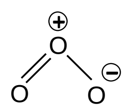
SOLUTIONS

CHEMISTRY

2. In the 8th period, the electrons will be filled in

8s, 8p, 7d, 6f and 5g orbitals (as per Aufbau rule)

So, total number of elements = 75.

3. 

4. Meq. of Cu = Meq. of I_2 = Meq. of $Na_2S_2O_3$

= 20 × 0.5 = 10

Meq. of Cu in 20 ml = 10

Meq. of Cu in 1000 ml = 500

Wt. of Cu $= \frac{500 \times 63.5}{1000} = 31.75$

$$2CuSO_4 + 4KI \longrightarrow Cu_2I_2 + 2K_2SO_4 + I_2$$ (n_f of Cu = 1)

% Cu $= \frac{31.75}{50} \times 100 = 63.5$

5. Diameter of bubble at the bottom = 3.6 mm

Volume of bubble at the bottom V_b = $\frac{4}{3}\pi\left(\frac{3.6}{2}\right)^3$

Volume of bubble at the surface V_s = $\frac{4}{3}\pi\left(\frac{4.0}{2}\right)^3$

$$\frac{V_b}{V_s} = \left(\frac{3.6}{4.0}\right)^3 = 0.729$$

Now, at the surface pressure is only due to atmospheric pressure, i.e. P_s = 76 cm, T_s = 273 + 40 = 313 K

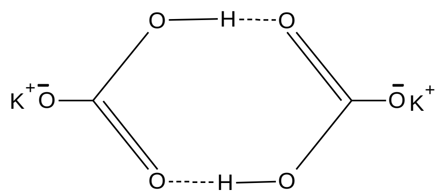
In the bottom pressure is due to atmospheric pressure + water in the lake, i.e.

$$P_b = 1 + \frac{250 \times 980}{1.01 \times 10^6} = 0.24 + 1 = 1.24 \text{ atm}$$

$$\frac{P_s V_s}{T_s} = \frac{P_b V_b}{T_b} \Rightarrow T_b = \frac{P_b V_b T_s}{P_s V_s} = 283.36 \text{ K}$$

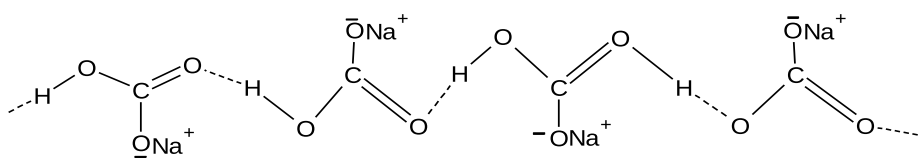
$= 10.36^{o}C$

6. The bicarbonate ions tend to be held together in crystal structures by hydrogen bonding giving layers of polymeric anions. The potassium salt, contains a dimeric anion (shown below).



(Structure of $KHCO_3$)

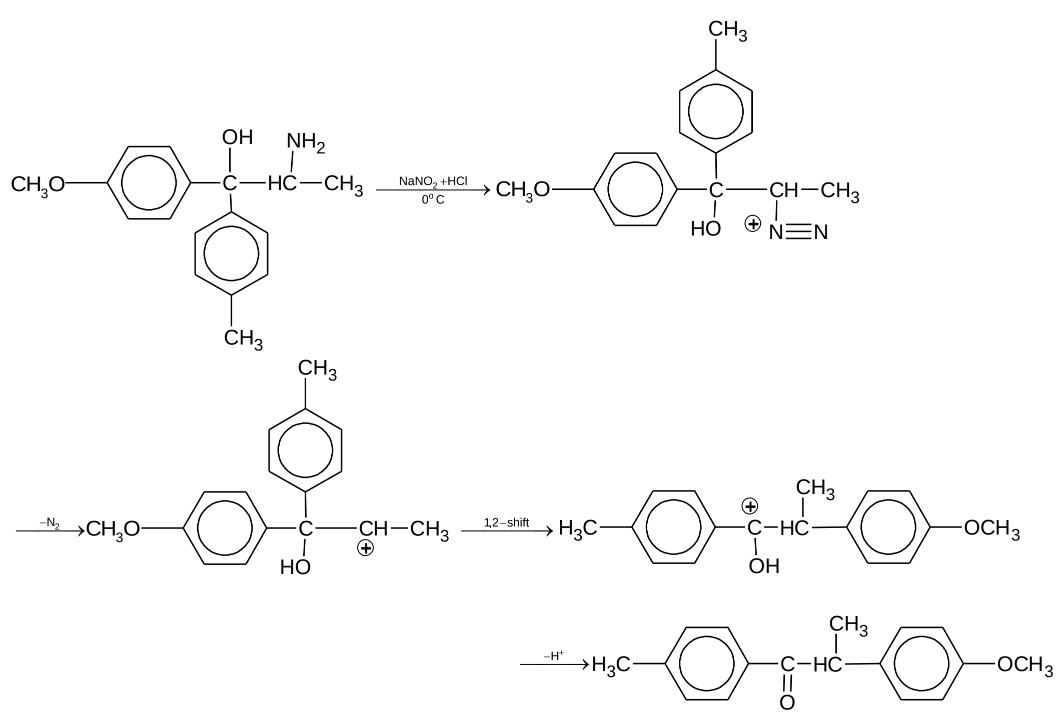
In sodium salt, however, the bicarbonate anions form an infinite chain (shown below)



(Structure of $NaHCO_3$)

Hence, (C).

7.



8. Baeyer-Villiger oxidation.

11. Less is the steric hindrance, higher is the rate of S_N2 reaction.

12. More stable the carbocation and better leaving group ability favours S_N1 reaction.

13. More the number of alkyl substitute at double bond, greater its thermodynamic stability.

14. C – H bond is broken in non rate determining step, therefore, substitution of α-H by deuterium doesn't affect the rate of reaction.

15. For a given pressure, Z is minimum for CO_2. Therefore CO_2 can be easily liquefied.

1. $SnCl_2 + Na_2CO_3 \rightarrow SnO + 2NaCl + CO_2$

   $H_2O_2 + H_2S \rightarrow 2\ H_2O + S$

   $SnO + 2\ NaOH \rightarrow Na_2SnO_2 + H_2O$

   $H_2O_2 + N_2H_4 \rightarrow N_2 + 4H_2O$

MATHEMATICS

1. Hint : S + λL = 0 is the equation of family of circles through Q and R.

Solution : Given circle is $x^2 + y^2 = 4$ ….(1)
QR is the chord of P(-3, 4) w.r.t. circle
$\therefore$ Equation of QR is x(-3) + 4(y) – 4 = 0
$\Rightarrow$ -3x + 4y – 4 = 0 …..(2)
$\therefore$ Equation of family of circles through the intersection of (1) and (2) is $\left(x^2 + y^2 - 4\right) + \lambda\left(-3x + 4y - 4\right) = 0$
this passes through P(-3, 4)
(9 + 16 – 4) + λ(9 + 16 – 4) = 0
$\Rightarrow$ 21 + 21λ = 0 $\Rightarrow$ λ = -1
$\therefore$ Equation of circumcircle of ΔPQR is $\left(x^2 + y^2 - 4\right) - 1\left(-3x + 4y - 4\right) = 0$

$\Rightarrow$ $x^2 + y^2 + 3x - 4y = 0$

2. Hint : On substituting the points in the given line we should get opposite sign

Solution : Let f(x, y) = x + y + 1, f(3, -2) = 3 – 2 + 1 = > 0

$f(a^2, a) = a^2 + a + 1 < 0$

This quadratic will have complex roots $a^2 + a + 1$ will always be positive $\because \Delta < 0 \therefore$ the two parts will never be on opposite

3. Hint : Equation of the chord with midpoint (x, y) is $T = S_1$.

Solution : y – 3x + 3 = 0 …..(1)

$T = S_1$

$yy_1 - 2\left(x + x_1\right) = y_1^2 - 4x_1$.

$yy_1 - 2x - y_1^2 + 2x_1 = 0$ ……..(2)

(1) and (2) represent the same line.

$$\frac{y_1}{1} = \frac{-2}{-2} = \frac{2x_1 - y_1^2}{3}$$

$y_1 = 1, \ 2x_1 - y_1^2 = 3$

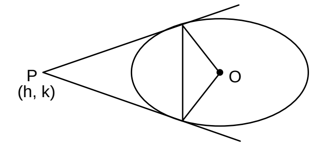
$2x_1 - 1 = 3$

$x_1 = 2$

$\left(x_1, y_1\right) = \left(2, 1\right)$.

4. Hint: Use Homogenization method.

Solution: Let P(h, k) be the point.



Then equation of chord of contact of P w.r.t. ellipse $x^2 + 2y^2 = 1$ ….(1)

is xh + 2yk = 1 ….(2)

Then the joint equation of lines joining origin to points of intersection of (1) and (2) is

$x^2 + 2y^2 = (xh + 2yk)^2$ [making (1) homogenous with the help of 2)]

The lines will be perpendicular if

$\left(1-h^2\right)+\left(2-4k^2\right)=0$

$\Rightarrow\ h^2 + 4k^2 = 3$

$\therefore$ Required locus is $x^2 + 4y^2 = 3$

5. $f(x) = \frac{x^2+3x+8}{x^2+6x+29}$

$= \frac{x^2+6x+29-3x-21}{x^2+6x+29}$

$= 1 - \frac{3(x+7)}{x^2+6x+29}$

$= 1 - \frac{3(x+7)}{(x+3)^2+20}$

f(x) is not one-one

f(x) is continuous:

domain of function is the set of real number

8. $a\sin x + 2\cos\left(x+\frac{\pi}{3}\right)$

$= a\sin x + 2\left[\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3}\right] = a\sin x + 2\left[\cos\frac{1}{2} - \sin x\frac{\sqrt{3}}{2}\right]$

$= \left(a-\sqrt{3}\right)\sin x + \cos x$

maximum value $\sqrt{\left(a-\sqrt{3}\right)^2 + 1^2} = 1$

$\left(a-\sqrt{3}\right)^2 + 1 = 1$

$a = \sqrt{3}$

9. Equation of the tangent is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

normal is ax cos θ + by cot θ = $a^2 + b^2$.

The normal at P meets the co-ordinate axes at $G\left(\frac{a^2+b^2}{a}\sec\theta,\ 0\right)$ and $g\left(0,\ \frac{a^2+b^2}{a}\tan\theta\right)$

$\therefore\ PG^2 = \left(\frac{a^2+b^2}{a}\sec\theta - a\sec\theta\right)^2 + (b\tan\theta - 0)^2$

$PG^2 = \frac{b^2}{a^2}\left(b^2\sec^2\theta + a^2\tan^2\theta\right)\cdot$

10. When $\tan\theta = 0$

$PG = \frac{b^2}{a}$.

11–12. Since the perpendicular bisectors of AD and BC become same line x = 1

$\Rightarrow$ x = 1 is the axis of the parabola

$\Rightarrow$ equation of the parabola is $y = ax^2 - 2ax + c$.

(– 1, 3), (0, 1) lies on it $\Rightarrow$ c = 1 and a = $\frac{2}{3}$

the parabola is $y = \frac{2}{3}x^2 - \frac{4}{3}x + 1 \Rightarrow$ vertex is $\left(1, \frac{1}{3}\right)$

directrix is $y + \frac{1}{4} = 0$.

Slope of the line AB is 1 $\Rightarrow t_3 = -1$

where t_3 is the point co-normal with A and B

$\Rightarrow$ the point of intersection of circles is (4, – 4).

13–14. $36\{x\}^2 = 6x\,[x] = 6([x] + \{x\})\,[x]$

$\Rightarrow (3\{x\} + [x])\,(2\{x\} - [x]) = 0$

$\therefore \{x\} = \frac{[x]}{2} \quad \because\ 2\{x\} + [x] \neq 0$

$\therefore 0 \le \frac{[x]}{2} < 1 \Rightarrow [x] = 1$

$\therefore \{x\} = \frac{1}{2}$ so $x = \frac{3}{2}$

$\therefore$ Terms are 3, 3, 3.

15. $x^2 - 2x + a = x - 1,\ x > 1$

$\Rightarrow x^2 - 3x + a + 1 = 0$

$\Delta > 0$

$x^2 - 2x + a + x - 1 = 0,\ x < 1$

$x^2 - x + a - 1 = 0$

$\Delta > 0$

$a > 4(a + 1)$ | $1 > 4(a - 1)$

$\Rightarrow a < 5/4$ | $a < 5/4$

$1 \cdot f(1) > 0$ | $1 \cdot f(1) < 0$

$a > 1$ | $a > 1$

$\Rightarrow a \in (1, 5/4)$

16. Three roots occur when the above set of equations has a root x = 1 as common root.

i.e. when a = 1.

1. $C_1C_2 < r_1 + r_2 \Rightarrow$ circles are intersecting

$\Rightarrow$ two common tangents equation of common chord $6x - 9 = 0 \Rightarrow x = \frac{3}{2}$

length of common chord = $2\sqrt{9 - \frac{9}{4}} = 3\sqrt{3}$

length of common tangent = $\sqrt{(C_1C_2)^2 - (r_1 - r_2)^2} = 3$.

Required greatest distance = $C_1C_2 + r_1 + r_2 = 9$.

2. (C) The point (6, 8) divides the line segment joining (2, 4) and (8, 10) in the ratio 2 : 1 internaly.

The harmonic conjugate of (6, 8) divides the line segment joining (2, 4) and (8, 10) in the ratio 2 : 1 externally.

$\therefore$ the point is (14, 16)

(D) Let θ be the angle of which line segment AB taken in the direction from A to B makes with the positive direction of x–axis then

$\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

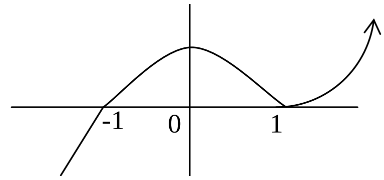
Let the coordinates of the new positions of B be (x, y) then

$$\frac{x-2}{4\sqrt{2}} = \cos(\theta - 90^o) = \sin\theta = -\frac{1}{\sqrt{2}}$$

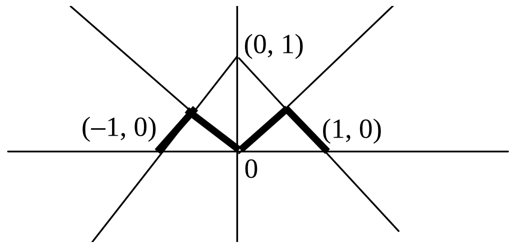
$\therefore$ x = –2 and $\frac{y-4}{4\sqrt{2}} = \sin(\theta - 90^o) = -\cos\theta = -\frac{1}{\sqrt{2}}$

$\therefore$ y = 0

3(A) $f(x) = (x + 1)\,|x - 1| = \begin{cases} x^2 - 1, & x \geq 1 \\ 1 - x^2, & x < 1 \end{cases}$



(B) $f(x) = \min\{|x|, 1 - |x|\}$



(C) $f(x) = \{x\} + 2\,[x] = x + [x]$

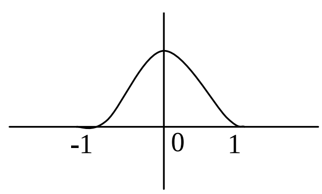
at $x = I$,

$LHL = I + (I - 1) = 2I - 1$

$RHL = I + I = 2I = f(1)$

$\therefore$ Not continuous hence not differentiable at integral points but increasing

(D) $f(x) = \sqrt{\cos^2 \frac{\pi x}{2}} = \left|\cos\frac{\pi x}{2}\right|$

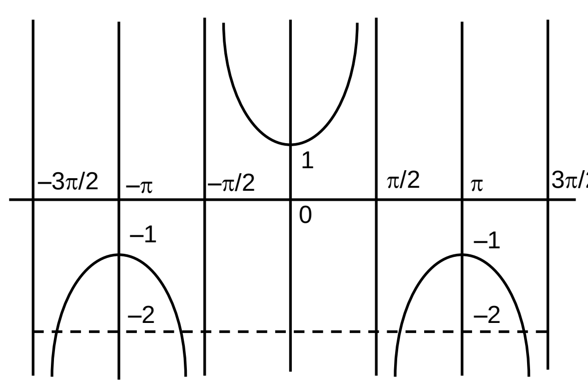


4. $f(x) = \begin{cases} [x], & -2 \leq x < 0 \\ |x|, & 0 \leq x \leq 2 \end{cases}$

$\Rightarrow f(x) = \begin{cases} -2, & -2 \leq x < 1 \\ -1, & -1 \leq x < 0 \\ x, & 0 \leq x \leq 2 \end{cases}$

$g(x) = \sec x, \quad x \in R - (2n + 1)\frac{\pi}{2}$

$fog = \begin{cases} -2, & -2 \leq \sec x < -1 \\ -1, & -1 \leq \sec x < 0 \\ \sec x, & 0 \leq \sec x \leq 2 \end{cases}$



$$\therefore \text{ fog} = \begin{cases} -2, & x \in \left[-\frac{4\pi}{3}, -\frac{2\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] - \{-\pi, \pi\} \\ -1, & x = -\pi, \pi \\ \sec x, & x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right] \end{cases}$$

limit of fog exist at x = – π, π, – 1, points of discontinuity of fog are – π, π

points of differentiability of fog are – 1, $\frac{5\pi}{6}$.

$$\text{gof} = \begin{cases} \sec(-2), & x \in [-2, -1) - \left\{-\frac{\pi}{2}\right\} \\ \sec(-1), & x \in [-1, 0) \\ \sec x & x \in [0, 2] - \left\{\frac{\pi}{2}\right\} \end{cases}$$

limit of gof does not exist at x = – 1.

PHYSICS

2. $mg\sin\theta - T = ma$

$T = ma$

3. $y = \frac{C}{6}t^6 \Rightarrow v = \frac{dy}{dt} = Ct^5$

$a = \frac{dv}{dt} = 5Ct^4$

$\frac{a}{v} = \frac{5}{t}$

So, at t = 5 sec, $\frac{a}{v} = 1 \Rightarrow$ a = v

4. $v_x = u_x + a_x t$

$v_x = 0 + gt$ ..... (1)

$v_y = u_y + a_y t$

$0 = u - gt$

$t = \frac{u}{g}$ ..... (2)

By (1) and (2) we get

$v_x = u$ and $v_y = 0$

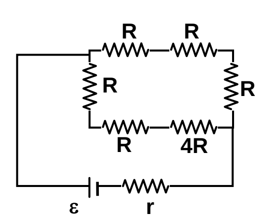
Hence net velocity = u

5. Conceptual.

6. $E_i = \frac{k(2Q)}{(2R)^2} - \frac{k(Q)}{(4R)^2} = \frac{7kQ}{16R^2}$

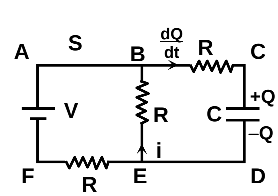
7. Circuit is forming a wheatstone bridge R_{eq} = 2R

For maximum power transfer 2R = r.



13. Apply KVL in ABEFA and BCEDB to get

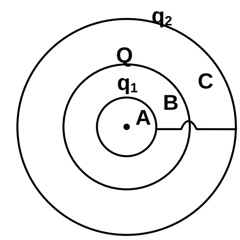
$Q = \frac{VC}{2}\left(1 - e^{-2t/3RC}\right)$



14. To calculate current in ED find dQ/dt

$$\frac{dQ}{dt} = \frac{VC}{2} \times \frac{2}{3RC} e^{-2t/3RC} = \frac{V}{3R} e^{-2t/3RC}$$

15. $q_1 + q_2 = 0$



$v_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$

$v_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$

$v_A = v_C$

$\Rightarrow$ $q_1 = -Q/3$ and $q_2 = Q/3$

16. $v_A = k\left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R}\right] = \frac{Q}{16\pi\varepsilon_0 R}$

4. $i = a - \frac{bt}{2} + \frac{ct^2}{3}$