FIITJEE FARIDABAD

MOCK PRACTICE PAPER FOR

JEE -Mains- 2020

MOCK PRACTICE PAPER-20

Time: 3 hours ***Maximum marks: 360***

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.

2. The Test Booklet consists of **90** questions. The maximum marks are **360.**

3. There are ***Three*** parts in the question paper. **Part 1: Physics, Part 2: Chemistry** and **Part 3 is Mathematics**. Each question is allotted **4 (four)** marks for correct response.

4. *Candidates will be awarded marks as stated above in instruction No. 3 for correct response of each question. –1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*

5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

PRACTICE SET - 8

Time : 3 Hours **Maximum Marks : 360**

PHYSICS

1. Two coherent monochromatic light beams of intensities I_0 and $4I_0$ are superposed. The maximum and minimum possible intensities in the resulting beam will be

(a) $5I_0$ and I_0 (b) $5I_0$ and $3I_0$

(c) $9I_0$ and I_0 (d) $9I_0$ and $3I_0$

2. Two masses M_1 and M_2 are connected to the ends of a string passing over a smooth pulley. The tension in the string is T and the masses are moving with acceleration a. If the masses are inter changed, then

(a) Both a & T will change

(b) a will change but T will remain unchanged

(c) T will change but a will remain unchanged

(d) none of them will change

3. If a heavy and a light body are dropped from the same height, the time taken by them to reach the ground is

(a) $t = \sqrt{\frac{2h}{g}}$ (b) $t = \sqrt{2hg}$

(c) $t = hg$ (d) None of these

4. A shell at rest explodes. The centre of mass of the fragments

(a) moves along a parabolic path

(b) moves along a straight line

(c) moves along an elliptical path

(d) remains at rest

5. In a long cylindrical tube, the water level is adjusted and the air column above it is made to vibrate in unison with a vibrating tuning fork kept at the open end. Maximum sound is heard when the air column lengths are equal to

(a) $\frac{\lambda}{4}, \frac{\lambda}{2}, \frac{3\lambda}{4}$

(b) $\frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$

(c) $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$

(d) $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$

6. A simple harmonic oscillator has an amplitude a and time period T. The time required by it to travel from $x = a$ to $x = \frac{a}{2}$ is

(a) $\frac{T}{6}$ (b) $\frac{T}{4}$

(c) $\frac{T}{3}$ (d) $\frac{T}{2}$

7. At 15°C, a wheel has diameter of 30 cm and inside diameter of a steel rim is 29.9 cm. Coefficient of thermal expansion of steel is 1.2×10^{-5} /°C. To what temperature, the rim must be heated so that it can slip over wheel?

(a) 2787°C (b) 2772°C

(c) 2529°C (d) 2802°C

8. Two bodies of masses m_1 and m_2 have equal momenta. If E_1 and E_2 be their respective kinetic energies, the ratio $E_1 : E_2$ is equal to

(a) $m_1 : m_2$ (b) $m_2 : m_1$

(c) $\sqrt{m_1} : \sqrt{m_2}$ (d) $m_1^2 : m_2^2$

9. Which of the following parameters does not characterize the thermodynamic state of matter?

(a) temperature (b) Pressure

(c) Work (d) Volume

10. Which one of the following represents the correct dimensions of the coefficient of viscosity?

(a) $ML^{-1}T^{-2}$ (b) MLT^{-1}

(c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$

11. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the times of flight in the two cases, then the product of the two times of flights is directly proportional to

(a) $\frac{1}{R^2}$ (b) $\frac{1}{R}$

(c) R (d) R^2

12. The angular frequency of a particle performing uniform circular motion is doubled & its kinetic energy halved, then the new angular momentum is

(a) $\frac{L}{4}$ (b) 2L

(c) 4 L (d) $\frac{L}{2}$

13. Consider the following two statements:

(A) Linear momentum of a system of particles is zero.

(B) Kinetic energy of a system of particles is zero.

Then

(*a*) A does not imply B and B does not imply A

(*b*) A implies B but B does not imply A

(*c*) A does not imply B but B implies A

(*d*) A implies B and B implies A

14. In concave mirror an object is placed at a distance x_1 from the focus, and the image is formed at a distance x_2 from the focus. Then the focal length of mirror is

(*a*) x_1x_2 (*b*) $\sqrt{x_1x_2}$

(*c*) $\frac{x_1+x_2}{2}$ (*d*) $\sqrt{\frac{x_1}{x_2}}$

15. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass

(*a*) of the box remains constant

(*b*) of the box plus the ball system remains constant.

(*c*) of the ball remains constant.

(*d*) of the ball relative to the box remains constant.

16. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(*a*) $T = t_1 + t_2$ (*b*) $T^2 = t_1^2 + t_2^2$

(*c*) $T^{-1} = t_1^{-1} + t_2^{-1}$ (*d*) $T^{-2} = t_1^{-2} + t_2^{-2}$

17. A body moves in a straight line along x-axis. Its distance from the origin is given by $x = 9t - 3t^2$. The average velocity between $t = 0$ to $t = 4$ is

(*a*) $-3\ ms^{-1}$ (*b*) $4\ ms^{-1}$

(*c*) $-16\ ms^{-1}$ (*d*) $-12\ ms^{-1}$

18. Two thermally insulated vessels 1 and 2 are filled with air at temperature (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(*a*) $T_1 + T_2$ (*b*) $\frac{(T_1+T_2)}{2}$

(*c*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (*d*) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$

19. Which of the following is the expression of Laden berg's correction used in the measurement of terminal velocity?

(*a*) $\upsilon' = \upsilon\left[1+2.4\frac{r}{R}\right]$ (*b*) $\upsilon' = \upsilon\left[2.4\frac{r}{R}\right]$

(*c*) $\upsilon' = 2.4\upsilon\left[1+\frac{r}{R}\right]$ (*d*) $\upsilon' = \upsilon\left[1+\frac{r}{R}\right]$

20. Two particles execute SHM which can be represented as $y_1 = a\sin\omega t$ and $y_2 = \frac{a}{\sqrt{2}}\sin\omega t + \frac{a}{\sqrt{2}}\cos\omega t$. The ratio of the amplitudes of the oscillations will be

(*a*) 1 : 1 (*b*) $1:\sqrt{2}$

(*c*) 1 : 2 (*d*) $\sqrt{2}:1$

21. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance will be :

(*a*) 24 m (*b*) 6 m

(*c*) 12 m (*d*) 18 m

22. A solid sphere is rotating in free space. If the radius of the sphere is increased, keeping mass same, which one of the following will not be affected?

(*a*) Moment of inertia

(*b*) Angular momentum

(*c*) Angular velocity

(*d*) Rotational kinetic energy

23. Five identical charges each of magnitude q are placed at the five corners of a regular hexagon of side a. The field at the centre of the hexagon is

(*a*) $\frac{kq}{a^2}$ (*b*) $\frac{5kq}{a^2}$

(*c*) $\frac{6kq}{5a^2}$ (*d*) $\frac{5kq}{6a^2}$

24. If a capillary tube is dipped in a liquid and the liquid levels inside and outside the tube are same, then the angle of contact is

(*a*) 0° (*b*) 90°

(*c*) 45° (*d*) cannot be obtained

25. In a screw gauge the screw moves 1 mm during half full rotation of the circular scale, If the circular scale has 100 divisions, what is the maximum possible error that can creep in while measuring the thickness of a thin sheet of paper?

(*a*) 0.02 cm

(*b*) 0.01 cm

(*c*) 0.05 cm

(*d*) depends on the thickness of the paper.

26. What are the orders of drift velocity for normally applied potentials (0 – 5V) and that of random thermal speed at room temperature say (300k)?

(*a*) 10 m/s, 10^2m/s

(*b*) 10^2 m/s, 10m/s

(*c*) 10^4 m/s, 10m/s

(*d*) 10m/s, 10^5 m/s

27. In a vernier calliper 9 main scale divisions is equal to 10 vernier scale divisions and one main scale division is equal to 2 mm. The least count of the calliper is

(*a*) 0.01 cm (*b*) 0.02 cm

(*c*) 0.0005 cm (*d*) 0.05 cm

28. Newton's law of cooling is valid only if the difference between the temperature of a hot body and that of surroundings

(*a*) Is not too small

(*b*) Is not too large

(*c*) Varies linearly with time

(*d*) None of the above

29. **Statement 1 :** If linear Momentum of a System is Conserved then Angular momentum will also remain conserved.

Statement 2 : Angular momentum is defined as $\vec{L} = \vec{r} \times \vec{P}$

(*a*) Statement-1 is true, Statement-2 is false.

(*b*) Statmeent-1is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; and Statement-2 is a correct explanation for Statement-1.

(*d*) Statement-1 is true, Statement-2 is true; but Statement-2 is not a correct explanation for Statement-1.

30. **Statement 1 :** Work done by magnetic field on a moving charge is always zero

Statement 2 : Force exerted by magnetic field. is perpendicular to velocity of charge particular.

(*a*) Statement-1 is true, Statement-2 is false.

(*b*) Statmeent-1is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; and Statement-2 is a correct explanation for Statement-1.

(*d*) Statement-1 is true, Statement-2 is true; but Statement-2 is not a correct explanation for Statement-1.

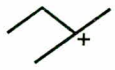
CHEMISTRY

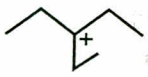
31. The amount of $(NH_4)_2SO_4$ in grams which must be added to 500 ml of 0.2 M NH_4OH to yield a solution pH 9.25 is [Given pKa of $NH_4^+ = 9.26$, Antilog $(-0.14) = 0.81$, Molar Mass of $(NH_4)_2SO_4 = 132$]

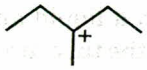
(*a*) 10.6 gm (*b*) 5.25 gm

(*c*) 20.4 gm (*d*) 2.04 gm

32. Which of the following carbocation is most stable?

(*a*) 

(*b*) 

(*c*) 

(*d*)

33. Which of the following reactions do not give dihydrogen?

(*a*) $Si + H_2O \longrightarrow$

(*b*) $Si + KOH + H_2O \longrightarrow$

(*c*) $NH_3 + CO_2 \xrightarrow[80\text{-}100\ atm]{200°C}$

(*d*) $AlN + NaOH + H_2O \xrightarrow{boil}$

34. Quantum number which is not obtained from Schrodinger wave equation?

(*a*) n (*b*) ℓ

(*c*) m (*d*) s

35. Which of the following oxide is unstable above 500°C?

(*a*) HgO (*b*) Al_2O_3

(*c*) Cr_2O_3 (*d*) MgO

36. Consider the following reaction:

$K_4[Fe(CN)_6] + O_2 \longrightarrow Fe_2O_3 + CO_2 + NO_3^- + K^+$

The equivalent mass of $K_4[Fe(CN)_6]$ will be

(*a*) M (*b*) $\frac{M}{2}$

(*c*) $\frac{M}{13}$ (*d*) $\frac{M}{61}$

37. Which is correct order for decreasing –I effect?

(*a*) Cl > Br > I > F (*b*) Br > I > F > Cl

(*c*) F > Cl > Br > I (*d*) F > Cl > Br ≈ I

38. The difference between oxidation numbers of two sulphur atoms in $H_2S_2O_3$ (thiosulphuric acid) is

(*a*) 6 (*b*) 4

(*c*) 5 (*d*) 0

39. Species having bond order different from that in CO is

(*a*) NO^- (*b*) NO^+

(*c*) CN^- (*d*) N_2

40. Which of the following cannot have sp^3 hybridization of central atom?

(a) $[NiCl_4]^{2-}$ (b) $[MnCl_4]^{2-}$

(c) $[CrO_4]^{2-}$ (d) $[CoCl_4]^{2-}$

41. 180 gm of water contains 6 gm urea (M = 60) and 68.4 gm sucrose (M = 342) at 298 K. If $p^0_{H_2O} = 23.8$ torr at 298 K, then vapour pressure of the solution will be

(a) 10.2 torr (b) 15.6 torr

(c) 23.1 torr (d) 40.6 torr

42. Which of the following is not the property of amino acids?

(a) Amino acids are non-volatile crystalline solids which melt with decomposition at high temperature.

(b) Their aqueous solutions behave like solutions of substances of high dipole moment.

(c) They are soluble in water but insoluble in non-polar solvent.

(d) Acidity and basicity constants are relatively high.

43. According to kinetic theory of gases, for a diatomic molecule

(a) pressure exerted by the gas is proportional to mean velocity of the molecule.

(b) pressure exerted by the gas is proportional to root mean velocity of the molecule.

(c) root mean square velocity of the molecule is inversely proportional to the temperature.

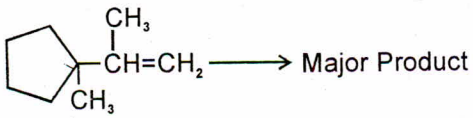
(d) mean translational kinetic energy of the molecule is proportional to the absolute temperature.

44. A white substance (A) reacts with dil. H_2SO_4 to produce a colourless gas (B) and a colourless solution (C). Addition of aqueous NH_3 or NaOH to (C) produces a precipitate which dissolves to give a clear solution (D) if NaOH is used in excess. The (D) in the above sequence is

(a) $[Sc(H_2O)_6]^{3+}$ (b) Na_2ZnO_2

(c) $NaAlO_2$ (d) $[Cu(NH_3)_2]SO_4$

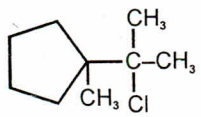
45.



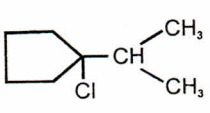
Major Product

Major product will be

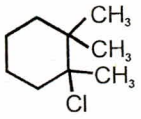
(a)



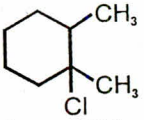
(b)



(c)



(d)



46. Which of the following do not have 90° bond angle?

(a) SF_4 (b) ClF_3

(c) IF_7 (d) PCl_5

47. Which of the following complexes don't exist as geometrical isomers?

(a) $[Pt(C_2H_4)(NH_3)(Cl)(Br)]$

(b) $[Pt(gly)_2]$

(c) $[Pd(Cl_2)(Br)(I)]$

(d) $[Pt(NH_3)Cl_5]^-$

48. By how much is the oxidizing power of MnO_4^-/Mn^{2+} couple decreased if H^+ conc. is decreased from 1 M to 10^{-4} M at 25°C? (Assume other species have no change in concentration)

(a) 0.22 V (b) 0.14 V

(c) 0.38 V (d) 1.0 V

49. In the following equilibrium reaction at 185°C,

$$CH_3COCH_3(g) \rightleftharpoons CH_3 - CH_3(g) + CO(g)$$

(a) increase of temperature favours formation of $CH_3COCH_3(g)$

(b) decrease of temperature favours formation of CO(g)

(c) Removal of CO(g) favours formation of CO(g)

(d) Addition of $CH_3 - CH_3(g)$ favours formation of CO(g)

50. Which of the following has least value of gold number?

(a) Potato starch (b) Haemoglobin

(c) Egg albumin (d) Gum Arabic

51. $H_3BO_3 \xrightarrow{100°C} A \xrightarrow{140°C} B \xrightarrow[\text{heat}]{\text{Strong}} \underset{\text{(glassy bead)}}{B_2O_3}$

B in the above sequence is

(a) HBO_2 (b) $H_4B_2O_7$

(c) $H_2B_4O_7$ (d) H_3BO_3

52. The difference between heat of reaction at constant pressure and heat of reaction at constant volume for the combustion of $C_2H_4(g)$ at 25°C is :

(a) –20.2 cal

(b) –30.4 cal

(c) –1.192 cal

(d) –5.4 cal

53. The correct order of nucelophilicity in a non-polar solvent is

(a) $CH_3^- > NH_2^- > OH^- > F^-$

(b) $F^- > OH^- > NH_2^- > CH_3^-$

(c) $OH^- > NH_2^- > CH_3^- > F^-$

(d) $NH_2^- > OH^- > F^- > CH_3^-$

54. Which of the following is basic unit of Pyrosilicates?

(a) SiO_4^{4-} (b) SiO_3^{2-}

(c) $Si_2O_6^{7-}$ (d) $Si_2O_7^{6-}$

55. During transformation of ${}^{a}_{c}X$ to ${}^{b}_{d}Y$ by α and β-decay, the number of β-particles emitted are

(a) $\frac{a-b}{4}$ (b) $d+\left(\frac{a-b}{2}\right)-c$

(c) $d+\left(\frac{a-b}{2}\right)+c$ (d) $2c-d+a-b$

56. Which conformer of following compound is most stable across C_2–C_3?

$$\underset{(2)}{HO-CH_2}-\underset{(3)}{CH_2-F}$$

(a) Staggered (b) Partially eclipsed

(c) Gauche (d) Fully eclipsed

57. If two solutions were mixed in an insulated bottle, one containing 400 ml of 0.2 M weak monoprotic acid solution and other one was 100 ml of 0.8 M NaOH, a 1.17°C change of temperature occurred in bottle. If density of solution is 1 gm/cm^3 and their specific heats are all 4.2 J $g^{-1}K^{-1}$, then the heat evolved in the neutralisation of acid will be :

(a) 30.71 kJ/mol

(b) 50.25 kJ/mol

(c) 70.31 kJ/mol

(d) 11.29 kJ/mol

58. Either which cannot be prepared by williamson's synthesis is

(a) $CH_3-CH=CH-O-CH=CH_2$

(b) $(CH_3)_3C-O-C_2H_5$

(c) $CH_3-O-C_2H_5$

(d) $C_6H_5-O-CH_2CH_3$

59. If equilibrium constants for

$$2CO+O_2 \rightleftharpoons 2CO_2,\ k=40\times10^{-3}$$

$$2H_2+O_2 \rightleftharpoons 2H_2O,\ k=36\times10^{-2}$$

Equilibrium constant for the reaction $CO+H_2O \rightleftharpoons CO_2+H_2$ will be

(a) 1.4 (b) 2.2

(c) 0.05 (d) 0.3

60. An aqueous solution containing 288 gm of a non-volatile (non-electrolyte) compound having stoichiometric composition $C_nH_{2n}O_n$ in 90 gm water boils at 110.24°C at 1.00 atm pressure. If $K_b(H_2O)=0.512\ k\ mol^{-1}kg$ and $T^0_{H_2O}=100°C$, then the molecular formula of compound will be

(a) $C_{12}H_{24}O_{12}$

(b) $C_6H_{12}O_6$

(c) $C_{44}H_{88}O_{44}$

(d) $C_{15}H_{30}O_{15}$

MATHEMATICS

61. The coordinates of the foot of the perpendicular from the point (3, –1, 11) on the line $\frac{x+2}{2}=\frac{y+1}{3}=\frac{z+1}{4}$ is

(a) (0, 2, 3)

(b) (2, 5, 7)

(c) (–2, –1, –1)

(d) (–4, –4, –5)

62. $\left[(\sqrt{2}+1)^6\right]+\left[(\sqrt{3}+1)^6\right]=?$ ([·] denotes greatest integer function)

(a) 611 (b) 612

(c) 613 (d) 614

63. An isosceles ΔABC is inscribed in $x^2+y^2=a^2$ with vertex at A($-a$, 0) and base angles B and C each equal to 75° then BC =

(a) a (b) $\frac{a}{2}$

(c) $\frac{2}{\sqrt{3}}a$ (d) $\frac{\sqrt{3}}{2}a$

64. The argument of $\frac{1+i}{\sqrt{3}-i}$ is

(a) $\frac{\pi}{12}$ (b) $\frac{7\pi}{12}$

(c) $\frac{-\pi}{12}$ (d) $\frac{5\pi}{12}$

65. Let $f(x)=\sin\left(\frac{x-a}{2}\right)\tan\left(\frac{\pi x}{2a}\right), x\neq a$. The value of $f(a)$ so that $f(x)$ is continuous at $x=a$ is

(a) $\frac{a}{\pi}$ (b) $-\frac{a}{\pi}$

(c) $\frac{\pi}{2a}$ (d) $-\frac{a}{2\pi}$

66. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^2}{r^3+n^3}=$

(a) $\frac{1}{3}\ell n2$ (b) $\frac{1}{3}\ell n3$

(c) $\ell n\,2$ (d) $\frac{1}{2}\ell n3$

67. If α, β, γ are the roots of $x^3+2x^2+1=0$, then $\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix}=$

(a) 8 (b) 5

(c) –8 (d) 3

68. Differential equation corresponding to the family of curves $y=c_1+c_2e^{3x}, c_1$ and c_2 are arbitrary constants is

(a) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=0$ (b) $\frac{d^2y}{dx^2}-3y=0$

(c) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}=0$ (d) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$

69. Which of the following functions is not onto?

(a) $f:R\to R, f(x)=2x+5$

(b) $f:[0,2\pi]\to[-5,5], f(x)=3\cos x+4\sin x$

(c) $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[1,7], f(x)=3\sin x+4$

(d) $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to[1,7], f(x)=3\cos x+4$

70. The line of intersection of the planes $3x-y+z=1$ and $x+4y-2z=2$ is given by

(a) $\frac{x-1}{2}=\frac{y-2}{-7}=\frac{z}{-13}$

(b) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-1}{-13}$

(c) $\frac{x}{-2}=\frac{y-2}{7}=\frac{z-3}{13}$

(d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{13}$

71. The point of contact of tangent to the ellipse $4x^2+9y^2=1$ parallel to the line $y=\frac{8}{9}x$ is

(a) $\left(\frac{2}{5},\frac{1}{5}\right)$ (b) $\left(-\frac{2}{5},\frac{1}{5}\right)$

(c) $\left(-\frac{2}{5},-\frac{1}{5}\right)$ (d) None of these

72. Let D denote the collection of the words in English dictionary. Define relation R on D as $R=\{(p,q): p,q\in D$ and p, q have at least one letter common$\}$, then R is

(a) reflexive, not symmetric, transitive

(b) reflexive, not symmetric, not transitive

(c) equivalence relation

(d) not equivalence relation

73. If imaginary part of $\frac{2z+1}{iz+1}$ is 4, then locus of z represents

(a) circle with centre at $\left(-\frac{1}{2},5\right)$

(b) circle with centre at $\left(-\frac{1}{12},\frac{5}{6}\right)$

(c) straight line passing through $\left(-\frac{1}{2},3\right)$

(d) parabola with vertex at $\left(-\frac{1}{2},3\right)$

74. Let $f(x)=\tan^{-1}x$ and $g(x)=\frac{x}{1+x^2}, x>0$ then

(a) $f(x)<g(x)$ in $(0,\infty)$

(b) $f(x)\le g(x)$ in $[1,\infty)$

(c) $g(x)<f(x)$ in $(0,\infty)$

(d) None of these

75. $\int e^{\sec x}\sec^3 x(\sin^2 x+\cos x+\sin x+\sin x\cos x)dx=$

(a) $e^{\sec x}(\sec x+\tan x)+c$

(b) $e^{\sec x}(\sec^2 x+\sec x\tan x)+c$

(c) $e^{\sec x}(\sec^2 x+\tan x)+c$

(d) None of these

76. Let $A_1A_2A_3A_4A_5A_6$ be regular hexagon with vertices A_1 and A_4 at (1, 0) and (–1, 0) respectively. The equations representing A_2A_5 are

(a) $y=\pm\sqrt{3}x$ (b) $x=\pm\sqrt{3}y$

(c) $y=\pm x$ (d) None of these

77. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+4x+3}{x^2+x+1}\right)^x =$

(a) e^{-4} (b) e^3

(c) e^2 (d) e^4

Directions (Q. 78 - 82) : *Read the following questions and choose:*

(a) Both statements are True, and Statement-2 explains Statement-1.

(b) Both statements are True, but Statement-2 does not explain Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

78. Statement-1:

$\hat{i}\times(\vec{a}\times i)+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$.

Statement-2:

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$.

79. Statement-1:

p : If $a,b\in R$ then either $a>b$ or $a=b$ or $a<b$.

then ~p : $a,b\in R$ and neither $a>b$ nor $a=b$ nor $a<b$.

Statement-2:

$\sim(p\to q)$ is $p\wedge\sim q$.

80. Statement-1:

If A is 3×3 matrix, $|A| = 1$ and $AA' = I$, then $|A-I| = 0$.

Statement-2:

If $AA' = I$ then $A' = A^{-1}$.

81. Statement-1:

The normal to the curve represented by $x = a(\cos t+t\sin t)$ and $y = a(\sin t-t\cos t)$ at any point t, makes a constant angle with x-axis.

Statement-2:

The normal to $x = a(\cos t+t\sin t)$, $y = a(\sin t-t\cos t)$ at any point t is at a constant distance from origin.

82. Statement-1:

The number of ways of dividing 15 men and 15 women into 15 couples, each consisting a man and a woman is 1240.

Statement-2:

$1^2+2^2+.....+n^2=\dfrac{n(n+1)(2n+1)}{6}\ \forall\ n\in N$.

83. The length of the chord of the parabola $y^2 = 4ax$, whose slope is $\sqrt{2}$ and which passes through (4a, 0) is

(a) $2\sqrt{11}a$ (b) $4\sqrt{2}a$

(c) $8\sqrt{2}a$ (d) $6\sqrt{3}a$

84. $\sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\dfrac{2-\sqrt{3}}{4}}+\cos^{-1}\dfrac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right)=$

(a) 0 (b) 15°

(c) 30° (d) 45°

85. If the letters of the word 'ASSASSIN' are written down at random in a row, the probability that no two 'S' occurs together is

(a) $\dfrac{1}{35}$ (b) $\dfrac{1}{14}$

(c) $\dfrac{13}{14}$ (d) $\dfrac{1}{7}$

86. $\dfrac{4}{19}+\dfrac{44}{19^2}+\dfrac{444}{19^3}+.....$ upto ∞ terms $=$

(a) $\dfrac{38}{81}$ (b) $\dfrac{4}{19}$

(c) $\dfrac{36}{171}$ (d) None of these

87. Two dice are thrown simultaneously until a sum of 2 or 12 is obtained. If X denotes number of tosses required then $P(X\geq6\,|\,X\geq3) =$

(a) $\left(\dfrac{17}{18}\right)^3$ (b) $\left(\dfrac{17}{18}\right)^3\dfrac{1}{18}$

(c) $\left(\dfrac{17}{18}\right)^5$ (d) $\left(\dfrac{1}{18}\right)^3$

88. If $\sin\dfrac{A}{2},\sin\dfrac{B}{2},\sin\dfrac{C}{2}$ are in A.P. then distance of incentre of ΔABC from the vertices are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

89. The standard deviation of first 20 natural numbers is

(a) $\dfrac{\sqrt{133}}{2}$ (b) $\dfrac{133}{2}$

(c) $\sqrt{133}$ (d) $\dfrac{\sqrt{1015}}{2}$

90. $\overrightarrow{PQ}=3\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{QR}=-\hat{i}-2\hat{k}$ are adjacent sides of a parallelogram PQRS. Angle between its diagonals is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{2}$ (d) $\dfrac{2\pi}{3}$

ANSWERS

1. (c)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (c)
11. (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (a)
21. (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (c)
31. (b)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (c)
41. (c)	**42.** (d)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (b)
51. (c)	**52.** (c)	**53.** (a)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (d)	**60.** (c)
61. (b)	**62.** (b)	**63.** (a)	**64.** (d)	**65.** (b)	**66.** (a)	**67.** (a)	**68.** (c)	**69.** (d)	**70.** (c)
71. (b)	**72.** (d)	**73.** (b)	**74.** (c)	**75.** (a)	**76.** (a)	**77.** (b)	**78.** (a)	**79.** (a)	**80.** (c)
81. (d)	**82.** (d)	**83.** (d)	**84.** (a)	**85.** (b)	**86.** (a)	**87.** (a)	**88.** (c)	**89.** (a)	**90.** (a)

EXPLANATIONS

PHYSICS

1. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

$I_{max} = 9I_o$

$I_{min} = I_o$

2. $a = \left|\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\right| g$; $T = \frac{2 m_1 m_2 g}{(m_1 + m_2)}$

3. When a body is released from some height, its initial velocity $u = 0$. So the second equation of motion $h = ut + \frac{1}{2}gt^2$ reduces to $h = \frac{1}{2}gt^2$.

Therefore, $t = \sqrt{\frac{2h}{g}}$

It is independent of the mass.

4. As the shell is at rest initially and the explosion is under internal forces only, the centre of the mass remains at rest.

5. $f = \frac{nv}{4L}$, $n = 1, 3, 5, \ldots$

$\Rightarrow L = \frac{nv}{4f} = \frac{n\lambda}{4}$, $n = 1, 3, 5, \ldots$

6. $x = a\sin\left(\frac{2\pi}{T}\right)t$

For $x = \frac{a}{2}$, $\frac{a}{2} = a\sin\left(\frac{2\pi}{T}\right)t$

$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{6}$

$t = \frac{T}{12}$

Similarly, for $x = a$, $t = \frac{T}{4}$

So time $\frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

7. Rim will slip if its diameter increases to 30 cm. Hence $\Delta l = 0.1$ cm

$\Delta T = \frac{\Delta l}{l \times \alpha} = \frac{0.1}{29.9 \times 1.2 \times 10^{-5}} = 2787°C$

Rim to be heated to temperature

$2787 + 15 = 2802°C.$

8. $\frac{E_1}{E_2} = \frac{P^2}{2m_1} \div \frac{P^2}{2m_2}$

$\Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$

9. Temperature, Pressure and volume characterize the thermodynamic state of matter.

10. From Newton's law of viscosity,

$|F| = \eta A \frac{\Delta v}{\Delta y}$

$\therefore$ Dimension of

$\eta = \frac{\text{Dimensions of force}}{\text{Dimensions of area} \times \text{Dimensions of velocity gradient}}$

$[\eta] = \frac{MLT^{-2}}{(L^2)(T^{-1})} = ML^{-1}T^{-1}$

11. We know that range of the projectile is same for complementary angles.

$\therefore T_1 = \frac{2u\sin\theta}{g}$

$$T_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

and $R = \frac{u^2 \sin 2\theta}{g}$

Therefore, $T_1 T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2R}{g}$

$\therefore \; T_1 T_2 \propto R$

12. Angular momentum

$$\propto \frac{1}{\text{Angular frequency}} \propto \text{Kinetic energy}$$

$$\Rightarrow \; L = \frac{2K.E}{w}$$

$$\frac{L_1}{L_2} = \left(\frac{K.E_1}{w_1}\right) \times \frac{w_2}{KE_2} = 4$$

$$\Rightarrow \; L_2 = \frac{L}{4}$$

13. Linear momentum is a vector quantity whereas kinetic energy is scalar. Therefore, the kinetic energy of a system of particles is zero implies that the linear momentum of the system is zero but reverse is not necessarily true.

14. Here $u = x_1 + f$, $v = x_2 + f$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

So, $\frac{1}{x_1 + f} + \frac{1}{x_2 + f} = \frac{1}{f}$

So, $f = \sqrt{x_1 x_2}$

15. There are the forces acting separately on the ball and the walls being exerted by one on other during collisions, but the net force on the box and the ball system is zero.

Therefore, the velocity of the box plus the ball system remains same.

16. For first spring,

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \qquad \ldots (i)$$

For second spring,

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \qquad \ldots (ii)$$

The effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$\therefore$ Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$$\Rightarrow \qquad T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \qquad \ldots (ii)$$

From equations (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

$$\Rightarrow \; t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow \; t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$\therefore \; t_1^2 + t_2^2 = T^2$ [From equation (iii)]

17. At $t = 0, x = 0$

At $t = 4, x = 9 \times 4 - 3(4)^2 = 36 - 48 = -12$ m

So the average velocity $= -\frac{12}{4} = -3 \text{ ms}^{-1}$

18. There will be no change in number of moles if the vessels are joined by valve.

$$\Rightarrow \; \frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{P(V_1 + V_2)}{RT}$$

$$\Rightarrow \; \frac{P_1 V_1 T_2 + P_2 V_2 T_1}{T_1 T_2} = \frac{P(V_1 + V_2)}{T}$$

$$\Rightarrow \; T = \frac{P(V_1 + V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$$

Now according to Boyle's law,

$$P_1 V_1 + P_2 V_2 = P(V_1 + V_2)$$

Hence, $T = \frac{(P_1 V_1 + P_2 V_2) T_1 T_2}{(P_1 V_1 T_2 + P_2 V_2 T_1)}$

20. $y_1 = a\sin\omega t$

$$y_2 = \frac{a}{\sqrt{2}}\sin\omega t + \frac{a}{\sqrt{2}}\cos\omega t$$

$$= a\left(\cos\frac{\pi}{4}\right)\sin\omega t + a\left(\sin\frac{\pi}{4}\right)\cos\omega t$$

$$= a\sin\left(\omega t + \frac{\pi}{4}\right)$$

Now, $A_1 : A_2 = a : a = 1 : 1$

21. $u_1^2 = 2as_1$

$u_2^2 = 2as_2$

$$\frac{u_1^2}{u_2^2} = \frac{s_1}{s_2}$$

$$\Rightarrow s_2 = \frac{u_2^2 \cdot s_1}{u_1^2} = 24 \text{ m}$$

22. In free space, neither acceleration due to gravity nor external torque acts on the rotating solid sphere. Therefore, taking the same mass of sphere if radius is increased then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, the angular momentum will not change.

23. Net electric field will be equal to electric field due to single charge.

24. $h = \dfrac{2T\cos\theta}{R\rho g}$

$\theta = 90°$ $h = 0$

25. error = Least Count = 0.02 cm.

27. 9 main scale divisions (9 S)

= 10 vernier scale divisions (10 V)

$9\,S = 10\,V$

$\Rightarrow 10\,S - S = 10\,V$

$\Rightarrow 10\,(S - V) = S$

$$\text{Least Count} = S - V = \frac{S}{10} = \frac{2\text{ mm}}{10} = 0.2\text{ mm}$$

$= 0.02$ cm.

CHEMISTRY

31. pKb of $NH_4OH = 14 - 9.26 = 4.74$

$pOH = 14 - 9.35 = 4.65$

Now 1 mole of $(NH_4)_2SO_4$ gives two moles of NH_4^+,

Let $[(NH_4)_2SO_4] = x$ mol/lt.

$[NH_4^+] = 2x$ mol/lt.

$[NH_4OH] = 0.2$ mol/lt.

Now, $pOH = pK_b + \log\dfrac{[NH_4^+]}{[NH_4OH]}$

$$4.65 = 4.79 + \log\left(\frac{2x}{0.2}\right)$$

$$-0.14 = \log\left(\frac{2x}{0.1}\right) \Rightarrow x = 0.081 \text{ mol/lt.}$$

$= 0.0405$ mol in 500 ml

$= 0.0405 \times 132 = 5.35$ gm

32. Maximum number of hyper conjugating structures will be in case of (d) option.

33. (a) $Si + H_2O \longrightarrow SiO_2 + 2H_2$

(b) $Si + 2KOH + H_2O \longrightarrow K_2SiO_3 + 2H_2$

(c) $NH_3 + CO_2 \xrightarrow[80\text{-}100\text{ atm}]{200°C} NH_2CONH_2 + H_2$

(d) $AlN + NaOH + H_2O \xrightarrow{\text{boil}} NaAlO_2 + NH_3$

35. $\Delta G°$ for combustion of Hg becomes positive above this temperature, hence reverse reaction $HgO \longrightarrow Hg + \dfrac{1}{2}O_2$ becomes spontaneous.

36. $Fe^{2+} \longrightarrow Fe^{3+} + e^-$

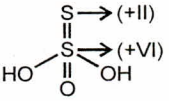
$6C^{2+} \longrightarrow 6C^{+4} + 12e^-$

$6N^{-3} \longrightarrow 6N^{+5} + 48e^-$

$\therefore\ n = 61$

37. More electronegative atom, the more –I effect.

38.



Difference = 6 – 2 = 4

39. $NO^- = 7 + 8 + 1 = 16$

$$\text{Bond order} = \frac{10 - 6}{2} = 2$$

$\sigma\,1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2$

$\pi 2p_x = \pi 2p_y^2, \pi^*2p_x^1 = \pi^*2p_y^1$

Bond order of isoelectronic CO, CN^-, N_2, NO^+ is same and each equal to three.

40. In $[CrO_4]^{2-}]$, chromium has d^3s hybridization.

41. $$\frac{p^0_{H_2O} - P_{\text{Solution}}}{p^0_{H_2O}} = \frac{n(\text{urea} + \text{sucrose})}{n_{\text{Total}}}$$

$$n_{\text{urea}} = \frac{6}{60} = 0.1$$

$$n_{\text{sucrose}} = \frac{68.4}{342} = 0.2$$

$$n_{H_2O} = \frac{180}{18} = 10$$

$$1 - \frac{P_{\text{solution}}}{23.8} = \frac{0.3}{10.3}$$

$P_{\text{solution}} = 23.1$ torr

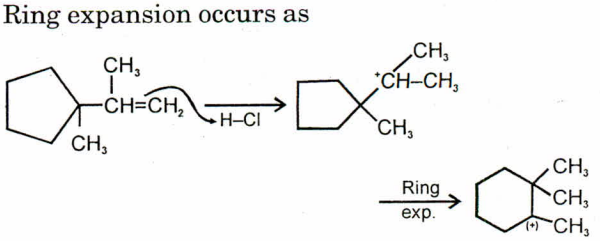
42. Acidity and basicity constant are lower.

44. $\underset{(A)}{ZnS} + H_2SO_4 \longrightarrow \underset{\text{Colourless}\ (C)}{ZnSO_4} + \underset{\text{Gas(colourless)}\ (B)}{H_2S}$

$\underset{(C)}{ZnSO_4} + 2NaOH \longrightarrow \underset{\text{White ppt.}}{Zn(OH)_2} + Na_2SO_4$

$Zn(OH)_2 + 2NaOH \longrightarrow \underset{\text{Clear solution (D)}}{Na_2ZnO_2} + 2H_2O$

45. Ring expansion occurs as



46. Due to lone pair-bond pair repulsion SF_4 don't have 90° bond angle.

47. No isomers are possible for such complex.

48. $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

$$E - E^\circ = -\frac{0.0591}{n}\log k$$

$$= -\frac{0.0591}{5}\log\frac{[Mn^{2+}][H_2O]^4}{[MnO_4^-][H^+]^8}$$

$$= -\frac{0.0591}{5}\log\frac{1}{(10^{-4})^8} = -0.38\ V$$

49. On removal of CO(g), equilibrium shifts towards right.

50. Potato starch = 25 (Gold No.)

Haemoglobin = 0.03 (Gold No.)

Egg albumin =Gum Arabic = 0.15 – 0.25 (gold no.)

51. $H_3BO_3 \xrightarrow{100°C} HBO_2 \xrightarrow{140°C}$

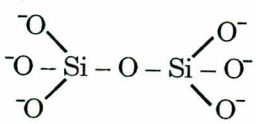
$$H_2B_4O_7 \xrightarrow[\text{heat}]{\text{Strong}} Br_2O_3$$

52. $C_2H_4(g) + 3O_2(g) \longrightarrow 2CO_2(g) + 2H_2O(\ell)$,

$\Delta n_g = -2$

$$\Delta H - \Delta E = \Delta n_g RT = \frac{-2\times 2\times 298}{1000} = -1.198\ \text{cal}$$

54.



55. ${}^a_c X \longrightarrow {}^b_d Y + x\,{}^4_2He + y\,{}^0_{-1}e$

$x \rightarrow \alpha$-particles emitted,

$y \rightarrow \beta$-particles emitted.

Now $a = b + 4x$ or $x = \dfrac{a-b}{4}$

$c = d + 2x - y$

$y = d - c + \dfrac{a-b}{2}$

56. Due to intramolecular H-bond.

57. Moles of monobasic acid in

$$400\ \text{ml} = \frac{0.2\times 400}{1000} = 0.08$$

$$\text{Moles of NaOH in 100 ml} = \frac{0.8}{1000}\times 100 = 0.08$$

$$\text{Mass of solution} = 500\ \text{ml} \times 1\ \frac{\text{gm}}{\text{ml}} = 500\ \text{gm}$$

$q = mC\Delta T = 500\times 4.2\times 1.17 = 2.457\times 10^3$ J

Heat evolved per mole of neutralisation

$$= \frac{2.457\times 10^3}{0.08}\ \text{J} = 30.71\ \text{kJ/mol}$$

59. $2CO + O_2 \rightleftharpoons 2CO_2 \qquad k = 40\times 10^{-3}$

$2H_2O \rightleftharpoons 2H_2 + O_2 \qquad k' = \dfrac{1}{36\times 10^{-2}}$

$2CO + 2H_2O \rightleftharpoons 2CO_2 + 2H_2 \qquad k'' = k\times k' = \dfrac{40}{360} = \dfrac{1}{9}$

Now k''' for $CO + H_2O \rightleftharpoons CO_2 + H_2$

$$= \sqrt{\frac{1}{9}} = 0.3$$

60. $\Delta T_b = 101.24 - 100 = 1.24, i = 1$

$$\Delta T_b = \frac{1000\times 0.512\times 288\times 1}{90\times M_A} = 1.24$$

$M_A = 1321.2$ gm/mol

Mol. mass of $C_nH_{2n}O_n = 30n$

So, $30n = 1321.2, n = 44$

So, molecular formula = $C_{44}H_{88}O_{44}$.

MATHEMATICS

61. Let the point be $(2\lambda - 2, 3\lambda - 1, 4\lambda - 1)$ then

$2(2\lambda - 2 - 3) + 3(3\lambda - 1 + 1) + 4(4\lambda - 1 - 11) = 0$.

$\Rightarrow 4\lambda - 10 + 9\lambda + 16\lambda - 48 = 0$

$\Rightarrow 29\lambda = 58 \Rightarrow \lambda = 2$

Hence foot of perpendicular is (2, 5, 7).

62. $I_1 = (\sqrt{2}+1)^6,\ F_1 = (\sqrt{2}-1)^6$

$I_1 + F_1 = (\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$= 2\left({}^6C_0(\sqrt{2})^6 + {}^6C_2(\sqrt{2})^4 + {}^6C_4(\sqrt{2})^2 + {}^6C_6(\sqrt{2})^0\right)$

$= 2(8 + 15\times 4 + 15\times 2 + 1) = 198$

$\Rightarrow$ As $[F_1] = 0,\ F_1 = \{F_1\} = 1 - \{I_1\}$

$\Rightarrow I_1 + 1 - \{I_1\} = 198$

$\Rightarrow [I_1] = 197$

$I_2 = (\sqrt{3}+1)^6, F_2 = (\sqrt{3}-1)^6$

$I_2 + F_2 = (\sqrt{3}+1)^6 + (\sqrt{3}-1)^6$

$= 2\left({}^6C_0(\sqrt{3})^6 + {}^6C_2(\sqrt{3})^4 + {}^6C_4(\sqrt{3})^2 + {}^6C_6\right)$

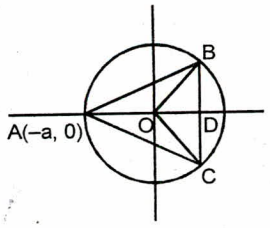
$= 2(27 + 15\times 9 + 15\times 3 + 1) = 416$

$\Rightarrow F_2 = \{F_2\} = 1 - \{I_2\}$

$[I_2] = 415$

$[I_1] + [I_2] = 197 + 415 = 612$

63.



$\angle A = 180° - 75° - 75° = 30°$

$\Rightarrow \angle BOC = 2\angle A = 60°$

$\frac{BD}{a} = \sin 30° \Rightarrow BD = \frac{a}{2}$

$\Rightarrow$ BC = a

64. $z = \frac{1+i}{\sqrt{3}-i}$

$\arg(z) = \arg(1+i) - \arg(\sqrt{3}-i) + 2n\pi$

$= \frac{\pi}{4} - \left(-\frac{\pi}{6}\right) + 2n\pi = \frac{5\pi}{12} + 2n\pi$

65. $f(a) = \lim_{x\to a} f(x) = \lim_{x\to a} \sin\left(\frac{x-a}{2}\right)\tan\left(\frac{\pi x}{2a}\right)$

$= \lim_{h\to 0} \sin\left(\frac{h}{2}\right)\tan\left(\frac{\pi}{2a}(a+h)\right)$

$= \lim_{h\to 0} \sin\left(\frac{h}{2}\right)\tan\left(\frac{\pi}{2} + \frac{\pi h}{2a}\right)$

$= -\lim_{h\to 0} \sin\left(\frac{h}{2}\right)\cot\left(\frac{\pi h}{2a}\right)$

$= -\lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\tan\left(\frac{\pi h}{2a}\right)} = -\lim_{h\to 0} \frac{\frac{h}{2}}{\frac{\pi h}{2a}} = -\frac{a}{\pi}$

66. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^2}{r^3+n^3}$

$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{nr^2}{r^3+n^3} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{\frac{r^2}{n^2}}{\frac{r^3}{n^3}+1}$

$= \int_0^1 \frac{x^2}{x^3+1} = \frac{1}{3}\left|\ell n\left|x^3+1\right|\right|_0^1 = \frac{1}{3}\ell n 2$

67. $x^3 + 2x^2 + 1 = 0$

$\alpha + \beta + \gamma = -2$

$\alpha\beta + \alpha\gamma + \beta\gamma = 0$

$\alpha\beta\gamma = -1$

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 3\alpha\beta\gamma - \alpha^3 - \beta^3 - \gamma^3$

$= -(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$

$= -(\alpha+\beta+\gamma)\left((\alpha+\beta+\gamma)^2 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)\right)$

$= -(\alpha+\beta+\gamma)^3 = 8$

68. $y = c_1 + c_2 e^{3x}$

$y' = 3c_2 e^{3x}$

$y'' = 9c_2 e^{3x} = 3y' \Rightarrow \frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$

69. (a) $x \in R \Rightarrow f(x) \in R$

(b) $x \in [0, 2\pi]$ $f(x) \in [-5, 5]$

(c) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $f(x) \in [1, 7]$

(d) $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $f(x) \in [4, 7] \neq$ co-domain

70. $3x - y + z = 1$

$x + 4y - 2z = 2$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix} = -2\hat{i} + 7\hat{j} + 13\hat{k}$

Let $z = 0$, $3x - y = 1$

$x + 4y = 2$

$\Rightarrow$ $x = \frac{6}{13}$, $y = \frac{5}{13}$

Equation of line is given by

$$\frac{x - \frac{6}{13}}{-2} = \frac{y - \frac{5}{13}}{7} = \frac{z}{13}$$

Let $x = 0$, $-y + z = 1$

and $4y - 2z = 2$

$y = 2$, $z = 3$

$\Rightarrow$ $\frac{x}{-2} = \frac{y-2}{7} = \frac{z-3}{13}$

71. $4x^2 + 9y^2 = 1 \Rightarrow \frac{x^2}{1/4} + \frac{y^2}{1/9} = 1$

$a^2 = \frac{1}{4}$, $b^2 = \frac{1}{9}$, $m = \frac{8}{9}$

Point of contact is given by $\left(-\frac{ma^2}{c}, \frac{b^2}{c}\right)$.

$c^2 = a^2m^2 + b^2 = \frac{1}{4} \times \frac{64}{81} + \frac{1}{9} = \frac{16+9}{81} = \frac{25}{81}$

$\Rightarrow$ $c = \pm\frac{5}{9}$

$\left(\frac{-\frac{8}{9}\times\frac{1}{4}}{\frac{5}{9}}, \frac{\frac{1}{9}}{\frac{5}{9}}\right)$ *i.e.*, $\left(-\frac{2}{5}, \frac{1}{5}\right)$ or $\left(\frac{2}{5}, -\frac{1}{5}\right)$.

72. R is not transitive as (abc, ade), (ade, dfg) $\in$ R but (abc, dfg) $\notin$ R .

R is reflexive as well as symmetric.

73. $\text{Im}\left(\dfrac{2z+1}{iz+1}\right)=4$

$z = x + iy$

$$\frac{2x+1+i2y}{-y+1+ix}\times\frac{-y+1-ix}{-y+1-ix}$$

$$\Rightarrow \frac{-x(2x+1)+2y(-y+1)}{(-y+1)^2+x^2}=4$$

$$\Rightarrow -2x^2-x-2y^2+2y=4x^2+4y^2-8y+4$$

$$\Rightarrow 6x^2+6y^2+x-10y+4=0$$

which represents circle with centre $\left(-\dfrac{1}{12},\dfrac{5}{6}\right)$.

74. $f(x)=\tan^{-1}x,\ g(x)=\dfrac{x}{1+x^2},\ x>0$

$$h(x)=f(x)-g(x)=\tan^{-1}x-\frac{x}{1+x^2}$$

$$h'(x)=\frac{1}{1+x^2}-\frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$=\frac{1+x^2-1+x^2}{(1+x^2)^2}=\frac{2x^2}{(1+x^2)^2}>0$$

$\Rightarrow h(x)$ is an increasing function.

$\Rightarrow x>0 \Rightarrow h(x)>h(0) \Rightarrow f(x)>g(x)$ in $(0,\infty)$.

75. $\int e^{\sec x}\sec^3 x(\sin^2 x+\cos x+\sin x+\sin x\cos x)dx$

$\sec x = t$

$\sec x \tan x\, dx = dt$

$$\int e^t t^2\left(\frac{\sin^2 x+\cos x+\sin x+\sin x\cos x}{\tan x}\right)dt$$

$$=\int e^t t^2\left(\sin x\cos x+\frac{\cos^2 x}{\sin x}+\cos x+\cos^2 x\right)dt$$

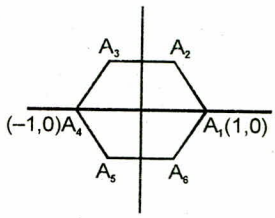
$$=\int e^t\left(\tan x+\frac{1}{\sin x}+\frac{1}{\cos x}+1\right)dt$$

$$=\int e^t\left(\sqrt{t^2-1}+t+1+\frac{1}{\sqrt{1-\frac{1}{t^2}}}\right)dt$$

$$=\int e^t\left(\sqrt{t^2-1}+t+1+\frac{t}{\sqrt{t^2-1}}\right)dt$$

$$=e^t\left(\sqrt{t^2-1}+t\right)+c=e^{\sec x}(\tan x+\sec x)+c$$

76.



$$A_2(\cos 60°, \sin 60°)=A_2\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$

Equation of A_2A_5 is $y=\sqrt{3}x$

If vertices taken in clockwise sense then equation is $y=-\sqrt{3}x$.

77. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+4x+3}{x^2+x+1}\right)^x$

$$=e^{\lim\limits_{x\to\infty}x\left(\frac{x^2+4x+3}{x^2+x+1}-1\right)}$$

$$=e^{\lim\limits_{x\to\infty}x\left(\frac{x^2+4x+3-x^2-x-1}{x^2+x+1}\right)}$$

$$=e^{\lim\limits_{x\to\infty}\frac{3x^2+2x}{x^2+x+1}}=e^3$$

78. $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})$

$$=\vec{a}-(\hat{i}\cdot\vec{a})\hat{i}+\vec{a}-(\hat{j}\cdot\vec{a})\hat{j}+\vec{a}-(\hat{k}\cdot\vec{a})\hat{k}$$

$$=3\vec{a}-\vec{a}=2\vec{a}$$

79. A : $a,b\in$ R

B : $a>b$

C : $a=b$

D : $a<b$

p : A $\to$ (B $\vee$ C $\vee$ D)

which is logically equivalent to $\sim$ A $\vee$ (B $\vee$ C $\vee$ D)

$\sim p = \sim$ ($\sim$ A $\vee$ (B $\vee$ C $\vee$ D))

$=$ A $\wedge \sim$ (B $\vee$ C $\vee$ D)

$=$ A $\wedge$ ($\sim$ B $\wedge \sim$ C $\wedge \sim$ D)

$\sim p$: $a,b\in$ R and neither $a>b$ nor $a=b$ nor $a<b$.

$p\to q$ is equivalent to $\sim p\vee q$

$\sim(p\to q)\equiv\sim(\sim p\vee q)\equiv(p\wedge\sim q)$

80. $A_{3\times3}$ matrix, $|A|=1$

$AA'=I$

$$|A-I|=|(A-I)'|=|A'-I|=|A'-AA'|$$

$$=|(I-A)A'|=|I-A|.|A'|=|-(A-I)|=-|A-I|$$

$\Rightarrow |A-I|=0$

$AA'=I\neq A'=A^{-1}$

Let $A=\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}$

$$\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}\cdot\begin{pmatrix}1&0\\0&1\\0&0\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

Here $A\cdot A'=I$ but A^{-1} does not exist.

81. $x=a(\cos t+t\sin t)$,

$y=a(\sin t-t\cos t)$

$$\frac{dy}{dx}=\frac{a(\cos t-\cos t+t\sin t)}{a(-\sin t+\sin t+t\cos t)}=\frac{\sin t}{\cos t}$$

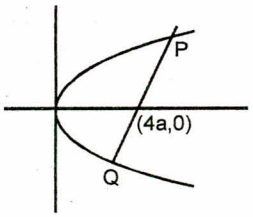
Equation of normal at t point is given by

$\cos t\,x+\sin t\,y=a$, which makes $\frac{\pi}{2}+t$ angle with x-axis, that varies with t.

But it is at constant distance from origin.

82. 15 men and 15 women can be divided into 15 couples each consisting of a man and a woman in 15! ways.

83. Equation of chord is $\sqrt{2}x-y=4a\sqrt{2}$



Let $P(at^2,2at)$

$$\Rightarrow \frac{2at}{at^2-4a}=\sqrt{2}$$

$$\Rightarrow \sqrt{2}t=t^2-4$$

$$\Rightarrow t^2-\sqrt{2}t-4=0$$

$$\Rightarrow t=\frac{\sqrt{2}\pm\sqrt{2+16}}{2}=\frac{\sqrt{2}\pm3\sqrt{2}}{2}=2\sqrt{2},-\sqrt{2}$$

$P\left(8a,4\sqrt{2}a\right)$, $Q\left(2a,-2\sqrt{2}a\right)$

$$PQ=\sqrt{36a^2+72a^2}=6\sqrt{3}\,a$$

84. $$\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}$$

$$=\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+\frac{\pi}{6}+\frac{\pi}{4}$$

$$=\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi}{4}=\frac{\pi}{2}$$

85. ASSASSIN

$$\text{Total arrangements}=\frac{8!}{4!2!}=\frac{8\cdot7\cdot6\cdot5}{2}=840$$

Arrangements in which no two S are together is

$$\frac{4!}{2!}\times{}^5C_4=12\times5=60$$

$$\text{Probability}=\frac{60}{840}=\frac{1}{14}$$

86. $$S=\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+\ldots.$$

$$\frac{1}{19}S=\frac{4}{19^2}+\frac{44}{19^3}+\ldots.$$

$$\frac{18}{19}S=\frac{4}{19}+\frac{40}{19^2}+\frac{400}{19^3}+\ldots..=\frac{\frac{4}{19}}{1-\frac{10}{19}}=\frac{4}{9}$$

$$S=\frac{4}{9}\times\frac{19}{18}=\frac{38}{81}$$

87. $$\frac{P(X\geq6\cap X\geq3)}{P(X\geq3)}=\frac{P(X\geq6)}{P(X\geq3)}$$

$$=\frac{\left(\frac{17}{18}\right)^5\frac{1}{18}+\left(\frac{17}{18}\right)^6\frac{1}{18}+\ldots.}{\left(\frac{17}{18}\right)^2\frac{1}{18}+\left(\frac{17}{18}\right)^3\frac{1}{18}+\ldots}$$

$$=\left(\frac{17}{18}\right)^3$$

88. $IA=r\operatorname{cosec}\frac{A}{2}$, $IB=r\operatorname{cosec}\frac{B}{2}$, $IC=r\operatorname{cosec}\frac{C}{2}$

$\Rightarrow$ IA, IB, IC are in H.P.

89. $$\text{Variance}=\frac{1}{20}\sum_{r=1}^{20}r^2-\left(\frac{1}{20}\sum_{r=1}^{20}r\right)^2$$

$$=\frac{1}{20}\times\frac{20\times21\times41}{6}-\left(\frac{1}{20}\times\frac{20\times21}{2}\right)^2$$

$$=\frac{287}{2}-\frac{441}{4}=\frac{133}{4}$$

$$\text{Standard deviation}=\frac{\sqrt{133}}{2}$$

90. Diagonals are given by $2\hat{i}-2\hat{j}$ and $4\hat{i}-2\hat{j}+4\hat{k}$.

$$\cos\theta=\frac{8+4}{\sqrt{4+4}\sqrt{16+4+16}}=\frac{12}{12\sqrt{2}}$$

$$\Rightarrow\theta=\frac{\pi}{4}$$

■■