

# HINTS & SOLUTIONS

## PAPER-1

### PART-I : (PHYSICS)

1. (C)

Sol.  $\frac{hc^2}{x}$  must be dimensionless putting down all the dimensions

will get  $x = ML^4T^{-3}$

2. (C)

Sol. Path AC is  $y = x$ ; thus  $dW = \vec{F} \cdot d\vec{r}$

$$W = (2x\hat{i} + y^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int_0^1 2xy dx + \int_0^1 y^2 dy = \int_0^1 2x^2 dx + \int_0^1 y^2 dy = \frac{2}{3} + \frac{1}{3} = 1J$$

3. (A)

Sol. at the corner

$$mg = \mu N_1 + N_2 \quad \dots(1)$$

$$N_1 = \mu N_2 \quad \dots(2)$$

and torque

$$\tau_{net} = \mu N_1 R + \mu N_2 R$$

$$\frac{2}{5}mR^2\alpha = \mu R(N_1 + N_2) \quad \dots(3)$$

$$\text{from equation (3)} \quad \alpha = \frac{5\mu R(N_1 + N_2)}{2mR^2}$$

$$0^2 - \omega^2 = 2\alpha\theta$$

hence calculate ' $\theta$ '.

4. (B)

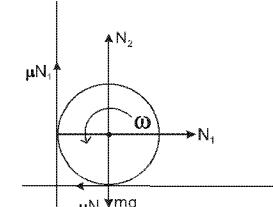
Sol. The frequency heard by the observer is

$$f' = f_0 \left[ \frac{v}{v - u \cos \theta} \right]$$

from the figure its clear that

$$\cos \theta = \frac{u}{v}$$

$$\text{hence } f' = f_0 \left[ \frac{v^2}{v^2 - u^2} \right]$$



5. (A)

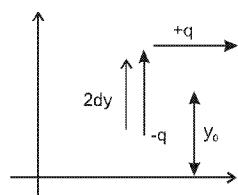
$$\text{Sol. } \tau = q(y_0 + dy)^2 \alpha$$

$$= q \alpha [4y_0 dy]$$

$$= q (2dy) \alpha 2y_0$$

$$= P \times 2 \alpha y$$

$$= 2 \alpha y_0 p$$



6. (B)

7. (B)

8. (C)

$$\text{Sol. } I_{net} = 4I_0 \cos^2 \phi / 2$$

75% of max means

$$3I_0 = 4I_0 \cos^2 \phi / 2$$

$$\frac{\phi}{2} = \frac{\pi}{6} \Rightarrow \phi = \frac{\pi}{3}$$

$$3I_0 = 4I_0 \cos^2 \left( \frac{\pi y}{\beta} \right)$$

$$\frac{\beta}{2} = \cos \left( \frac{\pi y}{r} \right)$$

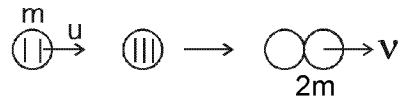
$$\frac{\pi y}{\beta} = \frac{\pi}{6}$$

$$y = \frac{\beta}{6} = \frac{\beta l}{6d} = \frac{1 \times 6 \times 10^{-7}}{6 \times 2 \times 10^{-3}} = \frac{1}{2} \times 10^{-4}$$

$$\text{hence } y = \frac{\lambda D}{6d}$$

9. (A,D)

Sol. If the collision is perfectly in elastic, then



$$v = \frac{u}{2}$$

$$K_{Lost} = \frac{1}{2}mu^2 - \frac{1}{2}(2m)v^2 = \frac{1}{4}mu^2$$

10. (B,C,D)

$$\text{Sol. } \vec{r} = (\hat{i} + 2\hat{j}) \sin \omega_0 t$$

$$x = \sin \omega_0 t$$

$$y = 2 \sin \omega_0 t$$

$$\text{eliminating } \sin \omega_0 t \Rightarrow x = \frac{y}{2}$$

equation of straight line

as its sinusoidal function hence periodic and simple harmonic.

11. (A,B,D)



Conserving the angular momentum about the point of contact

$$\frac{mR^2}{2}\omega_0 = \frac{mR^2}{2}\omega_1 + mv_1R$$

$$\omega_0 = \omega_1 + 2v_1 \quad [\text{as } u_1 = \omega_1 R]$$

$$\omega_1 = \frac{\omega_0}{3}$$

12. (C,D)

$$\text{Sol. } \frac{\frac{d}{2} \times d}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$$

13. (B,C)

$$\text{Sol. Acceleration } f = a - bV = \frac{dV}{dt} = \frac{dV}{a - bV} = dt, \int_0^V \frac{dV}{b - 2V} = \int_0^t dt$$

$$-\frac{1}{2} \left\{ \ln(6 - 2V) \right\}_0^V = t \Rightarrow \ln \frac{(6 - 2V)}{6} = -2t$$

$$\Rightarrow V = 3(1 - e^{-2t}) \quad e^{-2t} = \frac{1}{V}$$

$$\text{acceleration } f = 6e^{-2t}$$

14. (B,C)

15. (A)

**Sol.** from the figure

$$\sin \theta = \frac{d}{R}$$

$$\text{as } R = \frac{mu_0}{qB_0}$$

$$\text{hence } \sin \theta = \frac{d(qB_0)}{mu_0}$$

16. (B)

$$\text{Sol. Time spent} = \frac{\text{Arc length}}{u_0} = \frac{R\theta}{u_0}$$

17. (C)

**Sol.** average impulse is always equal to change in momentum

$$\vec{J} = \Delta \vec{p}$$

$$\text{as } d = \frac{\sqrt{3}mu_0}{2qB_0} \quad \text{hence } \theta = \frac{\pi}{3}$$

$$\Delta \vec{p} = \left( mu_0 \cos \frac{\pi}{3} \hat{i} + mu_0 \sin \frac{\pi}{3} \hat{j} \right) - mu_0 \hat{i}$$

$$\Delta \vec{p} = -\frac{mu_0}{2} \hat{i} + \frac{mu_0 \sqrt{3}}{2} \hat{j}$$

thus  $|\Delta \vec{p}| = mu_0$

18. (D)

**Sol.** A – only true if the mirror is at rest

R – always true

19. (C)

$$\text{Sol. A} - \text{Always true as } \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

R – Only true if transformer is ideal that is the efficiency is 100%

20. Ans. (A) – q,r,t ; (B) – q,s ; (C) – q,s,t ; (D) – q,r,t

$$\text{Sol. (A)} \quad P \propto \frac{1}{V} \Rightarrow PV = C$$

isothermal process. Hence  $\Delta u = 0$

as the process is isothermal hence Specific Heat is infinite.

(B) As  $T \neq C$  hence  $\Delta u \neq 0$  and

as process is isochoric hence C is  $= C_v$

(C)  $P = C$  isobaric hence  $\Delta T \neq 0$

Thus  $\Delta u \neq 0$

As process is isobaric hence  $C = C_p$

(D) as cyclic process hence  $\Delta u = 0$ . as  $dQ$  is non zero hence C is  $\neq 0$

## PART-II : (CHEMISTRY)

21. (B)

$$\text{Sol. } C_{p,m} = C_{v,m} + R$$

$$= 21.686 + 10^{-3}T + 8.314 = (30 + 10^{-3}T)$$

$$\Delta H = \int_{T_1}^{T_2} n C_p dT = 1 \int_{300}^{400} (30 + 10^{-3}T) dT = 3035J$$

22. (C)

$$\text{Sol. } t_1 = \frac{(t_{1/2})_1}{0.693} \ln \left( \frac{1}{1 - \frac{1}{4}} \right)$$

$$t_2 = \frac{(t_{1/2})_2}{0.693} \ln \left( \frac{1}{1 - \frac{3}{4}} \right)$$

$$\frac{t_1}{t_2} = \frac{1}{0.602} = \frac{(t_{1/2})_1}{(t_{1/2})_2} \frac{\ln \left( \frac{4}{3} \right)}{\ln(4)} \Rightarrow \frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{8}{1}.$$

23. (B)

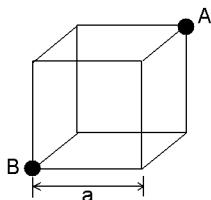
$$\text{Sol. } m = \frac{\Delta T}{k_b} \Rightarrow \frac{1}{0.51} = \frac{\Delta T_f}{k_f} \Rightarrow \Delta T_f = 3.647$$

Since freezing point undergoes a depression, hence freezing point is  $-3.647^\circ\text{C}$ .

24. (B)

Sol. Octahedral void is present at the center of cube while tetrahedral void is present at  $\frac{1}{4}th$  distance along each

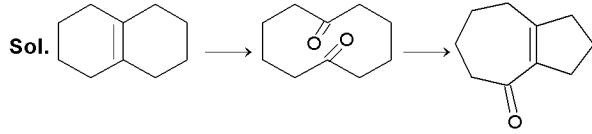
body diagonal. So distance between both =  $\frac{\sqrt{3}a}{4}$  :  $AB = \frac{\sqrt{3}a}{4}$



distance between octahedral & tetrahedral void  $\frac{AB}{4} = \frac{\sqrt{3}a}{4}$

25. (C)

26. (B)



27. (D)

Sol. Due to linear movement of electron by hyper conjugation of  $-\text{CH}_3$

28. (B)

Sol. In compound I : 1<sup>st</sup> and 4<sup>th</sup> carbon atom have cis configuration.  
In compound II : 1<sup>st</sup> and 4<sup>th</sup> carbon atom have trans configuration.

29. (ABC)

Sol. Smelting of iron in a blast furnace does not involve sublimation.

30. (AC)

31. (BCD)

Sol. Anode : Pb(s)

Cathode :  $\text{PbO}_2(\text{s})$

$\text{H}_2\text{SO}_4$  (conc.) about 38% solution of  $\text{H}_2\text{SO}_4$  is taken.

Anode :  $\text{Pb}(\text{s}) \rightarrow \text{Pb}^{2+}(\text{aq}) + 2\text{e}^-$

$\text{Pb}^{2+}(\text{aq}) + \text{SO}_4^{2-}(\text{aq}) \rightarrow \text{PbSO}_4(\text{s})$

$\text{Pb}(\text{s}) + \text{SO}_4^{2-}(\text{aq}) \rightarrow \text{PbSO}_4 + 2\text{e}^-$

most of the  $\text{PbSO}_4(\text{s})$  ppt sticks to the lead rod.

Cathode :  $2\text{e}^- + 4\text{H}^+ + \text{PbO}_2(\text{s}) \rightarrow \text{Pb}^{2+}(\text{aq}) + 2\text{H}_2\text{O}(\ell)$

$\text{Pb}^{2+}(\text{aq}) + \text{SO}_4^{2-}(\text{aq}) + 4\text{H}^+ + 2\text{e}^- \rightarrow \text{PbSO}_4(\text{s}) + 2\text{H}_2\text{O}(\ell)$

$\text{PbSO}_4(\text{s})$  sticks to cathode rod.

Overall reaction :  $\text{Pb}(\text{s}) + \text{PbO}_2(\text{s}) + 2\text{H}_2\text{SO}_4(\text{aq}) \rightarrow 2\text{PbSO}_4(\text{s}) + 2\text{H}_2\text{O}(\ell)$ ,  $E_{\text{cell}} = 2.05 \text{ V}$

32. (ABD)

33. (ABCD)

34. (ABC)

Sol. Ambident nucleophiles have more than one site of attack.

35. (A)

Sol. Electrophiles are  $\text{E}^-$  deficient species

36. (C)

Sol. III have maximum partial positive centre.

37. (B)

Sol. Due to 2 nitro groups at o- & p- position carbon directly attach to chlorine have max positive charge.

38. (D)

39. (B)

40. A - p, q, t ; B - p, r, t ; C - p, q, t ; D - s

Sol. (A) Beckmann rearrangement  
(B) Pinacol-pinacolone rearrangement  
(C) Dienone-phenol rearrangement  
(D) Oxymercuration demercuration

### PART-III : (MATHEMATICS)

41. (A)

Product of rational & irrational is integer only if rational is zero.  
Hence  $a = 0$  is only possible.

42. (D)

Arranging slopes in decreasing order.

$$3 > -\frac{1}{2} > -2$$

$$\tan A = \frac{3 + \frac{1}{2}}{1 - \frac{3}{2}} = -7 \Rightarrow A = \pi - \tan^{-1} 7$$

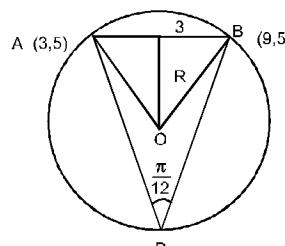
This is largest angle as it is obtuse.

43. (A)

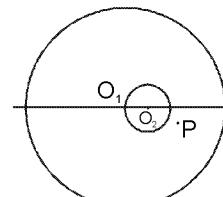
Chord AB subtend  $30^\circ$  at center

$$R = \frac{3}{\sin 15^\circ} = \frac{3 \cdot 2\sqrt{2}}{\sqrt{3}-1} \approx 6 \times 1.95 \sim 11.70$$

Thus  $[R] = 11$



44. (D)



$$PO_1 + PO_2 = r_1 - r_2 = 5 - 1 = 4$$

$$O_1 \equiv (0,0), O_2 \equiv (1,0)$$

$$2a = 4, 2ae = 1$$

$$e = 1/4$$

45. (C)

$$[\bar{a} \bar{b} \bar{c}] = -1 \text{ for unit vectors } \bar{a}, \bar{b}, \bar{c}$$

$\Rightarrow \bar{a}, \bar{b}, \bar{c}$  are mutually perpendicular.

$$\bar{a} = x(\bar{b} \times \bar{c}) + y(\bar{c} \times \bar{a}) + z(\bar{a} \times \bar{b})$$

Taking dot with  $\bar{a}$ ,

$$1 = x[\bar{a} \bar{b} \bar{c}] \Rightarrow x = -1$$

Taking dot with  $\bar{b}$  &  $\bar{c}$  we get  $y = 0 = z$ .

46. (B)

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \frac{1}{e} \text{ are in A.P.}$$

$$\Rightarrow \frac{a+b+c+d+e}{a}, \frac{a+b+c+d+e}{b}, \frac{a+b+c+d+e}{c},$$

$$\frac{a+b+c+d+e}{d}, \frac{a+b+c+d+e}{e} \text{ are in A.P.}$$

$$\Rightarrow \frac{b+c+d+e}{a}, \frac{a+c+d+e}{b}, \frac{a+b+d+e}{c},$$

$$\frac{a+b+c+e}{d}, \frac{a+b+c+d}{e} \text{ are in A.P.}$$

47. (B)

$$\sum_{r,s,k \in W, r+s+k=3} {}^3C_r \cdot {}^4C_s \cdot {}^5C_k = \text{number of ways of selection of 3 persons out of 12}$$

$$= {}^{12}C_3 = 220$$

48. (A)

Let  $P_n$  denotes the number of ways in which 'n' persons can be repeated as per condition,

$$\text{then } P_n = P_{n-1} + P_{n-2}$$

since  $P_1 = 1, P_2 = 2, P_3 = 3, P_4 = 5, P_5 = 8 \dots$  it is a fibonacci sequence. Total number of ways = 120

49. (B,C,D)

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

$$\Rightarrow 3 - \cos 2A - \cos 2B - \cos 2C = 4$$

$$\Rightarrow \cos 2A + \cos 2B + \cos 2C + 1 = 0$$

$$\Rightarrow 2\cos(A+B)\cos(A-B) + 2\cos^2 C = 0$$

$$\Rightarrow \cos A \cos B \cos C = 0$$

$$\Rightarrow A = \frac{\pi}{2}$$

50. (B,C)

$$A^2 = \begin{bmatrix} -iw & -iw^2 \\ iw^2 & iw \end{bmatrix} \begin{bmatrix} -iw & -iw^2 \\ iw^2 & iw \end{bmatrix}$$

$$= \begin{bmatrix} -w^2 + w & 0 \\ 0 & w - w^2 \end{bmatrix}$$

$$A^2 + 2I = \begin{bmatrix} -w^2 + w + 2 & 0 \\ 0 & -w^2 + w + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 2w & 0 \\ 0 & 3 + 2w \end{bmatrix} = (3 + 2w)I = (2 + \sqrt{3}i)I$$

$$= (1 - 2w^2)I$$

51. (B,C,D)

$$\Delta(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cosec x \\ \cos^2 x & \cos^2 x & \cosec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

$$C_2 \rightarrow C_2 - \cos^2 x C_1$$

$$= \begin{vmatrix} \sec x & 0 & \sec^2 x + \cot x \cosec x \\ \cos^2 x & \cos^2 x & \cosec^2 x \\ 1 & 0 & \cos^2 x \end{vmatrix}$$

$$= \cos^2 x \sin^2 x \left( \cos x - \sec^2 x - \frac{\cos x}{\sin^2 x} \right)$$

$$= \cos^3 x \sin^2 x - \sin^2 x - \cos^3 x = -\cos^5 x - \sin^2 x$$

$$\Delta(x) + \cos^5 x = -\frac{1}{4}$$

$$\Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \text{Ans. B, C, D}$$

52. (A,C)

$$D_f = R - \left\{ \frac{n\pi}{2}; n \in I \right\}$$

$$f'(x) = \frac{1}{\tan x \sec^2 x} \cdot \sec^2 x + \frac{\cot x}{\cosec^2 x} \cdot (-\cosec^2 x) = 0$$

$\Rightarrow f(x)$  is constant

$$\text{Also, } f\left(\frac{\pi}{4}\right) = 1$$

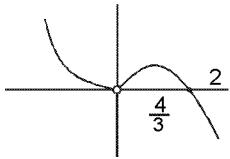
$$\therefore f(x) = 1 \forall x \in R - \left\{ \frac{n\pi}{2}; n \in I \right\}$$

53. (C,D)

$$\begin{aligned}
& \int \frac{3\cos^2 x + 2\cos x + 3\sin^2 x}{(2+3\cos x)^2} dx \\
&= \int \frac{3+2\cos x}{(2+3\cos x)^2} dx \quad (\text{dividing by } N_r \text{ & } D_r \text{ with } \sin^2 X) \\
&= \int \frac{3\operatorname{cosec}^2 x + 2\operatorname{cosec} x \cot x}{(2\operatorname{cosec} x + 3\cot x)^2} dx \\
&= \int \frac{1}{t^2} dt, t = 2\operatorname{cosec} x + 3\cot x \\
&= \frac{1}{t} + C = \frac{\sin x}{2+3\cos x} + C \\
&= \frac{\sin x}{2+3\cos x} - 2 + C = \frac{\sin x - 4 - 6\cos x}{2+3\cos x} + C \\
&= \frac{\sin x}{2+3\cos x} + 1 + C = \frac{\sin x + 2 + 3\cos x}{2+3\cos x} + C
\end{aligned}$$

54. (B,D)

$$\text{Vertex} = \left( -\frac{\lambda}{2t}, \frac{4t\mu - \lambda^2}{4t} \right) = (1, -2)$$



$$\Rightarrow \lambda = -2t, 4t\mu - \lambda^2 = -8t$$

$$4t\mu - 4t^2 = -8t \quad (\text{as } t \neq 0)$$

$$\therefore \mu = t - 2$$

$$\therefore f(t) = t(-2t)(t-2) = -2t^3 + 4t^2$$

$$f'(t) = -6t^2 + 8t = -2t(3t-4)$$

55. (D)

$$\frac{h(1)}{h(2)} \text{ is real} \Rightarrow \frac{f(1)}{g(1)} = \frac{f(2)}{g(2)}$$

56. (A)

$$\arg\left(\frac{h(1)+1}{h(1)-i}\right) = \frac{\pi}{4} \Rightarrow f^2(1) + g^2(1) = 1;$$

$$\arg\left(\frac{h(2)+2}{h(2)-2i}\right) = \frac{\pi}{4} \Rightarrow f^2(2) + g^2(2) = 4.$$

Consider the function  $y(x) = f^2(x) + g^2(x)$ . Applying LMVT to  $y(x)$  in  $[1, 2]$ , we get  $y'(c) = 3$  for atleast one  $c$  in  $(1, 2)$ .

But the first condition implies that  $y'(x)$  is strictly decreasing. Therefore the number of solutions is exactly one.

57. (A)

$\frac{h_1(x)}{h_2(x)}$  is purely imaginary. Thus  $f(x) = \frac{x}{16+x^3}$ . This function has one maxima and no minima.

58. (A)

Statement 2 is true

$$a+b+c=1, a^2+b^2+c^2=9$$

$$\rightarrow ab+bc+ca=-4$$

$$a^3+b^3+c^3-3abc=1.(9+4)$$

$$1-3abc=13$$

$$abc=-4$$

$$\Rightarrow \frac{ab+bc+ca}{abc}=1$$

59. (B)

Put  $x = y = 0$ , then

$$f(0) = \frac{f(0) + f(0) + 2011b}{2013}$$

$$\Rightarrow f(0) = b$$

$\Rightarrow$  Statement 2 is true (by substituting  $x = 2013$  and  $y = 0$ ).

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(2013h) - f(0)}{2013h} \quad \text{using the given relation}$$

$$= f'(0)$$

$$= a$$

$$\Rightarrow f(x) = ax + b \quad \text{Ans. B}$$

60. (A-r) ; (B-q) ; (C-s) ; (D-p)

(A) Foot of  $\perp$  from Q (1, 1, 1) to the line of intersection of

$$P_1, P_2 \text{ is } A\left(\frac{1}{2}, -1, \frac{1}{2}\right) \text{ as parallel vector of the line is}$$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{k} \equiv \hat{i} - \hat{k}$$

required plane is through A and  $\perp$  to AQ.

$$\overrightarrow{AQ} = \left( \frac{1}{2}, 2, \frac{1}{2} \right) \equiv (1, 4, 1)$$

Required plane is  $x + 4y + z = -3$

(B) required plane through Q has to be parallel to angle bisector planes.

$$\text{Angle bisector planes are } \frac{x+y+z}{\sqrt{3}} = \pm \frac{x-y+z-2}{\sqrt{3}}$$

i.e.,  $y = -1$  &  $x+z = 1$

$\therefore$  required plane is  $x + z = 2$  or  $y = 1$

(C) Plane through A line of  $P_1, P_2$  is

$$(x - y + z - 2) + \lambda(x + y + z) = 0$$

is  $\perp$  to  $x - y + z = 2$

$$1 + \lambda - (-1 + \lambda) + 1 + \lambda = 0$$

$$3 + \lambda = 0$$

$$\lambda = -3 \Rightarrow 2x + 4y + 2z + 2 = 0$$

(D)  $P_1(1,1,1) = 3$

$$P_2(1,1,1) = -1$$

$\therefore$  required bisector plane is

$$\frac{x+y+z}{\sqrt{3}} = -\frac{x-y+z-2}{\sqrt{3}}$$

$$\text{ie, } x+z=1$$

## PAPER-2

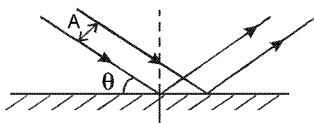
### PART-I : (PHYSICS)

1. (D)

Sol. For a perfectly reflecting surface the change in momentum for

$$\text{the light rays } \Delta p = \frac{2h}{\lambda} \cos(90^\circ - \theta)$$

$$\Delta p = \frac{2h}{\lambda} \sin \theta$$



$$\text{hence } F = \frac{2P}{c} \sin \theta \text{ (where } P \text{ is power)}$$

$$\text{Pressure} = \frac{\text{force}}{(\text{Area}_\perp)} = \frac{\frac{2P}{c} \sin \theta}{\frac{A}{\sin \theta}} = \frac{2I}{c} \sin^2 \theta$$

$$\text{as } \frac{\text{Power}}{\text{Area}} = \text{Intensity}$$

2. (D)

$$\text{Sol. } T = \frac{2\pi R}{v} \quad \dots(1)$$

$$\frac{Gm^2}{(2R)^2} = \frac{mV^2}{R}$$

$$m = \frac{4V^2 R}{G} = \frac{2V^3 T}{G\pi}$$

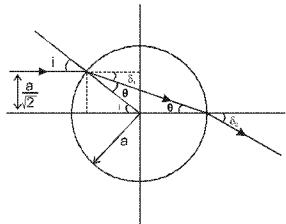
$m = \frac{2V^3 T}{G\pi}$

3. (C)

$$\text{Sol. } 1 \sin 45^\circ = \sqrt{2} \sin \theta$$

$$\theta = 30^\circ$$

at first refraction



$$\delta_1 = 45^\circ - 30^\circ = 15^\circ$$

and at second refraction

$$\delta_2 = 45^\circ - 30^\circ = 15^\circ$$

$$\text{thus } \delta_{\text{net}} = 15^\circ + 15^\circ = 30^\circ$$

4. (D)

Sol. As 'A' moves down by 'x' the spring get extended by 'x'

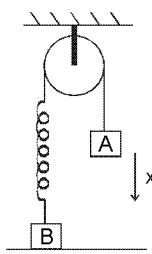
$$M_A g x = \frac{1}{2} K x^2$$

the block 'B' would get lifted if  $Kx = m_B g$

$$x = \frac{m_B g}{K}$$

$$\text{solving } \frac{M_A}{M_B} = \frac{1}{2}$$

5. (C)



$$\text{Sol. The time taken by the liquid to get empty is } t = \frac{A}{a} \sqrt{\frac{2h}{g}}$$

at any height 'y' the thrust force acting on the container is

$$\Rightarrow \rho a v^2 = \rho a (\sqrt{2gy})^2 = \rho a 2gy$$

This force accelerates the container ; thus

$$\rho a (2gy) = (\rho A y) \alpha \quad [\text{where '}\alpha\text{' is the acceleration}]$$

$$\alpha = \frac{a}{A} (2g)$$

$$\text{thus } v = 0 + \frac{a}{A} (2g) \frac{A}{a} \sqrt{\frac{2h}{g}} \Rightarrow v = 2\sqrt{2gh}$$

6. (D)

$$\text{Sol. } I = I_0 \cos^2 \frac{\pi y}{p}$$

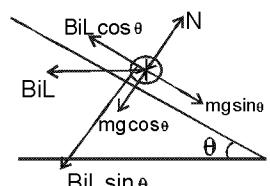
$$= I_0 \cos^2 \left( \frac{\pi \times \frac{B}{16}}{\beta} \right) = I_0 \times \frac{3}{4} \Rightarrow I = \frac{3I_0}{4}$$

7. (A)

$$\text{Sol. } N = mg \cos \theta + BiL \sin \theta$$

for balancing

$$mg \sin \theta = BiL \cos \theta + \mu N$$



$$\text{hence } B = \frac{mg}{il} \tan(\theta - \phi)$$

$$\text{as } \mu = \tan \phi$$

8. (C)

$$A_1 = A_0 e^{-\lambda t_1}$$

$$\text{Sol. } A_2 = A_0 e^{-\lambda t_2}$$

dividing

$$\frac{A_2}{A_1} = e^{-\lambda t_2 + \lambda t_1}$$

$$A_2 = A_1 e^{-\lambda(t_2 - t_1)} ; \left( \lambda = \frac{\ell n 2}{T} \right)$$

**9. (A,B,C,D)**

**Sol.** The potential inside the outer shell is same throughout hence (A)

The p.d between the shell decreases hence (B)

Earthing always is zero potential hence (C)

$$\text{If 'x' is the charge on inner sphere then } \frac{Kx}{R} + \frac{KQ}{2R} = 0$$

$$\therefore x = -\frac{Q}{2} \quad \text{Hence (D)}$$

**10. (B,C)**

**Sol.** By the weins law  $\lambda_m T = \text{constant}$

$$\lambda T = \frac{3}{4} \lambda T'$$

$$\boxed{T' = \frac{4}{3} T}$$

$$\text{Power } \propto (T^4)$$

**11. (B,C)**

$$\text{Sol. } \frac{2\pi}{\lambda} \times 1 = \frac{\pi}{8}$$

$$\lambda = 16 \text{ m}$$

**12. (B,C,D)**

$$\text{Sol. } r = \frac{mv}{qB}$$

$$\text{for He}^+ \Rightarrow r_1 = \frac{4mv}{qB}$$

$$\text{for O}^{2+} \Rightarrow r_2 = \frac{16mv}{2qB} = \frac{8mv}{qB}$$

neutron remains undeviated

**13. (A)**

**Sol.**  $S_1$  : The bodies would not collide at the mid point but at the centre of mass.

$S_2$  : Theory

$S_3$  : The momentum is a vector quantity thus the direction of motion can be different

$S_4$  : As kinetic energy before and after the collision is conserved and not during the collision.

**14. (C)**

**Sol.**  $S_1$  : True only if water is filled up to the brim in the vessel.

$S_2$  : Valid as  $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$S_3$  : Theory

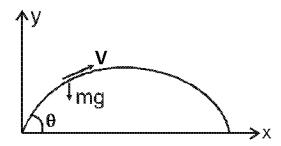
$S_4$  : Theory

**15.** The torque acting to the particle about the point of projection at any time would be

$$\tau = (mg \cos \theta) t$$

$$\text{as } \tau = \frac{dL}{dt}$$

$$\text{hence } \frac{dL}{dt} = mg \cos \theta t$$



$$L = mg \cos \theta \int_0^t t dt \quad L = \frac{mg \cos \theta}{2} \left[ \frac{9u^2}{16g^2} \right]$$

$$L = \frac{9\sqrt{3}mu^3}{64g}$$

$$\text{Ans. } x = 9$$

$$16. \quad x = a \cos \frac{\pi}{3}(t+1)$$

body comes to rest at the amplitude hence

$$x = \pm a \quad \Rightarrow \quad a = a \cos \frac{\pi}{3}(t+1)$$

$$\frac{\pi}{3}(t+1) = \pi$$

$$t+1 = 3 \quad \Rightarrow \quad t = 2 \text{ s} \quad \text{Ans. } t = 2$$

$$17. \quad f_1 - f_2 = 5$$

$$\sqrt{\frac{3RT}{M}} \left[ \frac{1}{2L_1} - \frac{1}{2L_2} \right] = 5$$

$$\sqrt{\frac{3R(3.24)}{M}} \left[ \frac{1}{2L_1} - \frac{1}{2L_2} \right] = f'$$

$$\text{dividing } \frac{1}{1.8} = \frac{5}{f'} \quad f' = 9$$

$$\text{Ans. } 9$$

**18.** The net resistance can be calculate by applying kirchoff's law applying in loop (1)

$$9(i - i_1) + 3(i_1) - V = 0$$

$$9i - 6i_1 = V \quad \dots(1)$$

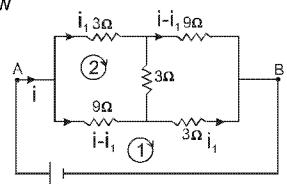
from loop (2)

$$3i_1 + 3(2i_1 - i) - 9(i - i_1) = 0$$

$$i_1 + 2i_1 - i - 3i + 3i_1 = 0$$

$$6i_1 = 4i$$

$$i_1 = \frac{2}{3}i \quad \dots(2)$$



$$9i - 6 \left[ \frac{2}{3} i \right] = V$$

$$5i = V \Rightarrow \frac{V}{i} = \frac{15}{3} \Omega$$

**Ans. 15**

19.  $x = 50 + 4t - t^2$

the particle stops when its velocity is zero.

$$\frac{dx}{dt} = 4 - 2t = 0$$

$$t = 2s$$

at 2s the x coordinate of particle is

$$x = 50 + 4(2) - (2)^2 = 54m$$

$$\text{at } t = 0$$

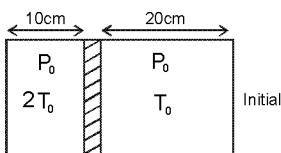
$$x = 50$$

hence displacement is 4m

**Ans. 4**

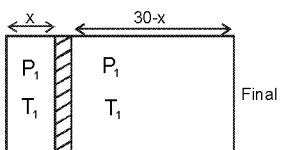
20. for chamber (1)  
equating number of moles

$$\frac{P_0(10)(A)}{R(2T_0)} = \frac{P_1 x(A)}{RT_1} \quad \dots(1)$$



for chamber (2)

$$\frac{P_0(20)(A)}{T_0} = \frac{P_1(30-x)A}{RT_1} \quad \dots(2)$$



dividing (1) by (2)

$$\frac{1}{4} = \frac{x}{30-x} \Rightarrow 30-x = 4x$$

$$5x = 30$$

Thus piston moves by 4cm

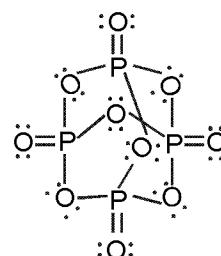
$$x = 6\text{cm} \quad \text{Ans. 4}$$

## PART-II : (CHEMISTRY)

21. (D)

$$\begin{aligned} \text{Sol. } P_T &= 200 \times 0.5 + 300 \times 0.5 \\ &= 100 + 150 = 250 \text{ torr.} \\ \text{If } P_{ex} &> 250, \text{ no vapour exist.} \\ \text{If } P_{ex} &\leq 250, \text{ vapour exist.} \end{aligned}$$

22. (B)



Sol.  $\text{P}_4\text{O}_{10}$

23. (B)

$$\text{Sol. n-factor} = \left( 3 - \frac{2}{0.9} \right) \times 0.9 = 0.7$$

$$\Rightarrow \text{eq.wt} = \frac{M}{0.7}$$

24. (A)

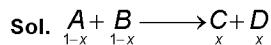
$$\text{Sol. } r_n \propto \frac{n^2}{z}$$

$$r_4(\text{He}^+) \propto \frac{4^2}{2} \text{ and } r_3(\text{He}^+) \propto \frac{3^2}{2} \Rightarrow (r_4 - r_3)_{\text{He}^+} \propto \left( \frac{16}{2} - \frac{9}{2} \right)$$

$$\text{Similarly for Li}^{2+}: (r_4 - r_3)_{\text{Li}^{2+}} \propto \left( \frac{16}{3} - \frac{9}{3} \right)$$

$$\text{so, ratio} = \frac{\left( \frac{16}{2} - \frac{9}{2} \right)}{\left( \frac{16}{3} - \frac{9}{3} \right)} = \frac{3}{2}$$

25. (A)



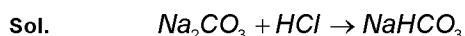
$$r = k(1-x)^{1/2} (1-x)^{1/2}$$

$$r = k(1-x) \Rightarrow \frac{dx}{dt} = k(1-x)$$

$$\Rightarrow t = \frac{1}{k} \ln \left( \frac{1}{1-x} \right) \Rightarrow$$

$$t = \frac{2.303}{2.303 \times 10^{-2}} \log \left( \frac{1}{0.01} \right) = 200 \text{ sec}$$

26. (B)



Initial mm	1	1.25	0
After reaction	0	0.25	1



Initial mm	1	0.25	0
After reaction	0.75	0	0.25

Its an acidic buffer  $pH = pK_{a_1} + \log \frac{[\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3]}$

$$= 6.4 + \log \frac{(0.75)}{(0.25)} = 6.7$$

27. (D)

Sol. carbon do not form 5 bonds.

28. (C)

Sol.  $\beta$ -diketone have higher enol content

29. (ACD)

30. (ABCD)

Sol. Self understood

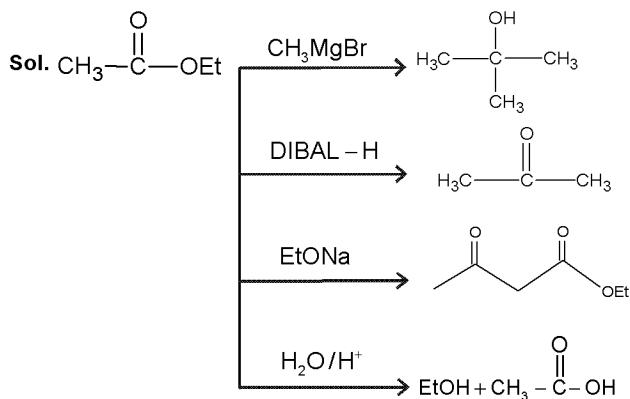
31. (ABCD)

Sol.  $\text{S}_{\text{N}1}$  and E1 reaction

32. (ABCD)

Sol. Self understood

33. (A)



34. (A)

35. (60)

Sol. Let us suppose total 100 volume of ozonised oxygen out of which  $V_1$  is of oxygen.

$$\text{O}_2 = V_1; \text{O}_3 = 100 - V_1$$

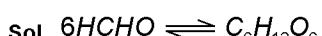
$$\text{Total mass} = V_1 \times 1.6 + (100 - V_1) 2.42$$

$$\text{nd, } \frac{V/220}{V/200} = \sqrt{\frac{d_{\text{O}_2}}{d_{\text{mix}}}} \Rightarrow d_{\text{mix}} = 1.93 \text{ gm/lit}$$

$$\Rightarrow 100 \times d_{\text{mix}} = V_1 \times 1.6 + (100 - V_1) 2.42$$

$$\Rightarrow V_1 = 60 \Rightarrow \text{O}_2 = 60\%$$

36. (03)



$$\begin{array}{ccc} \text{C} & & 0 \\ \text{C} (1-\alpha) & & \text{C}\alpha /6 \end{array}$$

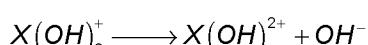
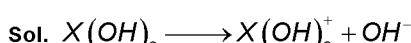
$$\text{Total mass} = [C(1-\alpha)]30 + \left(\frac{C\alpha}{6}\right)180$$

$$\text{Total moles} = C(1-\alpha) + \frac{C\alpha}{6}$$

$$\Rightarrow \text{mean molar mass} = \frac{[C(1-\alpha)]30 + \left(\frac{C\alpha}{6}\right)180}{C(1-\alpha) + \frac{C\alpha}{6}} = 150$$

$$\Rightarrow \text{Reported answer is } \frac{150}{50} = 3.$$

37. (50)



$$\Rightarrow \text{Total } (\text{OH})^- = (4 \times 10^{-3} + a)$$

$$\text{pH} = 11.78 \Rightarrow \text{pOH} = 2.22$$

$$\Rightarrow [\text{OH}^-] = 6 \times 10^{-3} \text{ M} \Rightarrow 4 \times 10^{-3} + a = 6 \times 10^{-3}$$

$$\Rightarrow a = 2 \times 10^{-3} \text{ M}$$

$\Rightarrow$  2<sup>nd</sup> dissociation is 50%

38. (34)

Sol. Volume strength =  $11.2 \times \text{Molarity}$

$$\Rightarrow 112 = 11.2 \times M$$

$$\Rightarrow M = 10$$

1lt of a solution contain 10moles  $\text{H}_2\text{O}_2$

1 mole of  $\text{H}_2\text{O}_2$  = 34gm

$$1\text{lt} \equiv 1000\text{ml}$$

$$1000\text{ml} \rightarrow 340\text{gm}$$

$$100\text{ml} \rightarrow 34\text{gm}$$

so  $34\%(\text{w/v})$

39. (10)

Sol. It shows geometrical, ionisation and linkage isomers.

$\text{NO}_2 - \text{NO}_2$  cis-trans.

$\text{NO}_2 - \text{ONO}$  cis-trans.

$\text{ONO} - \text{ONO}$  cis-trans.

$\text{NO}_2 - \text{NO}_3$  cis-trans.

$\text{ONO} - \text{NO}_3$  cis-trans.

Total number of isomers = 10.

40. (20)

$$\text{Sol. } \Delta G^{\circ}_{200} = \Delta H^{\circ}_{200} - T \Delta S^{\circ}_{200} \Rightarrow$$

$$\Delta H^{\circ}_{200} - 20 - 4 = 16 \text{ kJ/mol}$$

$$\Delta H_{T_2} = \Delta H_{T_1} + \Delta C_p (T_2 - T_1)$$

Putting the values, we get :  $\Delta C_p = 20 \text{ J/mol K}$

$$b\alpha + c\beta + a = 0$$

$c\alpha + a\beta + b = 0$  is consistent

$$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow a + b + c = 0$  as a,b,c are distinct

$\therefore ax + by + cz = 0$  passes through (0,0,0) and (1,1,1).

Thus it contain the line,  $x = y = z$ .

41. (D)

$$\frac{1}{h} = \frac{1}{t} + 1, k = 1 - \frac{1}{t}$$

$$\frac{1}{h} + k = 2$$

$$\text{Locus : } xy - 2x + 1 = 0$$

Which is a rectangular hyperbola .

42. (A)

43. (A)

$$\int_{\ln 2}^x \frac{e^{-\frac{x}{2}}}{\sqrt{1 - \left(e^{-\frac{x}{2}}\right)^2}} dx$$

$$= -2 \left[ \sin^{-1} \left( e^{-\frac{x}{2}} \right) \right]_{\ln 2}^x$$

$$-2 \sin^{-1} \left( e^{-\frac{x}{2}} \right) + 2 \sin^{-1} \left( e^{-\frac{\ln 2}{2}} \right) = \frac{\pi}{6}$$

$$-2 \sin^{-1} \left( e^{-\frac{x}{2}} \right) + 2 \sin^{-1} \left( \frac{1}{\sqrt{2}} \right) = \frac{\pi}{6}$$

$$-2 \sin^{-1} \left( e^{-\frac{x}{2}} \right) = -\frac{\pi}{3}$$

$$\sin^{-1} \left( e^{-\frac{x}{2}} \right) = \frac{\pi}{6}$$

$$e^{-\frac{x}{2}} = \frac{1}{2}$$

$$-\frac{x}{2} = -\ln 2$$

$$x = \ln 4 \quad \ln 4 = \frac{\ln a}{2} \quad \Rightarrow \quad a = 16$$

44. (A)

$$a\alpha + b\beta + c = 0$$

45. (B)

We have

$$\Delta = \pm \frac{i}{4} \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} \text{ is real for any } z_1, z_2, z_3$$

$$\Rightarrow \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix} \text{ is purely imaginary}$$

$\therefore$  Locus of z is y-axis.

46. (B)

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$$

$$2 + a = b$$

$$\text{and } 5 = b$$

$$\Rightarrow a^2 + b^2 = 34$$

47. (A)

ordered triplets =  $19 \times 7 = 133$

48. (A)

Let  $E_1 \rightarrow$  event that C's are separated

$E_2 \rightarrow$  event that all S's are separated

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$P(E_1) = \frac{300}{420}$$

$$P(E_1 \cap E_2) = \frac{96}{420}$$

$$\therefore P(E_2 | E_1) = \frac{96}{300} = \frac{8}{25}$$

49. (B,C)

$$2[x] + 64 = 3[x] - 192$$

$$[x] = 256 \Rightarrow p \in [256, 257)$$

$$\begin{aligned} q &= \sin \frac{\pi}{18} \sin \frac{3\pi}{18} \cdots \sin \frac{17\pi}{18} \\ &= \sin^2 \frac{\pi}{18} \sin^2 \frac{3\pi}{18} \sin^2 \frac{5\pi}{18} \sin^2 \frac{7\pi}{18} \\ &= \frac{1}{256} \quad \text{Ans. B,C} \end{aligned}$$

50. (A,B,D)

Solutions are  $y = mx + m^2$  where  $m$  is an arbitrary constant.

$$\text{Its singular solution is } y = \frac{-x^2}{4}.$$

Ans. A, B,D

51. (A,C)

$$y = g(x_0) \equiv C \equiv \frac{4}{e^2}$$

$$\text{Area} = \int_{-2}^0 \left( \frac{4}{e^2} - x^2 e^x \right) dx = \frac{18}{e^2} - 2. \Rightarrow A,C$$

52. (A,C)

$$\int_0^x [t] dt \quad x = n+f$$

$$= \int_0^n [t] dt + \int_n^{n+f} [t] dt$$

$$= \frac{(n-1)n}{2} + \int_n^{n+f} ndt$$

$$= \frac{(n-1)n}{2} + nf$$

$$= \frac{[x]^2}{2} - \frac{[x]}{2} + n(x-n) = -\frac{[x]^2}{2} - \frac{[x]}{2} + x[x]$$

Ans. A,C

53. (D)

$$S_1: |\text{adj } A| = |A|^2 \text{ if } A \text{ is } 3 \times 3$$

$$S_2: \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \Rightarrow \text{lines are concurrent}$$

when they are non parallel

$$S_3: \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$\Rightarrow$  System have infinitely many solutions other than (0,0,0), hence True

$S_4$ : If diagonal elements are zero the matrix is need not be skew symmetric.

54. (B)

$S_1$ : d.e of 2<sup>nd</sup> degree curves is of order 5 as number of independent arbitrary constants is 5.

$S_2$ : Between any two irrationals there are infinitely many rationals.

$$S_3: \lim_{n \rightarrow \infty} {}^n C_r p^r q^{n-r} = \frac{n!}{(n-r)! r!} p^r (1-p)^{n-r}$$

$$\lim_{n \rightarrow \infty} \frac{n(n-1)\dots(n-(r-1))}{r!} \frac{\lambda^r}{n^r} \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

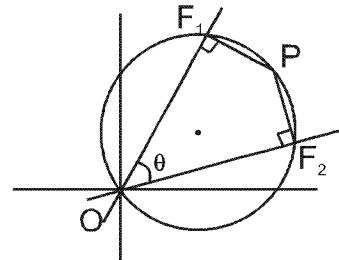
$$= \lim_{n \rightarrow \infty} 1 \cdot \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{(r-1)}{n}\right) \cdot \frac{\lambda^r}{r!} \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

$$= 1 \cdot 1 \cdot 1 \dots 1 \frac{\lambda^r}{r!} e^{-\lambda} = e^{-\lambda} \frac{\lambda^r}{r!}$$

$S_4$ : For the function  $f(x) = x^{\frac{2}{3}}$ ,  $f'(0^+)$  and  $f'(0^-)$  both does not exist, but  $f$  is continuous at 0.

55. (14)

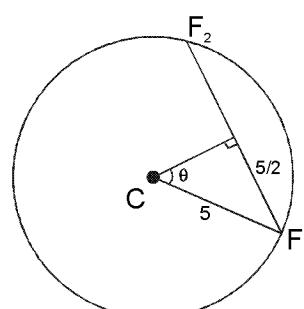
Consider circle with OP as diameter



$$x(x-6) + y(y-8) = 0$$

Let C be its centre &  $\theta$  be angle between lines

Radius = 5



hence  $CF_1F_2$  is equilateral triangle

Thus  $\angle F_1OF_2 = 30^\circ$

$$\text{Now } \tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \left| \frac{2\sqrt{4+a}}{a-1} \right|$$

$\Rightarrow a^2 - 14a - 49 = 0$ . Hence sum of all possible values of 'a' is 14.

56. (25)

$2x - y = 5$  is tangent to circle at  $(2, -1)$

$$\therefore S = (-2, 1)$$

$$\Rightarrow a = -2, b = 1$$

$$\Rightarrow d = 3$$

$$10^{\text{th}} \text{ term} = -2 + 9 \times 3 = 25$$

57. (03)

$$t_{n+1} = \sqrt{t_n + 6}$$

Taking limit  $n \rightarrow \infty$

$$\ell = \sqrt{\ell + 6} \Rightarrow \ell = 3$$

58. (21)

$$\left( x + 1 + \frac{1}{x} \right)^{10} = \frac{a_0}{x^{10}} + \frac{a_1}{x^9} + \dots + a_{20}x^{10}$$

Number of terms = 21

59. (00)

f is continuous every where

Therefore fof is also continuous every where

60. (00)

$$\text{Let } f(x) = \sin^{-1} x - x \quad x \in (0, 1]$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} - 1 > 0$$

So  $f(x)$  is increasing in  $(0, 1]$

$$\Rightarrow f(x) > f(0)$$

$$\Rightarrow \sin^{-1} x > x$$

$$\Rightarrow \frac{\sin^{-1} x}{x} > 1 \quad x \in (0, 1]$$

as  $\frac{\sin^{-1} x}{x}$  is even function so statement is also true for  $[-1, 0)$

$$\Rightarrow \frac{\sin^{-1} x}{x} + \frac{\sin^{-1} y}{y} + \frac{\sin^{-1} z}{z} > 3$$