

# Hints and Solutions

## CHEMISTRY (SECTION-I)

### PART-A

5. According to the given figure,  $A^+$  is present in the octahedral void of  $X^-$ . The limiting radius in octahedral void is related to the radius of sphere as

$$r_{\text{void}} = 0.414 r_{\text{sphere}}$$

$$r_{A^+} = 0.414 r_{X^-}$$

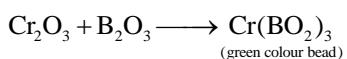
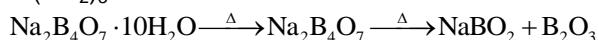
$$= 0.414 \times 250 \text{ pm} = 103.5$$

$$\approx 104 \text{ pm}$$

Hence (A) is correct.

7. Less the magnitude of  $\Delta G^\circ$  for a chemical reaction, less will be the feasibility

8.  $\text{Cr(BO}_2)_3$



9.  $M(s) + M_{(\text{aq})1M}^+ \longrightarrow M_{(\text{aq}),05M}^+ + M(s)$

According to Nernst equation ,

$$E_{\text{cell}} = 0 - \frac{2.303RT}{F} \log \frac{M_{,05M}^+}{M_{1M}^+}$$

$$= 0 - \frac{2.303RT}{F} \log (5 \times 10^{-2})$$

= +ve

Hence,  $|E_{\text{cell}}| = E_{\text{cell}} = 0.70 \text{ V}$  and  $\Delta G < 0$  for the feasibility of the reaction.

10. From above equation  $\frac{2.303RT}{F} = 0.0538$

$$\text{So, } E_{\text{cell}} = E_{\text{cell}}^o - \frac{0.0538}{1} \log 0.0025$$

$$= 0 - \frac{0.0538}{1} \log 0.0025$$

$$\approx 0.13988 \text{ V}$$

$$\approx 140 \text{ Mv}$$

### PART-C

1. At equilibrium relative lowering of vapour pressure of both the solution is same

$$\text{Then, } \frac{(1+\alpha)0.01}{60} = \frac{0.014}{60} \Rightarrow 1+\alpha = 1.4$$

$$\alpha = 0.4$$

$$10\alpha = 4$$

2. Two atoms of 'A' from corners and one atom of B from body diagonal

$$'A' remained  $\Rightarrow 6 \times \frac{1}{8} + 6 \times \frac{1}{2} = 3 + \frac{3}{4} = \frac{15}{4}$$$

$$'B' remained  $\Rightarrow 4 - 1 = 3$$$

$$A_{15/4}B_3 \Rightarrow A_{15}B_{12} \Rightarrow A_5B_4$$

3. en, ox, phen, dipy are bidentate ligands with identical donor atoms?

gly – is bidentate but with different donor atoms.  
 dien – tridentate  
 trien – tetradeinate  
 py – monodentate  
 SCN – ambidentate

4.  $[\text{Co}(\text{en})_2 \text{Cl}_2]^+$  → will show cis – trans isomerism  
 $[\text{CrCl}_2 (\text{C}_2\text{O}_4)_2]^{3-}$  → will show cis – trans isomerism  
 $[\text{Fe}(\text{H}_2\text{O})_4 (\text{OH})_2]^+$  → will show cis – trans isomerism  
 $[\text{Fe}(\text{CN})_4 (\text{NH}_3)_2]^-$  → will show cis – trans isomerism  
 $[\text{Co}(\text{en})_2 (\text{NH}_3)\text{Cl}]^{2+}$  → will show cis – trans isomerism  
 $[\text{Co}(\text{NH}_3)_4 (\text{H}_2\text{O})\text{Cl}]^{2+}$  → will show cis – trans isomerism
6. Black coloured sulphides {PbS, CuS, HgS, Ag<sub>2</sub>S, NiS, CoS}  
 \* Bi<sub>2</sub>S<sub>3</sub> in its crystalline form is dark brown but Bi<sub>2</sub>S<sub>3</sub> precipitate obtained is black in colour.
7. Number of ionisable Cl<sup>-</sup> in [Cr(H<sub>2</sub>O)<sub>5</sub>Cl]Cl<sub>2</sub> is 2  
 ∴ Millimoles of Cl<sup>-</sup> =  $30 \times 0.01 \times 2 = 0.6$   
 ∴ Millimoles of Ag<sup>+</sup> required = 0.6  
 ∴ 0.6 = 0.1 V  
 V = 6 ml

### MATHEMATICS (SECTION-II)

#### PART-A

1.  $A_1 = \int_1^{e^e} \ln y dy = [\ln y - y]_1^{e^e} = (e^e \cdot e - e^e + 1) \text{ sq. units}$   
 $A_3 = \int_1^e \ln x dx = [x \ln x - x]_1^e = (e - e) - (0 - 1) = 1. \text{ sq. unit}$   
 So,  $A_2 = e^e \cdot e - [e^e \cdot e - e^e + 1 + 1] = (e^e - 2) \text{ sq. units}$   
 $A_1 - A_2 = e^e \cdot e - e^e + 1 - e^e + 2 = e^e \cdot e - 2 \cdot e^e + 3 = e^e (e - 2) + 3$   
 Clearly  $A_1 > A_2 > A_3$

2. The given differential equation can be written as

$$\begin{aligned} \frac{x dx + y dy}{y dx - x dy} &= \frac{x \sin^2(x^2 + y^2)}{y^3} \\ \Rightarrow \frac{2x dx + 2y dy}{y dx - x dy} &= \frac{2x \sin^2(x^2 + y^2)}{y^3} \\ \Rightarrow \frac{d(x^2 + y^2)}{\sin^2(x^2 + y^2)} &= \frac{2x}{y^3} (y dx - x dy) \\ \Rightarrow \cosec^2(x^2 + y^2) d(x^2 + y^2) &= 2 \left( \frac{x}{y} \right) d \left( \frac{x}{y} \right) \end{aligned}$$

On integrating, we get

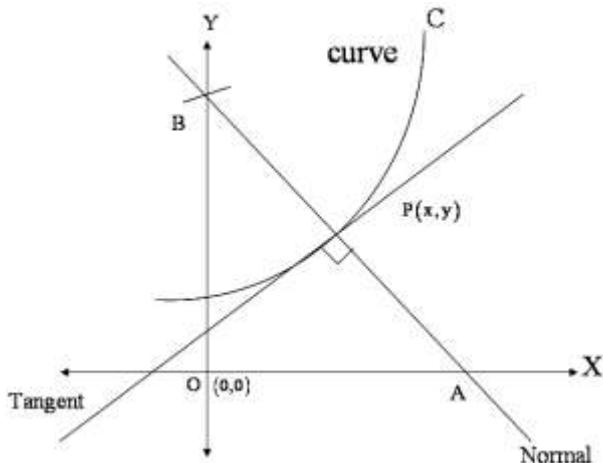
$$\int \csc^2(x^2 + y^2) d(x^2 + y^2) = 2 \int \left(\frac{x}{y}\right) d\left(\frac{x}{y}\right)$$

$$\Rightarrow -\cot(x^2 + y^2) = \left(\frac{x}{y}\right)^2 + C, \text{ which is the required solution.}$$

3. The equation of normal of  $P(x, y)$  is  $(Y - y) = \frac{-1}{\frac{dy}{dx}}(X - x)$

$$\therefore A\left(x + y \frac{dy}{dx}, 0\right) \text{ and } B\left(0, y + \frac{x}{\frac{dy}{dx}}\right)$$

$$\text{Now } \frac{1\left(x + y \frac{dy}{dx}\right) + 2(0)}{1+2} = x \Rightarrow x + y \frac{dy}{dx} = 3x$$



$$y \frac{dy}{dx} = 2x \quad \dots\dots(1)$$

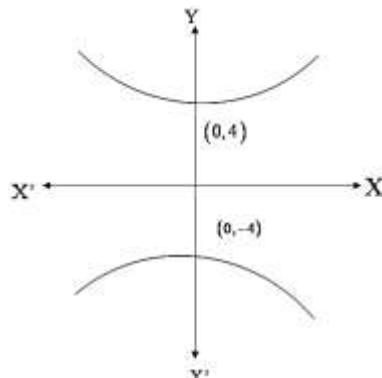
$$\Rightarrow \int y dy = \int 2x dx \Rightarrow \frac{y^2}{2} = x^2 + C$$

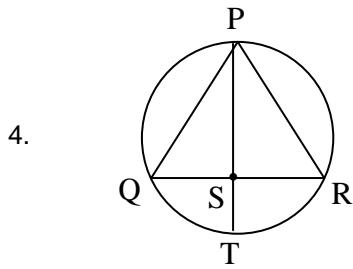
Also (0,4) satisfy it, so  $C = 8$

$$\therefore y^2 = 2x^2 + 16 \text{ (equation of curve)}$$

Which represent a hyperbola

$$\text{Also } \left. \frac{dy}{dx} \right|_{(4, 4\sqrt{3})} = \frac{2(4)}{4\sqrt{3}} = \frac{2}{\sqrt{3}}$$





H.M. < G.M.

$$\frac{2}{\frac{1}{PS} + \frac{1}{ST}} < (PS \cdot ST)^{1/2} \Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{PS \cdot ST}}$$

$$\text{As } PS \times ST = QS \times SR \Rightarrow \frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}} \dots \text{(i) Option (B) is correct}$$

A.M. > G.M

$$\frac{QS+SR}{2} > \sqrt{(QS \cdot SR)}$$

$$\therefore \sqrt{QS \times SR} < \frac{QR}{2} \dots \text{(ii)}$$

From (i) and (ii)

$$\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR} .$$

5. Here,  $(x-1)^2 - 3y^2 = 0$

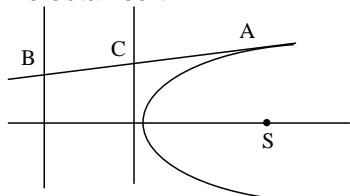
$$\Rightarrow (x + \sqrt{3}y - 1)(x - \sqrt{3}y - 1) = 0$$

$$\text{Now, } 4 \cos 30^\circ = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$\therefore$  from the figure, the equation of the third side is

$$x = 1 + 2\sqrt{3} \text{ or } x = -(2\sqrt{3} - 1)$$

6. For parabola  $y^2 = 4ax$ , eq. of tangent at A ( $at^2, 2at$ ) is  $ty = x + at^2$  comparing the line with the line  $x + y + 2 = 0$  we obtained  $t = -1$



$$\therefore C(0, at), B\left(-a, at - \frac{a}{t}\right)$$

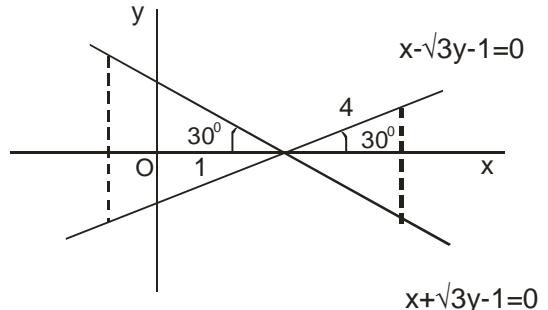
$$AC = at\sqrt{1+t^2}; \quad BC = \frac{a}{t}\sqrt{1+t^2}; \quad CS = a^2(1+t^2)$$

$$\Rightarrow AC \cdot BC = (CS)^2$$

$$\text{slope of } CS \times \text{slope of } AB = \frac{0-at}{a-0} \times \frac{1}{t} = -1$$

hence  $CS$  is perpendicular to  $AB$

$$CS = \frac{|2+2|}{\sqrt{2}} = 2\sqrt{2}$$



$$x + \sqrt{3}y - 1 = 0$$

7. Equation of tangent  $y = \frac{x}{t} + at$

$$x - yt + at^2 = 0 \quad \dots\dots\dots(1)$$

$$\text{normal at } \theta \text{ to } \frac{x^2}{5} + \frac{y^2}{4} = 1$$

$$\frac{x\sqrt{5}}{\cos\theta} - \frac{2y}{\sin\theta} = 1$$

$$2y\cos\theta = x\sqrt{5}\sin\theta - \sin\theta\cos\theta \quad \dots\dots\dots(2)$$

From 1 and 2

$$t = \frac{2\cos\theta}{\sqrt{5}\sin\theta} = \frac{2}{\sqrt{5}}\cot\theta, \quad t^2 = \frac{-\cos\theta}{\sqrt{5}} \Rightarrow -\frac{\cos\theta}{\sqrt{5}} = \frac{4\cos^2\theta}{5\sin^2\theta}$$

$$\cos\theta = 0 \text{ or } \sqrt{5}\sin^2\theta + 4\cos\theta = 0$$

$$\cos\theta = -\frac{1}{\sqrt{5}}, \quad \cos\theta = 0 \Rightarrow t = 0, \quad t^2 = \frac{1}{5}$$

8. Let  $(x_i, y_i) = \left(t_i, \frac{1}{t_i}\right); i = 1, 2, 3, 4$

any point on hyperbola  $\left(t, \frac{1}{t}\right)$  lie on  $x^2 + y^2 = 1 \Rightarrow t^2 + \frac{1}{t^2} = 1$

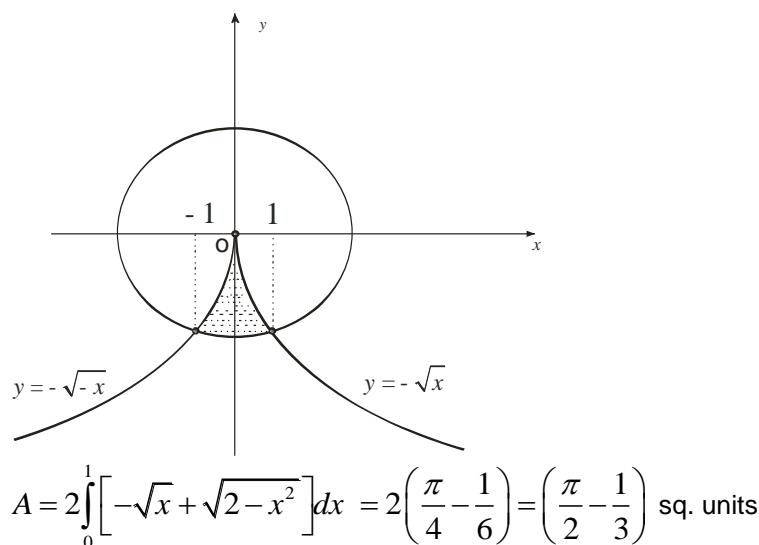
$$\Rightarrow t^4 - t^2 + 1 = 0$$

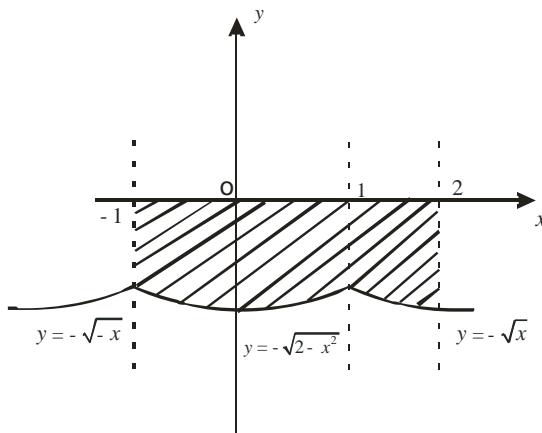
If the roots are  $t_1, t_2, t_3, t_4$  then  $\sum t_i = 0 \Rightarrow x_1 + x_2 + x_3 + x_4 = 0$

$$\sum t_i t_j = -1 \Rightarrow \sum t_1 t_2 t_3 = 0, \quad \sum t_1 t_2 t_3 t_4 = 1 \Rightarrow x_1 x_2 x_3 x_4 = y_1 y_2 y_3 y_4 = 1$$

$$\text{and } y_1 + y_2 + y_3 + y_4 = \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{\sum t_1 t_2 t_3}{\sum t_1 t_2 t_3 t_4} = 0.$$

9-10.





Required area bounded between  
 $y = f(x)$ ,  $x = -1$ ,  $x = 2$  and x-axis is given by

$$\begin{aligned} & \int_{-1}^1 \sqrt{2-x^2} dx + \int_1^2 \sqrt{x} dx \\ &= 2 \int_0^1 \sqrt{2-x^2} dx + \int_1^2 \sqrt{x} dx = \left( \frac{3\pi + 8\sqrt{2} + 2}{6} \right) \text{ sq. units} \end{aligned}$$

$$\text{Now; } \frac{A}{2} = \int_a^0 [y^2 - (-y^2)] dy = \frac{2}{3} [y^3]_a^0$$

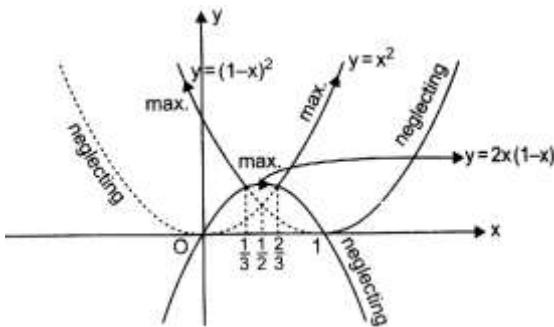
$$\frac{\pi}{4} - \frac{1}{6} = \frac{-2}{3} a^3 \Rightarrow a^3 = \frac{3}{2} \left[ \frac{1}{6} - \frac{\pi}{4} \right] = \frac{1}{4} - \frac{3\pi}{8}$$

$$a = \left[ \frac{1}{4} - \frac{3\pi}{8} \right]^{1/3}.$$

11. There will exist exactly one circle if the line passing through  $A(2, -3)$  and  $B(\lambda, 2\lambda - 1)$  is parallel to the given line  $16x - 2y + 27 = 0$   
 Also, if the point  $B(\lambda, 2\lambda - 1)$  lies on the line  $16x - 2y + 27 = 0$ , then we will have exactly one circle.  
 Thus two values of  $\lambda$  are possible.
12. The line joining  $(3, -5)$  and  $(5, -3)$  has slope 1 and thus it is perpendicular to  $2x + 2y + 13 = 0$ . Hence the two circles will have same radii.

### PART-C

1. We have,  
 $f(x) = \max\{x^2, (1-x)^2, 2x(1-x)\}$



$$f(x) = \begin{cases} (1-x)^2, & \text{for } 0 \leq x \leq 1/3 \\ 2x(1-x), & \text{for } 1/3 \leq x \leq 2/3 \\ x^2, & \text{for } 2/3 \leq x \leq 1 \end{cases}$$

Hence the area bounded by the curve  $y = f(x)$ ;  $x$ -axis and the lines  $x = 0$  and  $x = 1$  is given by

$$= \int_0^{1/3} (1-x)^2 dx + \int_{1/3}^{2/3} 2x(1-x) dx + \int_{2/3}^1 x^2 dx = \frac{17}{27} \text{ sq. unit.}$$

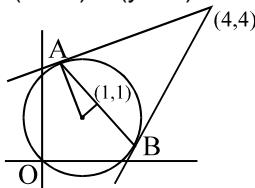
$$\therefore \frac{A}{54} = \frac{17}{27} \Rightarrow A = 34 \text{ Sq. unit.}$$

2.  $\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} - \frac{A}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2} y' = 2\sin^{-1}x - A \Rightarrow (1-x^2)y'' - xy' = 2$

3. length of C.O.C. =  $\frac{2RL}{\sqrt{R^2+L^2}}$

equation of circle  $x^2 + y^2 - 2x - 2y - 7 = 0$

equation of COC =  $4x + 4y - (x + 4) - (y + 4) - 7 = 0 = 3x + 3y - 15 = 0$



radius = 3 perpendicular from  $(1, 1) = \left| \frac{6-15}{\sqrt{18}} \right| = \frac{9}{\sqrt{18}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$

$$\therefore \text{length } \frac{AB}{2} = \sqrt{r^2 - \left( \frac{3}{\sqrt{2}} \right)^2} = \frac{3}{\sqrt{2}}$$

$$\therefore AB = 3\sqrt{2}$$

4.  $\Delta = \frac{1}{2} \left| \begin{vmatrix} \alpha & 2\alpha+3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ \alpha & 2\alpha+3 \end{vmatrix} \right|$

$$= \frac{1}{2} |2\alpha - (2\alpha + 3) + 3 - 4 + 4\alpha + 6 - 3\alpha| = \frac{1}{2} |\alpha + 2|$$

but  $[\Delta] = 2 \Rightarrow \left[ \frac{1}{2} |\alpha + 2| \right] = 2$

$$\Rightarrow 2 \leq \frac{|\alpha + 2|}{2} < 3$$

$$\leq 4 \leq |\alpha + 2| < 6$$

$$\Rightarrow 4 \leq \alpha + 2 < 6 \text{ and } -6 < \alpha + 2 \leq -4$$

$$\Rightarrow 2 \leq \alpha < 4 \text{ and } -8 < \alpha \leq -6$$

$$\therefore \alpha \in I$$

$$\therefore \alpha = 2, 3, -7, -6$$

5. Since both curves are symmetrical about the line  $y = x$  distance between any pair of points = 2(distance of  $(t, t^2 + 1)$  on the parabola  $y = x^2 + 1$  from  $y - x = 0$ )

$$2\left(\frac{t^2 + 1 - t}{\sqrt{2}}\right) = \sqrt{2}(t^2 - t + 1)$$

The minimum value of  $t^2 - t + 1 = \frac{3\sqrt{2}}{4} \Rightarrow$  Minimum distance =  $\frac{3\sqrt{2}}{4} \Rightarrow k = 4$

6. Any tangent to the ellipse is

$$\frac{x}{67} \cos \theta + \frac{y}{33} \sin \theta = 1$$

Sum of the squares of the lengths of the perpendicular from  $(0, \pm 10\sqrt{34})$  on this tangents is

$$\begin{aligned} & \left(\frac{10\sqrt{34}}{33} \sin \theta - 1\right)^2 + \left(\frac{10\sqrt{34}}{33} \sin \theta + 1\right)^2 = \frac{2[100 \times 34 \sin^2 \theta + (33)^2]}{(33)^2 \cos^2 \theta + (67)^2 \sin^2 \theta} \\ & = \frac{2 \times (67)^2 [((67)^2 - (33)^2) \sin^2 \theta + (33)^2]}{(33)^2 \cos^2 \theta + (67)^2 \sin^2 \theta} = 2 \times (67)^2 = 8978 \end{aligned}$$

$$\Rightarrow l = 2.$$

7. Let  $(x_1, y_1)$  be a point on  $y^2 = -8(x + 4)$

equation of chord of contact is

$2x - y_1 y + 2x_1 = 0$ , if  $p(h, k)$  be its mid point, then its equation will be

$$2x - ky + k^2 - 2h = 0$$

Compare both  $k = y_1$ ,  $2x_1 = k^2 - 2h$

$$\text{So, } k^2 = -4(k^2 - 2h + 8) \Rightarrow k^2 = \frac{8}{5}(h - 4). \text{ So, } \lambda = \frac{8}{5}$$

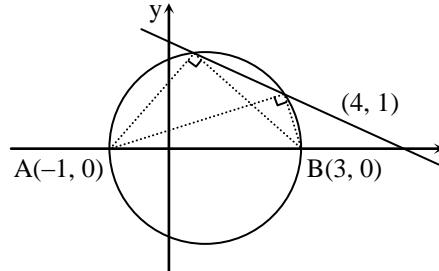
8. Centre =  $(1, 0)$ , Radius = 2

$$y - 1 = m(x - 4)$$

$$\left| \frac{-3m+1}{\sqrt{m^2+1}} \right| < 2$$

$$\Rightarrow m \in \left( \frac{6-\sqrt{96}}{10}, \frac{6+\sqrt{96}}{10} \right)$$

$$\lambda_1 + \lambda_2 = \frac{6}{5}$$



### PHYSICS (SECTION-III)

#### PART-A

7. As the focal length remains same but the amount of light passing through lens decreases.

$$\beta = \frac{D\lambda}{d}$$

8.  $\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$

Also  $m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$

$$\text{Also } 3\left(\frac{D}{6}\right)(600\text{nm}) = (2 \times 5 - 1)\left(\frac{D}{2d}\right)400\text{nm}$$

$$\text{Angular width } \theta = \frac{\lambda}{d}$$

11. Anywhere on screen because there is no relation between  $\theta, \mu$

12. Total path difference

$$\Delta x = (\mu - 1)t - ds \sin \theta$$

$\Delta x = 0$  for central maxima

### PART-C

$$2. \quad B_0 = \frac{E_0}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8} T$$

$$4. \quad v = k\sqrt{T}$$

$$\Rightarrow v = k\sqrt{27 + 273} \quad \dots(1)$$

$$2v = k\sqrt{T + 273} \quad \dots(2)$$

Dividing equation (2) and (1) we get

$$2 = \sqrt{\frac{T + 273}{300}}$$

$$\Rightarrow T = 927^\circ C$$

$$5. \quad \omega = \sqrt{\frac{YA}{mL}}$$

$$6. \quad n \sin \theta = (n - m\Delta n) \times \sin 90^\circ$$

$$\Rightarrow m = \frac{n}{2\Delta n}$$

$$\Rightarrow m = 8$$

7. The frequency of sound from the image is Doppler shifted due to its own motion and the motion of the observer itself.

$$8. \quad v = \frac{v}{4\ell_1} \text{ (for first harmonic)}$$

$$v = \frac{3v}{2\ell_2} \text{ (for first harmonic)}$$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{1}{6}$$