0}{1+\gamma}$$

$$\beta = \frac{v_{max/COM}}{\omega} = \frac{v_0}{\omega(1+\gamma)}$$

$$v_{max} = \frac{2v_0}{1+\gamma}$$

4. Ans. (A,B,C)

Sol. Since the process in chamber 2 is adiabatic

$\therefore \quad P_0V_0^{\gamma} = P_2V_2^{\gamma}$

$\therefore \quad P_0V_0^{5/3} = \frac{27}{8}P_0V_2^{5/3}$

$\therefore \quad V_2 = \left(\frac{8}{27}\right)^{3/5} V_0$

$\therefore$ Volume of chamber

$1 = 2V_0 - V_2 = \left[2 - \left(\frac{8}{27}\right)^{3/5}\right]V_0$

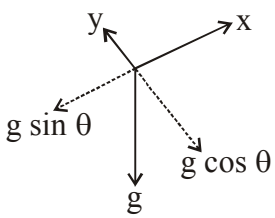
$P_0^{1-\gamma}T_0^{\gamma} = C$

$\therefore \quad T_2 = \left(\frac{27}{8}\right)^{2/5} T_0$

Work by the gas $= \frac{P_0V_0 - P_2V_2}{\gamma - 1}$

5. Ans. (A,C)

Sol.



$V_3^2 = V_1^2 - 2g \sin\theta\, x \quad$ so $V_3 < V_1$

$V_4^2 = V_2^2\, 2g\cos\theta\,(\Delta y) = V_2^2 - 2g\cos\theta(0) = V_2^2$

$V_4 = V_2$

6. Ans. (A,B,C)

Sol. $\gamma = \frac{n_1C_{p_1} + n_2C_{p_2}}{n_1C_{V_1} + n_2C_{V_2}} = \frac{\frac{n_1\gamma_1}{\gamma_1 - 1} + \frac{n_2\gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$

$(n_1 + n_2)\, C_V = n_1C_{V_1} + n_2C_{V_2}$

$(n_1 + n_2)\frac{R}{\gamma - 1} = \frac{n_1R}{\gamma_1 - 1} + \frac{n_2R}{\gamma_2 - 1}$

7. Ans. (A,B,C,D)

8. Ans. (A,C)

Sol. $F = \frac{dV}{dx}$

$\frac{dV}{dx} = 0$ at B,C,D

$\therefore$ F = 0 at B,C,D

so a = 0 at B,C,D

speed is maximum at B as potential energy at B is maximum

9. Ans. (A,B,C,D)

Sol. For reflection from densor medium, the reflected wave experience a phase difference of π & for refection from rarer medium, there is no phase difference.

10. Ans. (A,B,D)

Sol. For same heat flow rate,

$\left.\frac{d\theta}{dt}\right|_A = \left.\frac{d\theta}{dt}\right|_B$

$\Rightarrow \quad \frac{k_AA_A(100-0)}{\ell_A} = \frac{k_BA_B(100-0)}{\ell_B}$

In figure (ii), both rods are in series

$\therefore \quad R_{eq} = R_1 + R_2$

SECTION-II

1. Ans.(A)-(Q, T); (B)-(Q, R);(C)-(P, S); (D)-(P)

Sol. Rate of heat loss = 80 × 10.8 = 54 × 16 cal/sec.

(A) r = 1.6

$\Rightarrow$ rate of heat supplies by forming steam to water at 0° = 1.6 × 640 > 54 × 16

$\therefore$ additional ice will melt

(B) Rate of heat loss

= 54 × 16 = 64 × 13.5 cal/sec.

r = 1.35

= rate of heat supplied for converting steam to water at 0°C = 1.35 × 640 = 13.5 × 64.

no additional ice will melt or water will fuse.

(C) Rate of heat loss = 54 × 16 = 72 × 12 cal/sec.

Rate of heat supplied by converting steam to ice at 0°C = 1.20 × 720 = 12 × 72 cal/sec

no additional ice will melt or water will fuse.

(D) Additional water will fuse to ice.

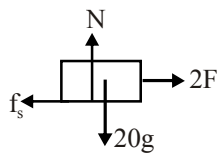
2. Ans. (A)-(P,R,S,T); (B)-(P,R,S,T); (C)-(Q,R,T); (D)-(Q,T)

Sol. From phase diagram, in case 'C' & 'D' particle are able to reach at heightest point, while in 'a' & 'b' they don't.

SECTION-IV

1. Ans. 6

Sol. $t_{max} = \mu N = 0.4 \times 20g = 80$



$F = 40\ t$

Sliding starts when

$F = f_{max}$

$80t = 80$

$\therefore\ t = 1s$

$$\int 2F\,dt - \int f_k dt = mv\ \ 0$$

$$\Rightarrow \int_1^3 80t\ dt - 80\int_1^3 dt = 20\ V$$

$\therefore\ v = 8m/s$

$P = \vec{F}\cdot\vec{v} = 40t \times 2v = 40 \times 3 \times 16 = 1920$

2. Ans. 5

Sol. Since 'B' completes the circle $\Rightarrow v_B$ at lowest point $= \sqrt{5gR}$

From work energy theorem for rod,

$$Mg\frac{\ell}{2} = \frac{1}{2}\times\frac{M\ell^2}{3}\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{\ell}}$$

For angular momentum conservation before & after collision,

$$\frac{M\ell^2}{3}.\omega = mv_B\ell$$

$$\Rightarrow \left(\frac{M}{m}\right) = \frac{3v_B}{\ell\omega} = \frac{3}{\ell}\frac{\sqrt{5gR}}{\sqrt{\frac{3g}{\ell}}} = N$$

$$\therefore\ N^2 = \frac{9\times 5gR}{\ell^2.\frac{3g}{\ell}} \qquad \text{since } \ell = R(\text{given})$$

$\therefore\ N^2 = 15$

$$\therefore\ \frac{N^2}{3} = 5$$

3. Ans. 4

Sol. From Bernoulli's equation,

$$P_0 + \rho g\,(h_1 - h_2) = \frac{1}{2}\rho v^2 + P_0$$

[Since cross section Area of beaker >> cross section area of faucet)

$$\therefore\ v = \sqrt{2g(h_1 - h_2)} = \sqrt{2\times 10\times\frac{8}{100}} = \frac{4}{\sqrt{10}}$$

4. Ans. 3

Sol. For floating

mg = Buoyant force

Since mass doesnot change with temperature

$$F_{B_1} = F_{B_2} \Rightarrow \rho_{w_1}V_1 g = \rho_{w_2}V_2 g$$

$$\rho_{w_1}(0.99\ V_0) = \left(\frac{\rho_{w_1}}{1+\gamma_w\Delta T}\right)\left(V_0(1+3\alpha\Delta T)\right)$$

$$\Rightarrow \left(1-\frac{1}{100}\right) = \left(\frac{1+3\alpha\Delta T}{1+\gamma_w\Delta T}\right)$$

By Binomial approx.

$$1 - \frac{1}{100} = (1 + (3\alpha - \gamma_w)\Delta T)$$

$$\frac{1}{100} = (\gamma_w - 3\alpha)\Delta T$$

$$\Rightarrow \Delta T = \frac{1}{100(\gamma_w - 3\alpha)} = 40°C$$

$$\Rightarrow T_f = 60°C$$

5. Ans. 7

Sol. We have $T_1 = 2\pi\sqrt{\frac{m}{k}}$

or

$$T_1^2 = 4\pi^2 \frac{m}{k} \qquad \text{... (1)}$$

After more weights Δm are added, we have

$$T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}}, \text{ or } T_2^2 = 4\pi^2 \frac{m+\Delta m}{k} \text{ ... (2)}$$

By subtracting Eq. (1) from Eq. (2), we get

$$T_2^2 - T_1^2 = 4\pi^2 \frac{\Delta\ell}{g}, \quad \text{or} \quad \Delta\ell = \frac{g}{4\pi^2}\left(T_2^2 - T_1^2\right).$$

Upon inserting the numerical data we obtain $\Delta\ell =$ 1.75 cm.

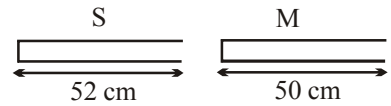
6. Ans. 1

Sol. For the semicircular plate of radius ℓ, the centre of mass lies at a distance of $\frac{4\ell}{3\pi}$ from the centre.

Taking σ to be the mass per unit area, the position of centre of mass of the remaining piece of the square would be at a distance of $\frac{\ell(3\pi-4)}{3(8-\pi)}$ from the centre of the original square plate. Now, taking the centre of the original square to the origin, the centre of mass of the new structure can be determined. This turns out to be at a distance of $\frac{\ell}{3}$ to the right of the origin.

7. Ans. 5

Sol.



$v = 325$ m/s

$$\nu_0 = \frac{V}{4L} \therefore \nu_{10} = \frac{325}{4\times 0.52}$$

$$\nu_{20} = \frac{325}{4\times 0.50}$$

$$\nu_{10} - \nu_{20} = \frac{325}{4}\left(1 - \frac{1}{0.264}\right)$$

$$= \frac{325}{4}\left(\frac{0.004}{0.260\times 0.264}\right)$$

$$= \frac{0.325}{0.260\times 0.264} = 6.25 \text{ Hz}$$

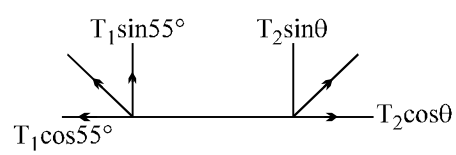
8. Ans. 4

Sol. $T_1 = 8 \times 10^8 \times 10^{-6} = 800$ N

$T_2 = 3 \times 10^8 \times 2 \times 10^{-6} = 600$ N

$$800 \times \frac{3}{5} = 600 \times \cos\theta$$

$$\Rightarrow \cos\theta = 4/5 \Rightarrow \theta = 37°$$



$$\Rightarrow 800 \sin 53° + 600 \sin 37° = F$$

$$\Rightarrow 800 \times \frac{4}{5} + 600 \times \frac{3}{5} = F$$

$$\Rightarrow F = 1000 \text{ N}$$

PART-2 : CHEMISTRY — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	B,C	B,D	D	B,D	D	D	A,C,D	A,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		Q,S	P,R,T	Q,R,T	Q,S		Q,S	Q,S,T	R,S	P,S,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	3	4	1	6	1	4	7		

SOLUTION

SECTION–I

1. Ans. (B)

2. Ans. (B)

3. Ans. (B, C)

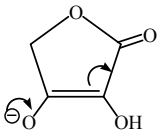
4. Ans. (B, D)

5. Ans. (D)

6. Ans. (B, D)

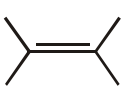
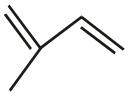
7. Ans. (D)

DOU = 3 (So, option A & C can not be answer)

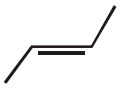


(Resonance stabilisation by extended conjugation)

8. Ans. (D)

HOC :  > 

(6c) (5c)

Stable :  > 

(4C, 1π) (4C, 1π)

In option (D) both compound has same number of carbon & π-bond. So, stability of compound is deciding factor for heat of hydrogenation as well as for heat of combustion.

9. Ans. (A,C,D)

Acid-base reaction favours in formation of weak acid & weak base. Acidic strength order is :

$CH_3COOH > Ph–OH > H_2O > HC\equiv CH$

10. Ans. (A,D)

In option (B) & (C), given pairs are not two different compounds they represent same ion whose resonance energy is fixed.

SECTION–II

1. Ans (A)-(Q,S); (B)-(P,R,T); (C)-(Q, R, T); (D)-(Q, S)

2. Ans (A) - (Q,S); (B) - (Q,S,T); (C) - (R,S); (D) - (P,S,T)

SECTION–IV

1. Ans. (1)

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2m.KE}} = \frac{\sqrt{h\lambda}}{\sqrt{2mC}}$$

$$= \sqrt{\frac{6\times10^{-3}\times9\times10^{-9}}{2\times9\times10^{-31}\times3\times10^{8}}} = 10^{-10} = 1Å$$

2. Ans. (3)

$$Z = \frac{V_{real}}{V_{ideal}}$$

$$1.5 = \frac{V_{real}}{2}$$

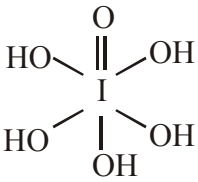
$$V_{real} = 3$$

3. Ans. (4)

4. Ans. (1)

5. Ans. (6)

Sol.



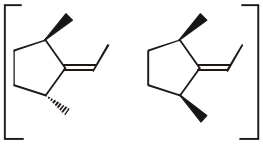
6. Ans. (1)

Sol. sp^3 hybridised P atom = 3

sp^2 hybridised N atom = 3

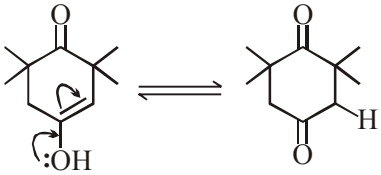
$$\text{ratio} = \frac{3}{3} = 1$$

7. Ans. (4)

(iii) , (iv)  , (v) , (vii)

8. Ans. (7)

(i) , (ii) , (iii) , (iv) , (vi) , (vii) , (ix)



PART-3 : MATHEMATICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	A,C	A,B,C,D	A,B	A,B	A,C	B,C	A,C	A,B,C,D	B,C
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R	Q	T	T		Q	S,T	Q	T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	7	6	3	7	3	4	8	3		

SOLUTION

SECTION-I

1. Ans. (A,D)

$$\left(\sqrt{x}+\frac{1}{2\sqrt[4]{x}}\right)^n$$

$$T_{r+1}={}^nC_r.\left(\sqrt{x}\right)^{n-r}\left(\frac{1}{2\sqrt[4]{x}}\right)^r$$

$$=\frac{{}^nC_r}{2^r}.x^{\frac{n-r}{2}}.\frac{1}{x^{r/4}}=\frac{{}^nC_r}{2^r}.x^{\frac{2n-3r}{4}}$$

$\frac{{}^nC_0}{2^0},\frac{{}^nC_1}{2^1},\frac{{}^nC_2}{2^2}$ are in A.P

$${}^nC_1=1+\frac{{}^nC_2}{4}\Rightarrow n=\frac{1+n(n-1)}{8}$$

$n^2-9n+8=0$

$\Rightarrow n=1,8$

$\therefore n=8$

$$\therefore\ T_{r+1}=\frac{{}^8C_r}{2^r}.x^{\frac{16-3r}{4}}$$

$\Rightarrow r=0,4,8$

2. Ans. (A,C)

(A) $S_1\equiv x^2+y^2+24x-10y+a=0$

for real circle, $g^2+f^2-c\geq 0$

$144+25-a\geq 0$

$a\leq 169$

Also $a\geq 0$

$\therefore$ Total non-negative integral values of $a=170$

(B) for no point in common $c_1c_2>r_1+r_2$

& $c_1c_2<|r_1-r_2|$

$c_1c_2=13$

$13>\sqrt{169-a}+6$

$\Rightarrow\ 169-a<49$

$a>120$ & $a\leq 169$

So in this condition we have 49 integral values of a

But from $c_1c_2<|r_1-r_2|$,

we will get additional values of a.

for orthogonal cut

$2.12.0+2(-5).0=-36+a\ \Rightarrow\ a=36$

If $a=0$, $c_1c_2=13$ & $r_1+r_2=19$

$c_1c_2<r_1+r_2$

No. of common tangent = 2

3. Ans. (A,B,C,D)

$$\sin x+\sin y=\frac{96}{65}$$

$$\Rightarrow\ 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{96}{65}\quad .....(1)$$

Also $\cos x+\cos y=\frac{72}{65}$

$$\Rightarrow\ 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{72}{65}\quad .....(2)$$

(1) ÷ (2), we get

$$\tan\left(\frac{x+y}{2}\right)=\frac{96}{72}=\frac{8}{6}=\frac{4}{3}$$

$$\text{Now, }\sin(x+y)=\frac{2.\frac{4}{3}}{1+\frac{16}{9}}=\frac{24}{25}$$

$$\cos(x+y)=-\frac{7}{25}$$

Square and add (1) & (2)

$$\cos^2\left(\frac{x-y}{2}\right).4=\frac{8^2(144+81)}{65^2}$$

$$\cos^2\left(\frac{x-y}{2}\right)=\frac{16.3^2.5^2}{13^2.5^2}$$

$$\therefore\quad \cos\left(\frac{x-y}{2}\right)=\pm\frac{12}{13}$$

4. Ans. (A,B)

$$S = \sum_{k=0}^{2014} {}^{2014}C_k . k = 2014.2^{2013}$$

$$= 1007.2^{2014} = 19 \times 53 \times 2^{2014}$$

& $E = 53 \times 2^{2014}$

$\therefore$ Highest exponent of $2 = 2014$

Number of divisors of $E = 2015 \times 2 = 4030$

5. Ans. (A,B)

$$\frac{1}{x-1} - \frac{4}{x-4} - \frac{5}{x-5} + \frac{8}{x-8} = \frac{6x^2 - 27x}{40}$$

Now,

$$\left(\frac{1}{x-1}+1\right) - \left(\frac{4}{x-4}+1\right) - \left(\frac{5}{x-5}+1\right)$$

$$+\left(\frac{8}{x-8}+1\right) = \frac{6x^2-27x}{40}$$

$x = 0$

Clubbing 1st & last &2nd & 3rd, we get

$$\frac{2x-9}{(x-1)(x-8)} - \frac{2x-9}{(x-4)(x-5)} = \frac{3}{40}(2x-9)$$

$$x = \frac{9}{2} \Rightarrow \frac{1}{(x-1)(x-8)} - \frac{1}{(x-4)(x-5)} = \frac{3}{40}$$

Solving, we get $x = 9$

Now, we can verify the options.

6. Ans. (A,C)

Put $2\sqrt{2}\,|\sin^3 x| = a,\ |\tan^3 x| = b,\ |\cot^3 x| = c$

we get $\dfrac{a}{b+c} + \dfrac{b}{c+a} + \dfrac{c}{a+b} = \dfrac{3}{2}$

$$\Rightarrow \quad \frac{a+b+c}{b+c} + \frac{b+c+a}{c+a} + \frac{c+a+b}{a+b} = \frac{9}{2}$$

$$(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) = \frac{9}{2}$$

Using AM $\geq$ HM,

$$\frac{(a+b)+(b+c)+(c+a)}{3} \geq \frac{3}{\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}}$$

$$(a+b+c)\left(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}\right) \geq \frac{9}{2}$$

$a = b = c$

$2\sqrt{2}\,|\sin^3 x| = |\tan^3 x| = |\cot^3 x|$

$\therefore \quad x = 2n\pi \pm \dfrac{\pi}{4}\ (n \in I)$

$\therefore$ Number of solution in $[0, 4\pi]$ is 8

(C) $n(A) = 6$; $n(B) = 10$

$\therefore$ Number of functions $= 10^6$

(D) Sum of 4 (+)ve solutions

$$\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} = 4\pi$$

7. Ans. (B,C)

Rationalise

$$f(x) = \frac{1}{2}\left[(x+1)^{1/3} - (x-1)^{1/3}\right]$$

$$f(1) = \frac{1}{2}\left[2^{1/3} - 0^{1/3}\right]$$

$$f(3) = \frac{1}{2}\left[4^{1/3} - 2^{1/3}\right]$$

$$f(5) = \frac{1}{2}\left[6^{1/3} - 4^{1/3}\right]$$

$$f(999) = \frac{1}{2}\left[(1000)^{1/3} - (999)^{1/3}\right]$$

$$\therefore\ E = \frac{1}{2} \times 10 = 5$$

Now, for (c) part

$T_{r+1} \gtreqless T_r$

$${}^7C_r \cdot \left(\frac{1}{3}\right)^r \gtreqless {}^7C_{r-1}\left(\frac{1}{3}\right)^{r-1}$$

$$\frac{1}{r} \cdot \frac{1}{3} \gtreqless \frac{1}{8-r}$$

$8 - r \gtreqless 3r$

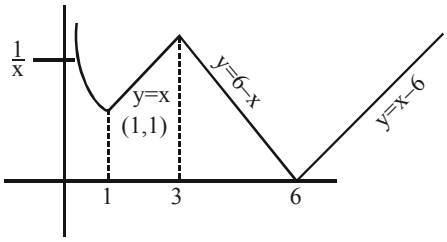
$2 \gtreqless r$

$\therefore$ Greatest terms are T_2 & T_3.

8. Ans. (A,C)

$$f(x) = \begin{cases} \dfrac{1}{x} & 0 < x \leq 1 \\ x & 1 < x \leq 3 \\ 6 - x & 3 < x \leq 6 \\ x - 6 & x > 6 \end{cases}$$

Plotting $f(x)$ we get



Clearly, if a,b,c,d are positive distinct numbers such that $f(a) = f(b) = f(c) = f(d)$, then
$y = t \in (1,3)$ must intersect
the graph of $y = f(x)$ at four points

$\therefore\ a = \frac{1}{t}, b = t, c = 6 - t, d = t + 6$

$\therefore\ abcd = 36 - t^2$

$t^2 \in (1,9)$

$\therefore$ Range of abcd is (27,35)

$\therefore$ Number of integral values in the range 7.

9. Ans. (A,B,C,D)

(A) $f(x)$ is odd $\therefore\ a + 2 = -b + 7 \Rightarrow a + b = 5$

(B) Clearly $x^2 \in (0,1) \Rightarrow [x^2] = 0$

$$\Rightarrow f(x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \text{even}$$

(C) for sum of coefficients; put x = 1 $\therefore\ f(0)=1$

10. Ans. (B,C)

$$\angle D + \angle E + \angle F = \frac{15\pi + \pi}{32} = \frac{\pi}{2}$$

$$\therefore \tan(D + E + F) = \frac{s_1 - s_3}{1 - s_2}$$

$\therefore\ s_2 = 1$

$\therefore$ tanD tanE + tanE tanF + tanF tanD = 1

& cotD + cotE + cotF = cotD cotE cotF

SECTION – II

1. Ans. (A)→(R); (B)→(Q); (C)→(T); (D)→(T)

(A) $\sum_{n=0}^{\infty} 2^{-n+(-1)^n} = 2 + 2^{-2} + 2^{-1} + 2^{-4} + \ldots\infty$

$$= \left(2 + \frac{1}{2} + \frac{1}{2^3} + \ldots\infty\right) + \left(\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots\infty\right)$$

$$= \frac{2}{1 - \frac{1}{2^2}} + \frac{\frac{1}{2^2}}{1 - \frac{1}{2^2}} = \frac{2.4}{3} + \frac{1}{3} = 3$$

(B) Put sinx = t, we get

$$16t^3 = 14 + \sqrt[3]{t+7}$$

$\therefore\ f(t) = f^{-1}(t)$

$\sin x = 1 \Rightarrow x = 2n\pi + \frac{\pi}{2} (n \in I)$

$\therefore$ 2 solutions.

(C) $\tan^{-1}(2 \sin x) = \cot^{-1}(\cos x)$

$$\tan^{-1}(2\sin x) + \tan^{-1}(\cos x) = \frac{\pi}{2}$$

$\therefore\ \sin x > 0,\ \cos x > 0\ \&\ 2\sin x\cos x = 1$

$$\therefore\ x = 2n\pi + \frac{\pi}{4}$$

$\Rightarrow$ 5 solutions

(D) Sum of roots

$$= \frac{a^2 - a + 4}{a - 1} = a + \frac{4}{a-1} = a - 1 + \frac{4}{a-1} + 1 \geq 5$$

Minimum value is 5

2. Ans.(A)→(Q);(B)→(S,T);(C)→(Q); (D)→(T)

(A) $$\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{2}{\left(\sqrt{n+2} + \sqrt{n}\right)\left(1 + \sqrt{n+2}\sqrt{n}\right)}\right)$$

$$= \sum_{n=0}^{\infty} \tan^{-1}\left(\frac{\sqrt{n+2} - \sqrt{n}}{1 + \sqrt{n+2}\sqrt{n}}\right)$$

$$= \sum_{n=0}^{\infty} \left(\tan^{-1}\sqrt{n+2} - \tan^{-1}\sqrt{n}\right)$$

$$= \frac{3\pi}{4} \qquad \therefore \left[\frac{3\pi}{4}\right] = 2$$

(B) $c > |a - b| \Rightarrow c^2 > a^2 + b^2 - 2ab$

Similarly, $a^2 > b^2 + c^2 - 2bc$

$b^2 > a^2 + c^2 - 2ac$

$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca)$

$$2\frac{(a^2 + b^2 + c^2)}{ab + bc + ca} < 4$$

(C) $a^2(b + c) = b^2(a + c)$

$a^2b - b^2a + a^2c - b^2c = 0$

$ab(a - b) + c(a - b)(a + b) = 0$

$a \neq b$

$\therefore\ ab + ac + bc = 0$

Multiply both sides by (a – c),

we get $(a - c)(ab + ac + bc) = 0$

$a^2b + a^2c - ac^2 - bc^2 = 0$

$a^2(b + c) = c^2(a + b) = 2$

(D) $P^2 + Q^2 + R^2 + S^2$

$= \sin^2A\sin^2B + \sin^2C\cos^2A + \sin^2A\cos^2B + \cos^2A\cos^2C$

$= \sin^2A + \cos^2A = 1$

SECTION-IV

1. Ans. 7

$\sin(x-y)\sin(x+y) = \sin^2x - \sin^2y$

$\therefore E = \sin^2(\sin^{-1}(0.5)) - \sin^2(\sin^{-1}(0.4)) = 0.09$

2. Ans. 6

Interpret the problem geometrically consider n right triangle joined at their vertices with bases $a_1, a_2, a_3, \ldots\ldots, a_n$ and heights 1,3,.....,2n–1. The sum of their hypotenusses is the value of S_n. The minimum value of S_n, then is the length of the straight line. Connecting the bottom vertex of first right triangle & the top vertex of the last right triangle, so

$$S_n \geq \sqrt{\left(\sum_{k=1}^{n}(2k-1)^2\right) + \left(\sum_{k=1}^{n} a_k\right)^2}$$

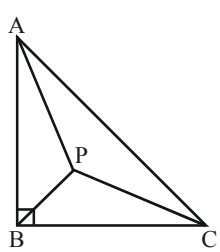
$\therefore S_n \geq \sqrt{17^2 + n^4}$

Now $17^2 + n^4 = m^2$ $(m \in I)$

$(m - n^2)(m + n) = 289.1$

$\therefore n^2 = 144 \therefore n = 12$

3. Ans. 3



using pythogoreas theorem

$AB^2 + BC^2 = CA^2$

$10^2 + 6^2 + 10.6 + 6^2 + PC^2 + 6PC$

$= 10^2 + PC^2 + 10PC$

$\Rightarrow 4PC = 132 \Rightarrow PC = 33$

4. Ans. 7

$$\sum_{n=1}^{2015}(-1)^n\left(\frac{n}{(n-1)!} + \frac{n+1}{n!}\right)$$

$$= \left(-1 - \frac{2}{1!}\right) + \left(\frac{2}{1!} + \frac{3}{2!}\right) - \left(\frac{3}{2!} + \frac{4}{3!}\right) + \ldots\ldots$$

$$-\left(\frac{2015}{(2014)!} + \frac{2016}{(2015)!}\right)$$

$$= -1 - \frac{2016}{(2015)!} \qquad \therefore \quad a+b+c = 4032$$

5. Ans. 3

Order at A,B,C is one

$${}^6C_3.3! = \frac{6!}{3!} \quad \therefore n = 3$$

6. Ans. 4

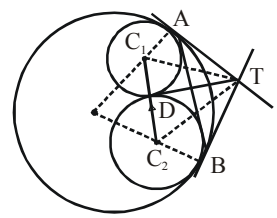
Let $x = k\sin\theta$ & $y = k\cos\theta$

$$\therefore \frac{\cos^4\theta}{\sin^4\theta} + \frac{\sin^4\theta}{\cos^4\theta} = \frac{194\sin\theta\cos\theta}{\sin\theta\cos\theta\left(\cos^2\theta + \sin^2\theta\right)} = 194$$

$$\therefore t = \frac{x}{y} + \frac{y}{x}, \text{ then } \left(t^2 - 2\right)^2 - 2 = 194$$

$\therefore t = 4$

7. Ans. 8



Let D be the point of tangency of C_1 and C_2. T will be radical center of 3 circles.

$\therefore TD = 4$

$$\text{Now, } \tan\left(\frac{\angle ATD}{2}\right) = \frac{1}{2} \text{ \& } \tan\left(\frac{\angle BTD}{2}\right) = \frac{3}{4}$$

$$\therefore \text{Radius of } C_3 = TA\tan\left(\frac{\angle ATB}{2}\right) = 8$$

8. Ans. 3

$${}^nC_m.{}^mC_p = \frac{n!}{m!(n-m)!} \times \frac{m!}{(m-p)!p!} \times \frac{(n-p)!}{(n-p)!}$$

$$= {}^nC_p.{}^{n-p}C_{m-p}$$

$$\therefore \sum_{p=1}^{n}\sum_{m=p}^{n} {}^nC_p\,{}^{n-p}C_{m-p} = \sum_{p=1}^{n} {}^nC_p\left({}^{n-p}C_0 + {}^{n-p}C_1 + \ldots + {}^{n-p}C_{n-p}\right)$$

$$= \sum_{p=1}^{n} {}^nC_p.2^{n-p} = \sum_{p=0}^{n} {}^nC_p 2^{n-p} - 2^n$$

$= 3^n - 2^n = 19$

$\therefore n = 3$