FIITJEE FARIDABAD

MOCK PRACTICE PAPER FOR

JEE -Mains- 2020

MOCK PRACTICE PAPER-24

Time: 3 hours *Maximum marks: 360*

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.

2. The Test Booklet consists of **90** questions. The maximum marks are **360.**

3. There are ***Three*** parts in the question paper. **Part 1: Physics, Part 2: Chemistry** and **Part 3 is Mathematics**. Each question is allotted **4 (four)** marks for correct response.

4. *Candidates will be awarded marks as stated above in instruction No. 3 for correct response of each question. –1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*

5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

Name of the Candidate :____

Batch :____________________ **Date of Examination :**______________________________________

Enrolment Number :____

PRACTICE SET - 9

Time : 3 Hours | **Maximum Marks : 360**

PHYSICS

1. A physical quantity a depends upon two other physical quantities b and c as follows:

$$a = b^4 c^{1/4}$$

In an experiment, the quantity b is determined by measuring a and c, and using the above equation. If the percentage of error in the measurement of a and c are 8% and 16% respectively, then the percentage of error in the determined value of b is

(*a*) 1% (*b*) 2%

(*c*) 3% (*d*) 4%

2. The method of dimensional analysis can be used to derive which of the following relations?

(*a*) $N_0 e^{-\lambda t}$ (*b*) $A\sin(\omega t + kx)$

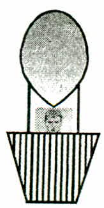
(*c*) $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ (*d*) none of these

3. Two particles are projected horizontally in opposite directions in the same vertical plane from the same height (5m) at $t = 0$ with velocities 12 m/s and 3 m/s. The instant when their velocities become mutually perpendicular is ($g = 10$ m/s^2)

(*a*) 0.4 s (*b*) 0.6 s

(*c*) 0.8 s (*d*) 1.0 s

4. A man in a balloon throws a stone downwards with a speed of 5 m/s with respect to balloon. The balloon is moving upward with a constant acceleration of 5 m/s^2. Then velocity of the stone relative to the man after 2 seconds is (Take $g = 10$ m/s^2)



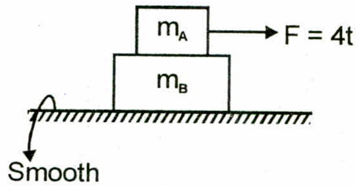
(*a*) 10 m/s

(*b*) 30 m/s

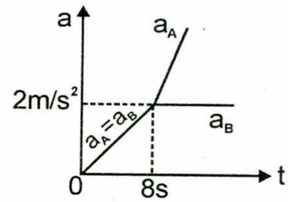
(*c*) 15 m/s

(*d*) 35 m/s

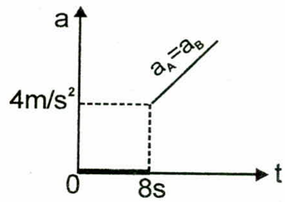
5. A block of mass $m_A = 4$ kg is kept over another block of mass $m_B = 8$ kg which is kept on a smooth horizontal surface. The coefficient of friction between the blocks is 0.4. A time varying horizontal force $F = 4t$ is applied at $t = 0$ on 4 kg mass. Then the acceleration time graph of the masses A and B is ($g = 10$ m/s^2)



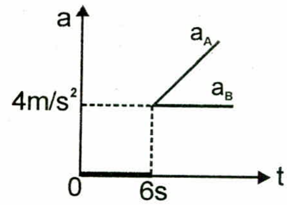
(*a*)



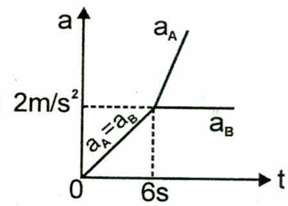
(*b*)



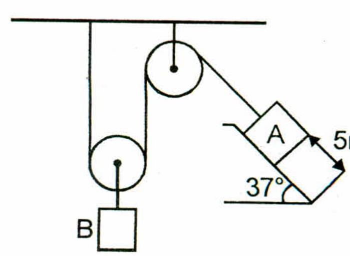
(*c*)



(*d*)



6. The blocks A and B shown in the figure have masses $M_A = 5$ kg and $M_B = 4$ kg. The system is released from rest. The speed of B after A has traveled a distance 1 m along the incline is



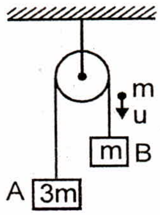
(*a*) $\frac{\sqrt{3}}{2}\sqrt{g}$

(*b*) $\frac{\sqrt{3}}{4}\sqrt{g}$

(*c*) $\frac{\sqrt{g}}{2\sqrt{3}}$

(*d*) $\frac{\sqrt{g}}{2}$

7. Two blocks A and B are connected by an inextensible massless string as shown. The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass m moving with a velocity u as shown hits the block B and gets embedded into it. The impulse imparted by tension force to the block of mass 3m is



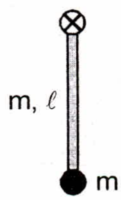
(a) $\frac{5mu}{4}$

(b) $\frac{4mu}{5}$

(c) $\frac{2mu}{5}$

(d) $\frac{3mu}{5}$

8. A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. The minimum speed given to the particle so that the rod performs circular motion in a vertical plane will be



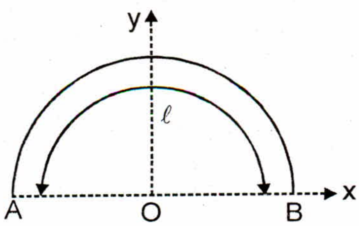
(a) $\sqrt{5g\ell}$

(b) $\sqrt{4g\ell}$

(c) $\sqrt{4.5g\ell}$

(d) $\sqrt{3.5g\ell}$

9. Gravitational field at the centre of a semi-circle formed by a thin wire AB of mass m and length ℓ is



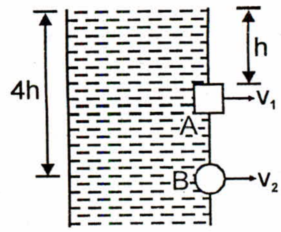
(a) $\frac{Gm}{\ell^2}$ along +x axis

(b) $\frac{Gm}{\pi\ell^2}$ along +y axis

(c) $\frac{2\pi Gm}{\ell^2}$ along +x axis

(d) $\frac{2\pi Gm}{\ell^2}$ along +y axis

10. A square hole of side l is made at a depth of h and a circular hole of radius r is made at a depth of 4 h from the surface of water in a water tank kept on a horizontal surface. If $l << h, r << h$ and the rate of flow of water through both the holes is the same, then r is equal to



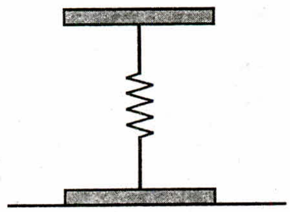
(a) $\frac{l}{3\pi}$

(b) $\frac{l}{\sqrt{3\pi}}$

(c) $\frac{l}{\sqrt{2\pi}}$

(d) $\frac{l}{2\pi}$

11. Two plates of same mass are attached rigidly to the two ends of a massless spring. One of the plates rests on a horizontal surface and the other results a compression y of the spring when it is in equilibrium state. The additional minimum compression required, so that when the force causing compression is removed the lower plate is lifted just off the surface, will be



(a) 0.5y (b) 3y

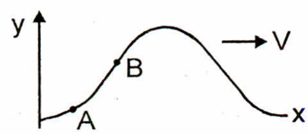
(c) 2y (d) y

12. A structural steel rod has a radius of 10 mm and length of 1.0 m. A 100 kN force stretches it along its length. Young's modulus of structural steel is 2×10^{11} Nm^{-2}. The percentage strain will be about

(a) 0.24% (b) 0.32%

(c) 0.08% (d) 0.16%

13. A wave pulse is generated in a string that lies along x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratio of wave speed to the particle speed respectively, then



(a) $R_A > R_B$

(b) $R_B > R_A$

(c) $R_A = R_B$

(d) Information is not sufficient to decide

14. Focal length of objective lens of an astronomical telescope is 1.5 m. To get minimum magnification of 25 times, the focal length of eyepiece should be

(a) 0.06 m (b) 2.5 m

(c) 1.07 m (d) 1.35 m

15. In Young's double slit experiment with light of wavelength λ, fringe pattern on the screen has fringe width β. When two thin transparent glass (refractive index μ) plates of thickness t_1 and t_2 ($t_1 > t_2$) are placed in the path of the two beams respectively, the fringe pattern will shift by a distance :

(a) $\frac{\beta(\mu-1)}{\lambda}\left(\frac{t_1}{t_2}\right)$ (b) $\frac{\mu\beta}{\lambda}\frac{t_1}{t_2}$

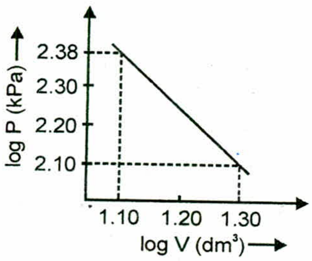
(c) $\frac{\beta(\mu-1)}{\lambda}(t_1-t_2)$ (d) $(\mu-1)\frac{\lambda}{\beta}(t_1+t_2)$

16. A pendulum clock keeps correct time at 0°C. Its mean coefficient of linear expansions is α °C^{-1}. The loss in seconds per day by the clock if the temperature rises by t°C is

(a) $\dfrac{\frac{1}{2}\alpha t\times 864000}{1-\frac{\alpha t}{2}}$ (b) $\frac{1}{2}\alpha t\times 86400$

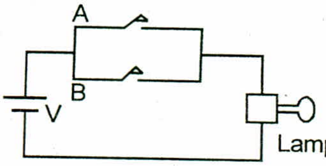
(c) $\dfrac{\frac{1}{2}\alpha t\times 86400}{\left(1+\frac{\alpha t}{2}\right)^2}$ (d) $\dfrac{\frac{1}{2}\alpha t\times 86400}{1+\frac{\alpha t}{2}}$

17. Logarithms of readings of pressure and volume for an ideal gas were plotted on a graph as shown in figure. By measuring the gradient, it can be shown that the gas may be



(a) monoatomic undergoing an adiabatic change
(b) monoatomic undergoing an isothermal change
(c) diatomic undergoing an adiabatic change
(d) triatomic undergoing an isothermal change

18. Which logic gate with inputs A and B performs the same operation as that performed by the following circuit?



(a) NAND gate (b) OR gate
(c) AND gate (d) NOR gate

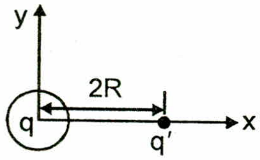
19. A point charge q is placed at the centre of a thin spherical metallic shell of radius less than R as shown in the figure. Another point charge q' is placed at a distance 2R from the centre of the shell. The force exerted by the induced charges of the shell on the charge q is

(a) 0

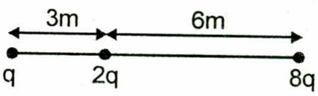
(b) $\frac{1}{4\pi\epsilon_0}\frac{qq'}{R^2}\hat{i}$

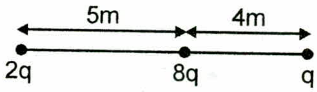
(c) $\frac{1}{16\pi\epsilon_0}\frac{qq'}{R^2}(-\hat{i})$

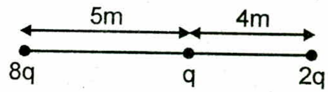
(d) $\frac{qq'}{16\pi\epsilon_0 R^2}\hat{i}$

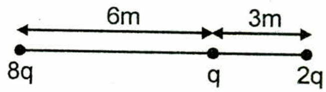


20. Three point charges q, 2q and 8q are kept on a straight line of 9 m length so as to make the electrostatic potential energy of the system minimum. Neglect gravitational potential energy. The charges should be arranged as

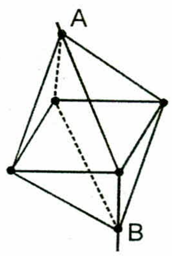
(a)


(b)


(c)


(d)


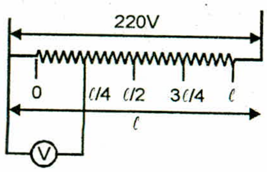
21. Each of the twelve wires of the circuit as two tetra pyramid skeletons as shown in the figure has a resistance R ohms. The equivalent resistance of the circuit between the junctions A and B, is



(a) R/2 (b) R
(c) R/3 (d) R/4

22. A 12 kΩ rheostat is used in an experiment when 220 V potential difference is maintained across it. Voltmeter shown in figure is of 6 kΩ. Main current in the circuit is

(a) 0.01 A
(b) 0.02 A
(c) 0.03 A
(d) 0.04 A



23. A charged particle of charge +q and mass m is projected with velocity, $\vec{v} = \frac{v_0}{\sqrt{2}}\hat{i} + \frac{v_0}{\sqrt{2}}\hat{j}$ in the external uniform magnetic field $\vec{B} = B_0\hat{i}$ and external uniform electric field $\vec{E} = \frac{-E_0}{\sqrt{2}}\hat{i} + \frac{E_0}{\sqrt{2}}\hat{j}$. At what time the speed of the particle will be minimum?

(a) $\frac{2mv_0}{qE_0}$ (b) $\frac{mv_0}{qE_0}$

(c) $\frac{mv_0\sqrt{2}}{qE_0}$ (d) $\frac{mv_0}{qE_0\sqrt{2}}$

24. Statement-1: Electromagnetism is relativistic.

Statement-2: When we move along with the charge so that there is no motion of charge relative to us, we find no magnetic field associated with the charge.

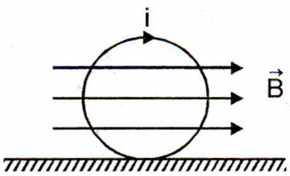
(a) Both statements are True and Statement-2 explains Statement-1.

(b) Both statements are True but Statement-2 does not explain Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

25. A conducting rigid ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane with the plane of ring in the vertical plane. The ring carries a current *i* = 4A. A horizontal magnetic field B = 10T is switched on at time *t* = 0 as shown in figure. The initial angular acceleration of the ring will be



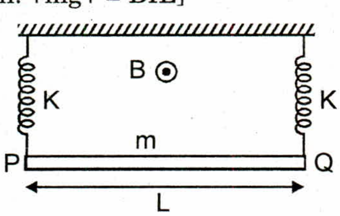
(a) 40π rad/s²

(b) 20π rad/s²

(c) 5π rad/s²

(d) 15π rad/s²

26. A conducting rod of mass m and length L is connected at the ends by two identical massless springs each of spring constant K as shown in the figure. Initially the system is in equilibrium. A uniform magnetic field of magnitude B directed perpendicular to the plane of the paper outwards also exists in the region. A constant current I is switched on that passes from P to Q through the rod. Further maximum elongation in the spring is [Given: |mg| = BIL]



(a) $\frac{BIL}{K}$ (b) $\frac{BIL}{4K}$

(c) $\frac{BIL}{8K}$ (d) $\frac{BIL}{16K}$

27. A sample originally contained 10^{20} radioactive atoms, which emits α particles. The ratio of α particles emitted in the third year to that emitted during the second year is 0.3. How many α particles were emitted in the first year?

(a) 5×10^{18}

(b) 3×10^{18}

(c) 7×10^{19}

(d) 3×10^{19}

28. A circular hole, of diameter R is cut from a disc of mass M and radius R; the circumference of the cut passes through the centre of the disc. The moment of inertia of the remaining portion of the disc about an axis perpendicular to the disc and passing through its centre is

(a) $\left(\frac{15}{32}\right)MR^2$ (b) $\left(\frac{3}{8}\right)MR^2$

(c) $\left(\frac{1}{8}\right)MR^2$ (d) $\left(\frac{13}{32}\right)MR^2$

29. Statement-1: Nuclear energy is due to the difference in the sum of the masses of the constituent nucleons and the nucleus.

Statement-2: The mass of the nucleus is more than the sum of the masses of the constituent nucleons.

(a) Both statements are True and Statement-2 explains Statement-1.

(b) Both statements are True but Statement-2 does not explain Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

30. Statement-1: Heavy water is preferred to normal water in reactors to slow down neutrons.

Statement-2: Deuterium in D_2O does not form stable nuclei on absorbing neutron, but protons in H_2O forms.

(a) Both statements are True and Statement-2 explains Statement-1.

(b) Both statements are True but Statement-2 does not explain Statement-1.

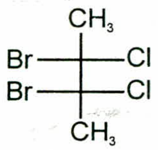
(c) Statement-1 is True, Statement-2 is False.

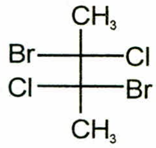
(d) Statement-1 is False, Statement-2 is True.

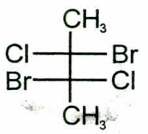
CHEMISTRY

31. $H_3C-C\equiv C-CH_3 \xrightarrow[\text{(ii) } Cl_2(1\text{ eq.})/CCl_4]{\text{(i) } Br_2(1\text{ eq.})/CCl_4}$

The final product of the reaction is

(a) 

(b) 

(c) 

(d) All of these

32. Equal volumes of 0.1M urea solution and 0.1M NaCl solutions are mixed, the resulting solution will have:

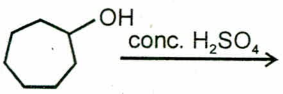
(a) Lower osmotic pressure than 0.1M urea solution

(b) Lower osmotic pressure than 0.1M urea solution

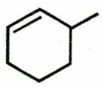
(c) Higher osmotic pressure than 0.1M urea solution as well as 0.1M NaCl solution

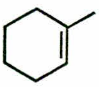
(d) Same osmotic pressure as 0.1M urea as well as 0.1M NaCl solution

33. The major product of the following reaction is



(a) 

(b) 

(c) 

(d) 

34. The circumference of an orbit in H-atom is 8.3053×10^{-9} metre. Electron jumps from this orbit to ground state energy level emitting a radiation of wavelength 'λ'. The value of 'λ' is ($R_H = 1.1 \times 10^7\ m^{-1}$)

(a) 950 Å (b) 970 Å

(c) 1022 Å (d) 1500 Å

35. 500 ml of 0.2 M CH_3COOH solution is mixed with 500 ml 0.2M NaOH solution. The pH of resulting solution if $K_a(CH_3COOH) = 10^{-5}$

(a) 7.5 (b) 6.9

(c) 8.5 (d) 9.5

36. The oxidation number of Fe in $Fe(CO)_5$ is

(a) 0 (b) +2

(c) +3 (d) +5

37. Consider the following reactions

(i) A $\longrightarrow$ P (I^{st} order)

(ii) B $\longrightarrow$ P (n^{th} order)

The initial concentration of B was two times to the initial concentration of A and initial $t_{1/2}$ of A and B was 17.5 and 5 minutes respectively. If [A] = [B] after 35 minutes then the value of 'n' is

(a) 0 (b) 2

(c) 3 (d) none of these

38. Which of the following compound gives positive test with Tollen's reagent?

(a) But-2-ene (b) 2-methylbut-2-ene

(c) But-2-yne (d) 3-methylpentyne

39. Which of the following test is related to carbohydrates?

(a) Beilstein Test (b) Molisch's Test

(c) Lassaigne's Test (d) Millon's Test

40. Consider the following ground state electronic configuration for nitrogen

(i) 

(ii) 

(iii) 

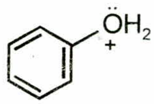
(iv) 

Aufbau principle is not followed by electronic configuration in

(a) (i) and (iv) (b) (i) and (ii)

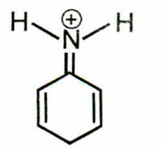
(c) (ii) and (iii) (d) (i) and (ii)

41. Which of the following molecule can not exhibit resonance?

(a) 

(b) $CH_3-\ddot{O}-CH=CH_2$

(c) 

(d) 

42. SrF_2 is dissolved in water to form saturated solution at 25°C. The concentration of F^- in this saturated solution is found to be 2×10^{-3} moles/L. The solubility product of SrF_2 at 25°C is-

(a) 16×10^{-9} (b) 64×10^{-9}

(c) 4×10^{-9} (d) 8×10^{-9}

43. Which of the following statement is correct for α – D – glucose when dissolved in water?

(*a*) Its specific rotation changes from +112° to 52.7° on standing solution.

(*b*) In solution it has equilibrium between open chain structure and cyclic structure of α-D-Glucose

(*c*) On dissolution in water glucose gets hydrolyse to give fructose

(*d*) Only (*a*) and (*b*) are correct

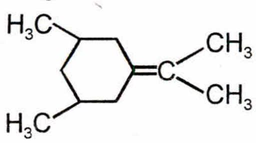
44. At 100°C and 1 atm pressure the density of water vapours is 5.97×10^{-4} gm/ml. The compressibility factor 'Z' is

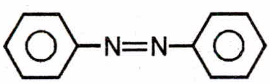
(*a*) 0.523 (*b*) 0.984

(*c*) 0.914 (*d*) 0.808

45. Among the following compounds, which can exhibit geometrical isomerism

(I) $CH_3CH{=}NHOH$

(II) 

(III) 

(*a*) Only I (*b*) Only III

(*c*) I & III (*d*) I, II, & III

46. Which of the following is correct match for the stoichiometric coefficient of reactants and products of the following balanced redox reaction.

$$aC_2O_4^{2-} + bMnO_4^- + cH^+ \longrightarrow xCO_2 + yMn^{2+} + zH_2O$$

(*a*) $a = 2, b = 5, c = 8$ (*b*) $x = 10, y = 2, z = 8$

(*c*) $a = 5, b = 2, c = 8$ (*d*) $x = 10, y = 1, z = 16$

47. Chemical reaction $2A + B_2 \longrightarrow 2AB$ follows the following reaction mechanism

$$A + A \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} A_2 \text{ (fast)};$$

$$A_2 + B_2 \xrightarrow{K_2} 2AB \text{ (slow)}$$

The rate expression of the reaction is

(*a*) $\text{Rate} = \frac{K_1}{K_{-1}}[A]^2[B_2]$

(*b*) $\text{Rate} = \frac{K_2}{K_{-1}K_1}[A][B_2]$

(*c*) $\text{Rate} = \frac{K_1K_2}{K_{-1}}[A]^2[B_2]$

(*d*) $\text{Rate} = K_2[A]^{1/2}[B_2]^2$

48. The IUPAC name of the compound $[Cr(NH_3)_4(H_2O)_2]Cl_3$ is

(*a*) Tetramminediaquachromium (III) chloride

(*b*) Diaquatetramminechromium (III) chloride

(*c*) Tetramminediaquachromium (V) chloride

(*d*) Diaquatetramminechromium (V) trichloride

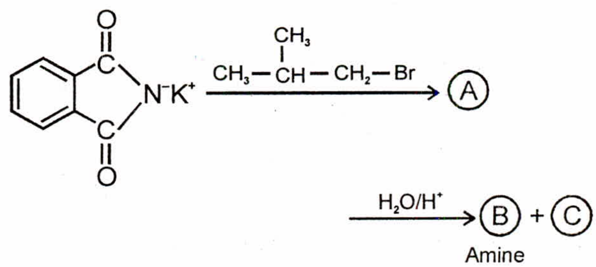
49. A V litre vessel contains a mixture of He and H_2 gases, at T kelvin temperature and 1 atm pressure. If the mass of H_2 is two times to the mass of He in the container then

(*a*) Partial pressure of He is 0.8 atm

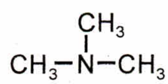
(*b*) Partial pressure of H_2 is 0.6 atm

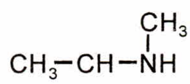
(*c*) Partial pressure of He is 0.2 atm

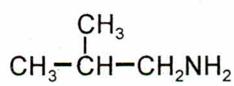
(*d*) Each has a partial of 0.5 atm.

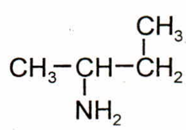
50. 

The amine (B) as the product of reaction is

(*a*) 

(*b*) 

(*c*) 

(*d*) 

51. At 25° and 1 atm pressure the heat evolved during combustion of 12 gm of C (graphite) is –393.5 kJ. The internal energy change of the reaction is

(*a*) –393.5 kJ/mole

(*b*) 393.5 kJ/mole

(*c*) Less than 393.5 kJ/mole

(*d*) Internal energy change is zero

52. 1 mole $CH_3COONH_4(s)$ is dissolved in 2 litre water to form an aq solution. If K_a of $CH_3COOH = 1.8 \times 10^{-5} = K_b$ of NH_4OH then pH of solution is (at 25°C)

(*a*) More than 7 (*b*) Less than 7

(*c*) 6.8 (*d*) 7

53. Which of the following statement about the nitration of benzene (C_6H_6) and duterobenzene (C_6D_6) is correct?

(*a*) The rate of nitration of C_6H_6 is faster than C_6D_6

(*b*) The rate of nitration of C_6H_6 is slower than C_6D_6

(*c*) The rate of nitration of both C_6H_6 and C_6D_6 are same

(*d*) Nitration of C_6H_6 is electrophilic addition reaction

54. An electron jumps from an n^{th} orbit of He^+ ion to $n = 2$ having wave number equal to the wave number of first line of Lyman series of H-atom. The total energy of electron in n^{th} orbit of He^+ ion is

(*a*) –3.4 eV (*b*) –13.6 eV

(*c*) –6.8 eV (*d*) –1.51 eV

55. $\underset{(1atm)}{Pt;\ H_{2(g)}}\ |\ \underset{(0.001M)}{H^+_{(aq)}}\ ||\ \underset{(0.0001M)}{H^+_{(aq)}}\ |\ \underset{(2atm)}{H_{2(g)}};Pt$

The E_{cell} of the above cell at 25°C is ($\log_{10}2 = 0.30$)

(*a*) 0.1773 V (*b*) –0.1773 V

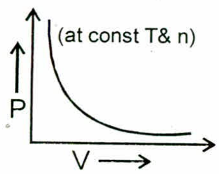
(*c*) 0.01679 V (*d*) –0.0679 V

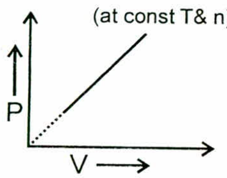
56. 1 kg 1 molal urea solution is mixed with 1 kg, 2 molal glucose solution. The freezing point of resulting solution (K_f of H_2O = 1.86 K kg mol^{-1}) is

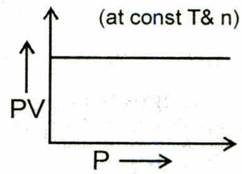
(*a*) 270.20 K (*b*) 269.47 K

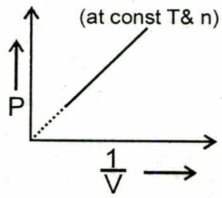
(*c*) 273 K (*d*) 271.14 K

57. Which of the following graph is not correct representation of Boyle's law for ideal gases?

(*a*)


(*b*)


(*c*)


(*d*)


58. The degree of dissociation of $PCl_{5(g)}$ into $PCl_{3(g)}$ and $Cl_{2(g)}$ has a relationship with Kp at equilibrium pressure P atm given by :

(*a*) $\alpha = \sqrt{\frac{Kp}{Kp + 4P}}$

(*b*) $\alpha = \sqrt{\frac{Kp}{Kp + P}}$

(*c*) $\alpha = \sqrt{\frac{Kp + P}{P}}$

(*d*) $\alpha = \sqrt{\frac{Kp}{4Kp + P}}$

59. The fuel having maximum antiknocking property is :

(*a*) 30% octane and 70% heptane

(*b*) 90% iso-octane and 10% n-heptane

(*c*) 50% iso-octane and 50% n-heptane

(*d*) 95% iso-octane and 5% n-heptane

60. Bond energies of $N \equiv N, H - H$ and $N - H$ are 945.1, 435.1 and 389.1 kJ per mole respectively. The enthalpy of formation of NH_3 is approximately

(*a*) 42.6 kJ/mole

(*b*) –42.6 kJ/mole

(*c*) 1146 kJ/mole

(*d*) –1146 kJ/mole

MATHEMATICS

61. The number of integeral value of k for which the roots of the equation $(k - 2)x^2 + 8x + k + 4 = 0$ are real, distinct and negative is

(*a*) 6 (*b*) 4

(*c*) 1 (*d*) 3

62. The global maximum value of the function $f(x) = \sum_{i=1}^{n} (x - x_i)^3$ for $x \in \{x_1, ..., x_n\}$, where $x_i \neq x_j \ \forall \ i \neq j$ occurs at

(*a*) $\sum_{i=1}^{n} x_i$

(*b*) max. $\{x_i : 1 \le i \le n\}$

(*c*) median of $\{x_i : 1 \le i \le n\}$

(*d*) none of these

63. $\sum_{i=0}^{20}\sum_{j=i+1}^{20} {}^{20}C_i\,{}^{20}C_j =$

(a) 2^{39} (b) $\frac{2^{40}-{}^{20}C_{10}}{2}$

(c) $\frac{2^{40}-{}^{40}C_{20}}{2}$ (d) $\frac{2^{40}-2^{20}}{2}$

64. A line has intercepts 4 and 3 on x and y axis respectively. When the axes are rotated anticlockwise through an angle α, keeping the origin fixed, the line makes equal intercepts on new axis, then $\tan\alpha =$

(a) 7 (b) 1/7

(c) 3/4 (d) 1

65. Number of circles of radius 5 touching the circle $x^2+y^2-2x-4y-20=0$ externally at (5, 5) is

(a) 1 (b) 2

(c) 0 (d) 4

66. If $z=\frac{7+i}{3+4i}$, then $z^{14}=$

(a) 2^7 (b) -2^7

(c) 2^7i (d) -2^7i

67. Which of the following parametric equation does not represent a parabola?

(a) $x=(t+1)^2, y=2(t+1)$

(b) $x=t^2+t+1, y=t^2-t+1$

(c) $x = a\cos t,\ y = b\sin^2 t$

(d) $x = a\cos t + b\sin t,\ y = a\sin t - b\cos t$

68. The solution of $(xdx+ydy)(x^2+y^2)+(xdy-ydx)=0$ is

(a) $x^2+y^2+\frac{y}{x}=c$

(b) $x^2+y^2+2\tan^{-1}\left(\frac{y}{x}\right)=c$

(c) $\frac{x^2+y^2}{2}+\tan^{-1}\left(\frac{x}{y}\right)=c$

(d) $x^2+y^2+\tan^{-1}\left(\frac{y}{x}\right)=c$

69. The equation of line passing through (2, –1, 4) and (–1, 1, 3) is

(a) $\vec{r}=2\hat{i}-\hat{j}+4\hat{k}+\lambda(3\hat{i}+\hat{k})$

(b) $\vec{r}=-\hat{i}+\hat{j}+3\hat{k}+\lambda(\hat{i}+7\hat{k})$

(c) $\frac{x+4}{3}=\frac{y-3}{-2}=\frac{z-2}{1}$

(d) None of these

70. $\int\frac{dx}{\sin^2 x+\tan^2 x}=$

(a) $-\frac{1}{2}\left(\tan x+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)\right)+c$

(b) $-\frac{1}{2}\left(\cot x+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)\right)+c$

(c) $-\left(\cot x+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)\right)+c$

(d) None of these

71. Lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ lie on a plane. Distance of that plane from origin is

(a) $\frac{4}{\sqrt{6}}$ (b) $\frac{3}{\sqrt{6}}$

(c) $\frac{2}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{6}}$

72. Number of five digit numbers using digits 0, 1, 2, 3, 4 and 8 without repetition and divisible by 6 is

(a) 98 (b) 116

(c) 150 (d) 216

73. Let $f(x)=\begin{vmatrix}\cos x & 2\sin x & \tan x\\ x & x^2 & x\\ 1 & 2x & 1\end{vmatrix}$, then $\lim_{x\to0}\frac{f(x)}{x^2}=$

(a) 0 (b) 1

(c) –1 (d) None of these

74. In a ΔABC, $a = 8$, $b = 7$, $c = 6$. Length of median from A to BC is

(a) $\frac{\sqrt{106}}{2}$ (b) $\sqrt{53}$

(c) $\frac{\sqrt{106}}{4}$ (d) $\frac{\sqrt{21}}{2}$

75. If (12, 5) and (7, 24) are foci of a hyperbola and if the hyperbola passes through origin, then eccentricity e =

(a) $\frac{38}{\sqrt{386}}$

(b) $\frac{25}{\sqrt{386}}$

(c) $\frac{\sqrt{386}}{13}$

(d) $\frac{\sqrt{386}}{12}$

76. If A denotes the area bounded by $y = \min(\sin x, \cos x)$, $0 \le x \le \frac{\pi}{2}$ and x-axis, then [A] =? ([.] denotes greatest integer function)

(a) 0

(b) 1

(c) 2

(d) None of these

77. If $x = a\cos 2t, y = b\sin^2 t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is

(a) 0 (b) 1

(c) $\frac{\sqrt{2}b}{a}$ (d) $\frac{b}{\sqrt{2}a}$

Directions (Q. 78 - 82) : *Read the following questions and choose:*

(a) Both statements are True and Statement-2 explains Statement-1.

(b) Both statements are True but Statement-2 does not explain Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

78. Statement -1:

For any two $m \times m$ matrices A and B and $n \in N, (A+B)^n = \sum_{r=0}^{n} {}^nC_r A^{n-r} B^r$.

Statement -2:

For three matrices A, B, C, AB = AC $\nRightarrow A = 0$ or $B = C$.

79. Statement -1:

m circles and n straight lines gives maximum of $2mn + m(m-1) + \frac{n(n-1)}{2}$ points of intersection.

Statement -2:

Different circles may intersect maximum at 4 distinct points.

80. If A, B, C are mutually independent events.

Statement -1:

A and $B \cup C$ are independent.

Statement -2:

A and $B \cap C$ are independent.

81. A fair dice is thrown twice. Let (x, y) denote the outcome where x denote first throw and y second throw. Let A = {(x, y) | x is odd}, B = {(x, y) | y is odd}

Statement -1:

C = {(x, y) | x + y is odd} then $P(A \cap B \cap C) = \frac{1}{8}$.

Statement -2:

D = {(x, y) | x + y is even} then

$P(A \cap B \cap D \mid A \cup B) = \frac{1}{3}$.

82. Statement -1:

If $a \neq 0$ and equation of $ax^2 + bx + c = 0$ has two roots α, β such that α < −2, β > 2, then $a(a \pm b + c) < 0$.

Statement -2:

If a > 0 then $ax^2 + bx + c < 0$ for all x lie between the roots of $ax^2 + bx + c = 0$.

83. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$ then $|\vec{a} \cdot \vec{b}| =$

(a) 4 (b) 6

(c) 5 (d) 8

84. If a, b, c are three distinct positive numbers in G.P. and if $x = \frac{a+c}{b}$, then range of x is

(a) $[2, \infty)$ (b) $(2, \infty) \cup \{1\}$

(c) $(2, \infty) \cup \{1, -1\}$ (d) $(2, \infty)$

85. Consider the following subsets of the plane $R \times R$

$S = \{(x, -y) : y = -x, x \in R\}$

$T = \{(x, -y) : x + y \text{ is integer}\}$ then

(a) Both S and T are equivalence relations on R.

(b) S is equivalence, T is not an equivalaence relation.

(c) T is equivalence, S is not an equivalence relation.

(d) Neither S nor T is equivalence relations on R.

86. The number of values of θ in [0, 3π] satisfying equation $2\cos^2\theta + 5|\cos\theta| - 3 = 0$ is

(a) 3 (b) 5

(c) 6 (d) 8

87. The range of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$ is

(a) $[0, \infty)$ (b) $\left(0, 2\log\frac{5}{4}\right]$

(c) $\left[0, \sqrt{2\log_{10}\frac{5}{4}}\right]$ (d) $\left[0, \log_{10}\frac{5}{4}\right]$

88. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.8, then median of the data is

(a) 5 (b) 6

(c) 7 (d) 8

89. $\cot^{-1}\left(\sqrt{\cos\alpha}\right)-\tan^{-1}\left(\sqrt{\cos\alpha}\right)=x$, then $\cos\alpha =$

(a) $\dfrac{1+\sin x}{1-\sin x}$ (b) $\dfrac{1-\sin x}{1+\sin x}$

(c) $\dfrac{1+\cos x}{1-\cos x}$ (d) $\dfrac{1-\cos x}{1+\cos x}$

90. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=|\vec{w}|=2$ if projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u}+\vec{v}-\vec{w}| =$

(a) 1 (b) 3

(c) 9 (d) None of these

ANSWERS

1. (c)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (c)
11. (c)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (d)
21. (a)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (a)
31. (d)	**32.** (a)	**33.** (c)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (c)
41. (c)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (c)
51. (a)	**52.** (d)	**53.** (c)	**54.** (a)	**55.** (d)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (b)
61. (c)	**62.** (b)	**63.** (c)	**64.** (b)	**65.** (a)	**66.** (c)	**67.** (d)	**68.** (b)	**69.** (c)	**70.** (b)
71. (a)	**72.** (c)	**73.** (c)	**74.** (a)	**75.** (d)	**76.** (a)	**77.** (a)	**78.** (d)	**79.** (c)	**80.** (a)
81. (d)	**82.** (a)	**83.** (b)	**84.** (d)	**85.** (a)	**86.** (c)	**87.** (c)	**88.** (a)	**89.** (b)	**90.** (b)

EXPLANATIONS

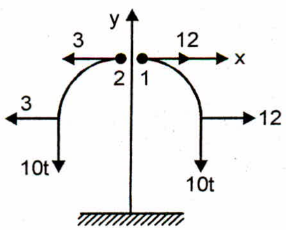
PHYSICS

1. $b = a^{1/4}c^{-1/16}$

$$\frac{\Delta b}{b} = \frac{1}{4}\frac{\Delta a}{a} + \frac{1}{16}\frac{\Delta c}{c}$$

$$= \frac{1}{4}\times 8\% + \frac{1}{16}16\% = 3\%$$

3.



$\vec{v}_1 = 12\hat{i} - 10t\,\hat{j},$

$\vec{v}_2 = -3\hat{i} - 10t\,\hat{j}$

$\vec{v}_1 \cdot \vec{v}_2 = 0$

$\Rightarrow\ 36 = 100t^2$

$\Rightarrow\ t = 0.6\ s$

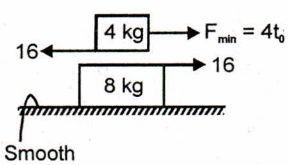
4. Relative velocity of stone = 5 m/s

Relative acceleration of stone = 10 + 5 = 15 m/s^2

$\therefore$ $v = u$ + at = 5 + 15 × 2 = 35 m/s

$\therefore$ Relative velocity after t = 2 second is 35 m/s.

5.



$a_A = a_B$

$$\Rightarrow \frac{F_{min} - 0.4\times 40}{4} = \frac{16}{8}$$

$\Rightarrow F_{min} = 24 = 4t_0$

$\Rightarrow t_0 = 6s$

$a_A = a_B = \dfrac{F}{12} = \dfrac{t}{3}$ for $0 < t < 6$ s

$a_B = \max = 2\,m/s^2$ for $t > 6$ s

$a_A = \dfrac{4t-16}{4} = t-4$ for $t > 6$ s

6. If A moves down the incline by 1 metre, B shall move up by 1/2 metre. If the speed of B is v then the speed of A will be $2v$.

From conservation of energy

Gain in K.E. = Loss in P.E.

$$\frac{1}{2}m_A(2v)^2 + \frac{1}{2}m_B v^2 = m_A g\frac{3}{5} - m_B g\times\frac{1}{2}$$

Solving we get $v = \dfrac{1}{2}\sqrt{\dfrac{g}{3}}$

7. By conservation of linear momentum along the string.

$$mu = (m + m + 3m)v$$

or $$v = u/5$$

and impulse on the block A = $3m(v - 0) = \dfrac{3mu}{5}$

8. Loss in KE = gain in PE

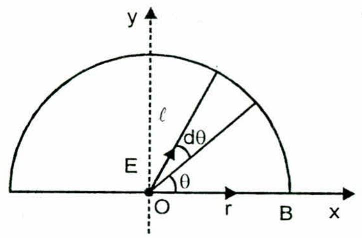
$$\frac{1}{2}I\omega^2 = mg2\ell + mg\ell$$

$$\Rightarrow \frac{1}{2}\left(\frac{m\ell^2}{3} + m\ell^2\right)\omega^2 = 3mg\ell$$

$$\omega = \sqrt{\frac{9g}{2\ell}}$$

$$\therefore v = \omega\ell = \sqrt{4.5g\ell}$$

9.



Let mass per unit length of wire, $\lambda = \frac{m}{\ell}$

and $\pi r = \ell,\ r = \frac{\ell}{\pi}$

mass of element, $dm = \lambda r d\theta$ then

$$dE = \frac{Gdm}{r^2}$$

$$\int_0^{\pi} d\vec{E} = \int_0^{\pi}\frac{G\lambda r d\theta}{r^2}(\hat{i}\cos\theta + \hat{j}\sin\theta)$$

$$\vec{E} = \frac{G\lambda}{r}\left[\int_0^{\pi}\hat{i}\cos\theta\, d\theta + \int_0^{\pi}\hat{j}\sin\theta\, d\theta\right]$$

$$= \frac{2G\lambda}{r}\hat{j} = \frac{2Gm}{\ell r}\hat{j} = \frac{2Gm\pi}{\ell^2}\hat{j} \quad \text{(along y-axis)}$$

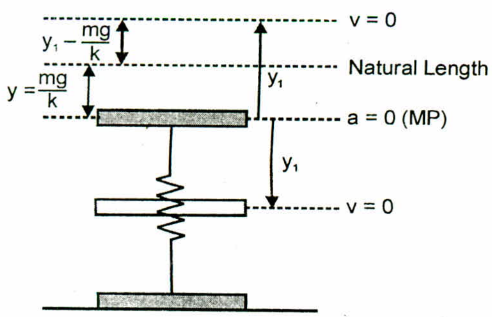
10. $\ell^2\sqrt{2gh} = \pi r^2\sqrt{2g(4h)}$;

$$\pi\frac{r^2}{\ell^2} = \sqrt{\frac{2gh}{8gh}}$$

$$\frac{r^2}{\ell^2} = \frac{1}{2\pi};$$

$$r = \frac{\ell}{\sqrt{2\pi}}$$

11.



$$k\left(y_1 - \frac{mg}{k}\right) \geq mg$$

$$\Rightarrow y_{1\,min} = \frac{2mg}{k} = 2y$$

12. $r = 10$ mm, $\ell = 1$ m, F = 100 kN, Y = 2×10^{11}

$$Y = \frac{F/A}{\Delta\ell/\ell};\ \frac{\Delta\ell}{\ell} = \frac{F}{AY} = \frac{100\times10^3}{\pi\times(10\times10^{-3})^2\times2\times10^{11}}$$

$$\frac{\Delta\ell}{\ell}\times100 = \frac{7}{44} = 0.16\%$$

13. Slope at any point on the string in wave motion represents the ratio of particle speed to wave speed.

$\because$ | slope B | > | slope A |

hence $R_A > R_B$.

14.

$$m_{min} = \frac{f_0}{f_e}$$

$$\Rightarrow f_e = \frac{1.5}{25} = 0.06\ m$$

15. Total path difference = $(\mu - 1)t_1 - (\mu - 1)t_2$

$$\therefore (\mu-1)(t_1 - t_2) = y\cdot\frac{\lambda}{\beta};\ y = \frac{(\mu-1)(t_1-t_2)\beta}{\lambda}$$

16. T = Time period

$$T \propto \ell^{1/2}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell} = \frac{1}{2}\alpha t;$$

$$\Delta T = \frac{1}{2}\alpha t T = \alpha t$$

$$\text{Total loss} = \frac{1}{2}\alpha t\times86400$$

17. log P = m log V + c, where m is slope

$$m = \frac{2.38-2.10}{1.1-1.3} = -1.4$$

$\Rightarrow$ log P = −1.4 log V + c

$\Rightarrow$ log $PV^{1.4} = c$

$\Rightarrow$ $PV^{1.4} = k$

18. OR gate

The current through the bulb (*i.e*, O/P = 1) will exist even if any one of the switches is closed or both are closed.

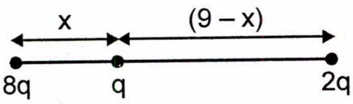
19. Net force on q must be zero. (Electro-static shielding)

Force due to induced charge + Force due to $q' = 0$

$\Rightarrow$ Force due to induced charge = – Force due to q'

$$= \frac{qq'}{16\pi\epsilon_0 R^2}\hat{i}$$

20. Let



$$\text{P.E.} = U = \frac{k8q^2}{x} + \frac{2kq^2}{9-x} + \frac{16kq^2}{9}$$

$$\frac{dU}{dx} = 0$$

$$\Rightarrow \frac{-8}{x^2} + \frac{2}{(9-x)^2} = 0$$

$\Rightarrow 2(9 - x) = x$

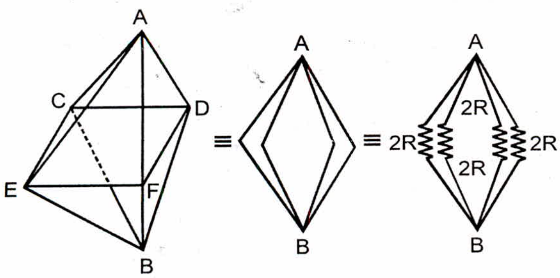
$\Rightarrow 18 = 3x$

$\Rightarrow x = 6\ m$

$$\frac{d^2U}{dx^2} = \left(\frac{16}{x^3} + \frac{4}{(9-x)^3}\right) > 0 \quad \text{for } x > 0$$

$\Rightarrow$ U is min at $x = 6$m

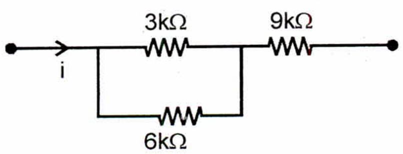
21.



By symmetry $V_C = V_D = V_E = V_F$;

$$R_{eq} = \frac{2R}{4} = \frac{R}{2}$$

22.



$$R_{eq} = \frac{3 \times 6}{3+6} k\Omega + 9\,k\Omega = 11\ k\Omega$$

$$\text{Current} = \frac{V}{R_{eq}} = \frac{220}{11000} = 0.02\ \text{A}$$

23. Speed is changed by electric field only.

$\therefore$ $\vec{F}$ (due to electric field only)

$$= -\frac{qE_0}{\sqrt{2}}\hat{i} + \frac{qE_0}{\sqrt{2}}\hat{j}$$

Speed is minimum when $v_x = 0$

$\Rightarrow v_x = u_x + a_x t$

$$\Rightarrow 0 = \frac{v_0}{\sqrt{2}} - \frac{qE_0}{m\sqrt{2}}t$$

$$\Rightarrow t = \frac{mv_0}{qE_0}$$

25. Torque $= iAB = 10\pi$ N-m

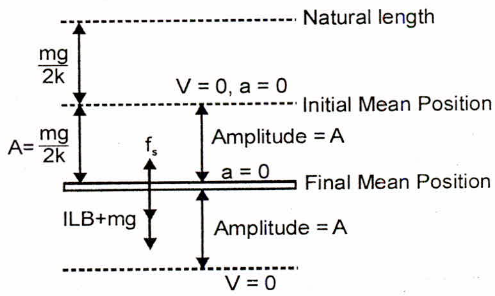
$I\alpha$ = Torque

Moment of inertia of ring about vertical diameter

$$= I = \frac{1}{2}mR^2 = \frac{1}{4}kg\text{-}m^2$$

$\therefore \quad \alpha = 40\pi\ rad/s^2$

26. Magnetic force = ILB vertically downward = mg



Further maximum elongation

= Distance between Initial M.P. to final rest position

$$= 2A = 2\left(\frac{mg}{2K}\right) = \frac{mg}{K} = \frac{ILB}{K}$$

27. Particles decayed in 2nd year

$$= N_0(e^{-\lambda} - e^{-2\lambda}) = N_2$$

Particles decayed in 3rd year

$$= N_0(e^{-2\lambda} - e^{-3\lambda}) = N_3$$

$$\frac{N_3}{N_2} = e^{-\lambda} = 0.3$$

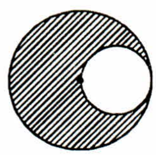
Particles decayed in Ist year

$$= N_0(1 - e^{-\lambda})$$

$$= 10^{20}(1 - 0.3)$$

$$= 7 \times 10^{19}$$

28.



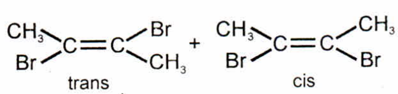
$$\text{Mass of Hole} = -\frac{M}{4} = M' \text{ and } R' = \frac{R}{2}$$

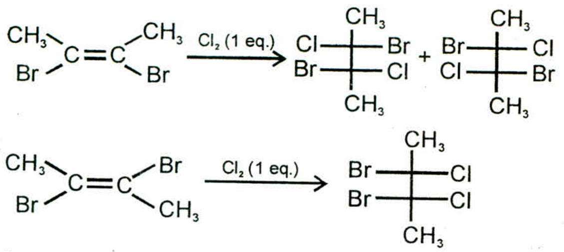
$$\text{M.I.} = \frac{MR^2}{2} - \left[\frac{1}{2}M' \cdot \frac{R^2}{4} + M'\frac{R^2}{4}\right]$$

$$= \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

CHEMISTRY

31. $CH_3 - C \equiv C - CH_3 \xrightarrow{1\ eq.Br_2}$





32. for urea solution and for NaCl solution

$$\pi_1 = i\left(\frac{n}{V}\right)RT$$

$$\pi_2 = i\left(\frac{n}{V}\right)RT$$

$$(i=1)\pi_1 = 1\times 0.1\times RT$$

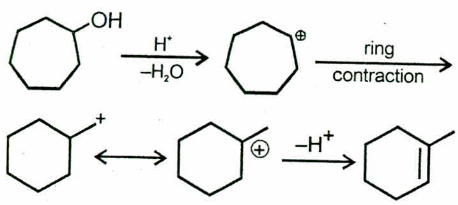
$$(i=2); \pi_2 = 2\times 0.1\times RT$$

On mixing equal volumes of these two solutions

$$\pi_3 = \left(\frac{0.1V+0.2V}{2V}\right)RT$$

$$\pi_3 = \frac{0.3}{2}RT = 0.15RT$$

$$\pi_2 > \pi_3 > \pi_1$$

33. 

34. $2\pi r = 8.3053\times 10^{-9} m = 83.053\times 10^{-10} m = 83.053 Å$

$$r = \frac{83.053}{2\times 3.14} = 13.225Å$$

$$r_n = 0.529\times\frac{n^2}{Z} \qquad \text{for H atom } z = 1$$

so $13.225 = 0.529 \times n^2$ or $n^2 = 25$; $n = 5$

$$\frac{1}{\lambda} = R_H\left[\frac{1}{1^2}-\frac{1}{5^2}\right]$$

$$\frac{1}{\lambda} = 1.1\times 10^7\left[\frac{25-1}{25}\right] = 1.1\times 10^7 \times\frac{24}{25}$$

$$\lambda = \frac{25}{24\times 1.1\times 10^7} = 0.9469\times 10^{-7}$$

$$\simeq 9.5\times 10^{-8} m$$

$$\simeq 950 Å$$

35. $CH_3COOH + NaOH \longrightarrow CH_3COONa \; pH_2O$

	CH_3COOH	$NaOH$	CH_3COONa	H_2O
Initially moles	0.5 × 0.2	0.5 × 0.2	0	0
After mixing moles	0	0	0.01	

$$pH = 7 + \frac{1}{2}pKa + \frac{1}{2}\log C$$

$$= 7 + \frac{1}{2}\times 5 + \frac{1}{2}\log\frac{0.01}{1}$$

$$pH = 7 + \frac{5}{2} - 1 = 7 + 2.5 - 1 = 8.5$$

37. $\underset{a \text{ mole/L}}{[A]} \xrightarrow{17.5\,min} \underset{\frac{a}{2}\text{mole/L}}{[B]} \xrightarrow{17.5\,min} \underset{\frac{a}{4}\text{mole/L}}{[A]}$

$\underset{2a \text{ mole/L}}{[B]} \xrightarrow{5\,min} \underset{a \text{ mole/L}}{[B]} \xrightarrow{10\,min} \underset{\frac{a}{2}\text{mole/L}}{[B]} \xrightarrow{20\,min} \underset{\frac{a}{4}\text{mole/L}}{[B]}$

for B; $t_{1/2} \propto \frac{1}{a}$ so order of reaction (*ii*) is 2.

38. Terminal alkynes give positive test with Tollen's reagent.

40. (*i*) → Pauli's principle is not followed.

(*ii*) → Pauli's principle as well as afbau principle are not followed.

(*iii*) → Afbau principle not followed.

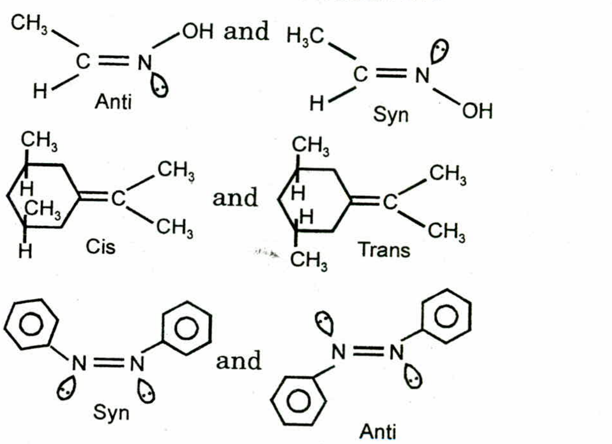
41. Lone pair of O atom and π e's are not in conjugation.

42. $\underset{}{SrF_{2(s)}} \rightleftharpoons \underset{s}{Sr^{2+}_{(aq)}} + \underset{2s}{2F^-_{(aq)}}$

$$2s = [F^-] = 2\times 10^{-3}; s = 10^{-3}\text{moles/L}$$

$$K_{sp} = 4s^3 = 4\times(10^{-3})^3 = 4\times 10^{-9}\text{mole}^3/\text{Litre}^3$$

44. $Z = \frac{PV}{nRT} = \frac{1\times 10^{-3}\times 18}{5.97\times 10^{-4}\times 0.0821\times 373} = 0.984$

45. 

46. $5C_2O_4^{2-} + 2MnO_4^- + 16H^+ \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

47. $A + A \underset{K_{-1}}{\overset{K_1}{\rightleftharpoons}} A_2$

$$A_2 + B_2 \xrightarrow{K_2} 2AB \text{ slow}$$

$$\text{Rate} = K_2[A_2][B_2] \;;$$

$$\frac{K_1}{K_{-1}} = \frac{[A_2]}{[A]^2}$$

or $[A_2] = \frac{K_1}{K_{-1}}[A]^2$

so rate law expression

$$\text{Rate} = \frac{K_1K_2}{K_{-1}}[A]^2[B_2]$$

49. Let 4 gm of He and 8 gm of H_2 are present in container

$n_{H_2} = \frac{8}{2} = 4$;

$n_{H_e} = \frac{4}{4} = 1$

$P_{H_2} = \frac{4}{5} \times 1 = 0.8\,\text{atm}$;

$P_{He} = 1 - 0.8 = 0.2$ atm

51. $C_{(gr)} + O_{2(g)} \rightarrow CO_{2(g)} \Delta H_r = -393.5\,kJ/mole$

$\Delta n_{(g)} = 0$ for the equation so

$\Delta H = \Delta E + \Delta n_g RT$

i.e. $\Delta H = \Delta E = -393.5$ kJ/mole.

52. $pH = 7 + \frac{1}{2}pKa - \frac{1}{2}pKb$

pH = 7 (because pKa = pKb)

53. The rate of nitration of benzene and C_6D_6 are same.

54. $R_H(1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R_H 2^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

$n = 4$, $E = -13.6 \times \frac{Z^2}{n^2} = -13.6 \times \frac{2^2}{4^2} = -3.4\,eV$

55. Cell reaction

$$\underset{(1atm)}{H_{2(g)}} + \underset{10^{-4}M}{2H^+_{(aq)}} \longrightarrow \underset{10^{-3}M}{2H^+_{(aq)}} + \underset{(2atm)}{H_{2(g)}}$$

$$E_{cell} = E^0_{cell} - \frac{0.0591}{2}\log\frac{(10^{-3})^2 \times 2}{(10^{-4})^2 \times 1}$$

$$E_{cell} = \frac{-0.0591}{2}\log 2 \times 10^2$$

$$= \frac{-0.0591 \times 2.30}{2}$$

$$= -0.0679\text{ V}$$

56. In resulting solution total moles of solutes = 3

mass of solvent in resulting solution

= 2000 – (60 × 1 + 180 × 2)

= 1580 gm

$m = \frac{3 \times 1000}{1580} = 1.9$

$\Delta T_f = K_f \times 1.9$

$= 1.86 \times 1.9 = 3.53$

Freezing point of solution

= 273 – 3.53

= 269.47 k.

58. $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

at eq $1-\alpha$ α α

$$Kp = \frac{\alpha^2}{(1-\alpha)} \times \frac{P}{1+\alpha};\ \alpha = \sqrt{\frac{Kp}{Kp+P}}$$

59. More is the % of iso-octane more is antiknocking property.

60. $\frac{1}{2}N_2 + \frac{3}{2}H_2 \longrightarrow NH_3 \quad \Delta Hr = ?$

$$\Delta Hr = \left[\frac{1}{2} \times 945.1 + \frac{3}{2} \times 435.1\right] - [3 \times 389.1]$$

$= 1124.7 - 1167.3$

$= -42.6$ kJ/mole

MATHEMATICS

61. For roots to be real and distinct.

$D > 0 \Rightarrow 64 - 4(k-2)(k+4) > 0$

$\Rightarrow k^2 + 2k - 24 < 0 \Rightarrow k \in (-6, 4)$ ... (*i*)

Also roots are negative.

$\Rightarrow \alpha\beta > 0$

$\Rightarrow \frac{k+4}{k-2} > 0 \Rightarrow k \in (-\infty, -4) \cup (2, \infty)$ ... (*ii*)

and $\alpha + \beta < 0$

$\Rightarrow \frac{-8}{k-2} < 0 \Rightarrow \frac{8}{k-2} > 0 \Rightarrow k > 2$ ... (*iii*)

From (*i*), (*ii*) and (*iii*)

$$k \in (2, 4)$$

62. $f'(x) = 3\sum_{i=1}^{n}(x - x_i)^2 > 0\ \forall\ x \in R$

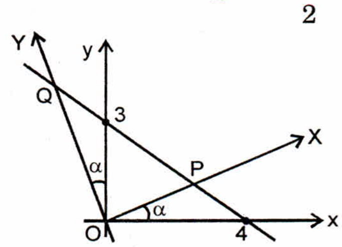
$\Rightarrow f(x)$ is strictly increasing function.

Hence, for $x \in \{x_1, x_2, x_3, ..., x_n\}$ $f(x)$ attains minimum value for minimum value of x_i and maximum value for maximum value of x_i.

63. $$\sum_{i=0}^{20}\sum_{j=i+1}^{20} {}^{20}C_i\,{}^{20}C_j = \frac{\left(\sum_{i=0}^{20}{}^{20}C_i\right)^2 - \sum_{i=0}^{20}\left({}^{20}C_i\right)^2}{2}$$

$$= \frac{(2^{20})^2 - {}^{40}C_{20}}{2} = \frac{2^{40} - {}^{40}C_{20}}{2}$$

64.



$$OP = OQ$$

Let P$(h, 0)$, Q$(0, h)$ in new frame.

$$x = X\cos\alpha - Y\sin\alpha,\ y = X\sin\alpha + Y\cos\alpha$$

Coordinates of P in old frame is $(h\cos\alpha, h\sin\alpha)$ and Q h as $(-h\sin\alpha, h\cos\alpha)$.

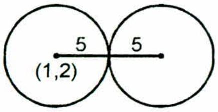
Slope of PQ is $\dfrac{\sin\alpha - \cos\alpha}{\cos\alpha + \sin\alpha} = -\dfrac{3}{4}$

$\Rightarrow$ $4\sin\alpha - 4\cos\alpha = -3\cos\alpha - 3\sin\alpha$

$\Rightarrow$ $7\sin\alpha = \cos\alpha$

$\Rightarrow$ $\tan\alpha = \dfrac{1}{7}$

65.



Centre of required circle is (9, 8).

66. $z = \dfrac{7+i}{3+4i}$; $|z| = \dfrac{\sqrt{50}}{\sqrt{25}} = \sqrt{2}$

$$\arg(z) = \tan^{-1}\left(\frac{1}{7}\right) - \tan^{-1}\left(\frac{4}{3}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{7}-\frac{4}{3}}{1+\frac{1}{7}\times\frac{4}{3}}\right)$$

$$= \tan^{-1}\left(\frac{-25}{25}\right) = -\frac{\pi}{4};$$

$$z = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

$$z^{14} = (\sqrt{2})^{14}\left(\cos\frac{14\pi}{4} - i\sin\frac{14\pi}{4}\right)$$

$$= 2^7(0+i) = 2^7 i$$

67. (*a*) $x = \left(\dfrac{y}{2}\right)^2 \Rightarrow y^2 = 4x$

(*b*) $t = \dfrac{x-y}{2}$

$$t^2 + 1 = \frac{x+y}{2}$$

$\Rightarrow$ $\dfrac{x+y}{2} = \left(\dfrac{x-y}{2}\right)^2 + 1$

$\Rightarrow$ $2(x+y) = (x-y)^2 + 4$

$\Rightarrow$ $(x-y)^2 = 2(x+y-2)$

(*c*) $1 - \dfrac{y}{b} = \left(\dfrac{x}{a}\right)^2$

(*d*) $x^2 + y^2 = a^2 + b^2$

68. $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{x^2+y^2} = 0$

$$d\left(\frac{x^2+y^2}{2}\right) + d\left(\tan^{-1}\frac{y}{x}\right) = 0\ ;$$

$$x^2 + y^2 + 2\tan^{-1}\left(\frac{y}{x}\right) = c$$

69. Equation of line is given by

$\dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z-4}{1}$ and $(-4, 3, 2)$ lies on it.

Hence, answer is (*c*).

70. $\displaystyle\int\frac{dx}{\sin^2 x + \tan^2 x} = \int\frac{\cos^2 x\,dx}{\sin^2 x\cos^2 x + \sin^2 x}$

$$= \int\frac{\cos^2 x}{\sin^2 x(\cos^2 x + 1)}dx$$

$$= \int\frac{\cos^2 x}{(1+\cos^2 x)(1-\cos^2 x)}dx$$

$$= \frac{1}{2}\int\frac{(1+\cos^2 x)-(1-\cos^2 x)}{(1+\cos^2 x)(1-\cos^2 x)}dx$$

$$= \frac{1}{2}\int\frac{1}{1-\cos^2 x}dx - \frac{1}{2}\int\frac{dx}{1+\cos^2 x}$$

$$= -\frac{1}{2}\cot x - \frac{1}{2}\int\frac{\sec^2 x}{\tan^2 x + 2}dx$$

$$= -\frac{1}{2}\cot x - \frac{1}{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$$

71. $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $1 + 2k + 1 + k^2 - 2 + k = 0$

$\Rightarrow$ $k^2 + 3k = 0$

$\Rightarrow$ $k = 0, -3$

Possible equation of planes are given by

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $x - 2 - y + 3 + 2z - 8 = 0$

$\Rightarrow$ $x - y + 2z = 7$

whose distance from origin is $\dfrac{7}{\sqrt{6}}$.

or $\begin{vmatrix} x-2 & y-3 & z-4 \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $-5x + 10 - 10y + 30 + 5z - 20 = 0$

$\Rightarrow$ $x + 2y - z = 4$ distance from origin is $\dfrac{4}{\sqrt{6}}$.

72. Sum of all digits 0 + 1 + 2 + 3 + 4 + 8 = 18

For number to be divisible by 3, sum of digits should be divisible by 3.

Hence we can leave either 0 or 3.

Using digits, 1, 2, 3, 4, 8 even numbers can be formed in $\overline{4}\times\overline{3}\times\overline{2}\times\overline{1}\times\overline{3}$ *i.e.*, 72 ways.

Using digits 0, 1, 2, 4, 8 even numbers can be formed in $\overline{4}\times\overline{3}\times\overline{2}\times\overline{1}\times\overline{1}$ *i.e.*, 24 ways or $\overline{3}\times\overline{3}\times\overline{2}\times\overline{1}\times\overline{3}$ *i.e.*, 54 ways.

Total 72 + 24 + 54 = 150 ways.

73. $f(0) = 0$, using L'Hopital rule.

$$\lim_{x\to 0}\frac{f(x)}{x^2} = \lim_{x\to 0}\frac{f'(x)}{2x}$$

$$f'(x) = \begin{vmatrix} -\sin x & 2\sin x & \tan x \\ 1 & x^2 & x \\ 0 & 2x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 2\cos x & \tan x \\ x & 2x & x \\ 1 & 2 & 1 \end{vmatrix} + \begin{vmatrix} \cos x & 2\sin x & \sec^2 x \\ x & x^2 & 1 \\ 1 & 2x & 0 \end{vmatrix}$$

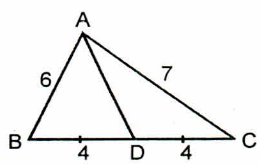
$$= -\sin x(x^2 - 2x^2) - (2\sin x - 2x\tan x) + \sec^2 x(2x^2 - x^2) - (2x\cos x - 2\sin x)$$

$$= x^2\sin x - 2\sin x + 2x\tan x + x^2\sec^2 x + 2\sin x - 2x\cos x$$

$$= x^2\sin x + x^2\sec^2 x + 2x\tan x - 2x\cos x$$

$$\lim_{x\to 0}\frac{f'(x)}{2x} = \lim_{x\to 0}\frac{x\sin x + x\sec^2 x + 2\tan x - 2\cos x}{2} = -1$$

74.



$$AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2} = \frac{\sqrt{2\times 7^2 + 2\times 6^2 - 8^2}}{2} = \frac{\sqrt{106}}{2}$$

75. $|PS - PS'| = 2a$

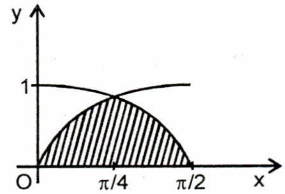
$\Rightarrow \left|\sqrt{144+25} - \sqrt{49+576}\right| = 2a = 25 - 13$

$\Rightarrow a = 6$

$SS' = 2ae = \sqrt{25+361} = \sqrt{386}$

$\Rightarrow e = \dfrac{\sqrt{386}}{12}$

76.



$$A = \int_0^{\pi/4}\sin x\,dx + \int_{\pi/4}^{\pi/2}\cos x\,dx$$

$$= 2\left(1 - \frac{1}{\sqrt{2}}\right) = 2 - \sqrt{2}$$

$[A] = 0$

77. $y = b\sin^2 t,\ x = a\cos 2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2a\sin 2t}{2b\sin t\cos t} = -\frac{2a}{b};$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = 0$$

78. $(A+B)^n = \sum_{r=0}^{n} {}^nC_r A^{n-r}B^r$ if $AB = BA$

$AB = AC \nRightarrow A = 0$ or $B = C$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

But $A \neq 0$ and $B \neq C$

79. m circles intersects at ${}^mC_2 \times 2$ points.

n straight lines intersect at ${}^nC_2$ points.

m circles and n straight lines intersects at ${}^mC_1 \times {}^nC_1 \times 2$

$$m(m-1) + \frac{n(n-1)}{2} + 2mn$$

From 3 non-collinear points a unique circle passes hence second statement is false.

80. $P(A\cap B) = P(A)\cdot P(B)$

$P(A\cap C) = P(A)\cdot P(C)$

$P(B\cap C) = P(B)\cdot P(C)$

$P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$

$P(A\cap(B\cup C)) = P((A\cap B)\cup(A\cap C))$

$= P(A\cap B) + P(A\cap C) - P((A\cap B)\cap(A\cap C))$

$= P(A)\cdot P(B) + P(A)\cdot P(C) - P(A)\cdot P(B)\cdot P(C)$

$= P(A)\cdot P(B\cup C)$

$P(A\cap(B\cap C)) = P(A)\cdot P(B)\cdot P(C)$

81. $P(A) = \dfrac{3\times 6}{6\times 6} = \dfrac{1}{2}$,

$P(B) = \dfrac{6\times 3}{6\times 6} = \dfrac{1}{2}$

$P(C) = \dfrac{3\times 3 + 3\times 3}{6\times 6} = \dfrac{1}{2}$

$P(A \cap B \cap C) = 0$ as $A \cap B \cap C = \phi$

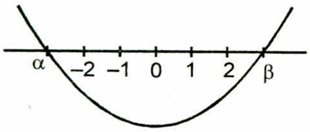
$$P(A \cap B \cap D \mid A \cup B) = \frac{P((A \cap B \cap D) \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{\frac{3\times 3}{6\times 6}}{\frac{3\times 6}{6\times 6} + \frac{6\times 3}{6\times 6} - \frac{3\times 3}{6\times 6}}$$

$$= \frac{9}{18+18-9} = \frac{1}{3}$$

82. $ax^2 + bx + c = 0,\ a \neq 0$

$a > 0$,



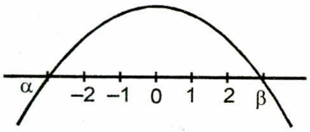
$f(-2) = 4a - 2b + c < 0$

$f(2) = 4a + 2b + c < 0$

$f(0) = c < 0$

Also $f(1) < 0$ and $f(-1) < 0$.

$a < 0$,



$f(-2) = 4a - 2b + c > 0$

$f(2) = 4a + 2b + c > 0$

$f(0) = c > 0$

Also $f(1) > 0$ and $f(-1) > 0$. Hence, $a(a \pm b + c) < 0$

83. $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$

$64 + |\vec{a}\cdot\vec{b}|^2 = 4\times 25$

$\Rightarrow |\vec{a}\cdot\vec{b}|^2 = 36$

$\Rightarrow |\vec{a}\cdot\vec{b}| = 6$

84. $x = \dfrac{a + ar^2}{ar} = r + \dfrac{1}{r}$; $r > 0$ then $x > 2,\ r \neq 1$.

85. $S = \{(x, -y) : y = -x,\ x \in R\}$

$= \{(x, x) : x \in R\}$

Which is an identity relation hence equivalence.

$T = \{(x, -y) : x + y$ is integer$\}$

$(x, x) \in T\ \forall\ x \in R$ as $x - x = 0$ is integer.

$(x, y) \in T$ if $(x - y)$ is integer then $(y, x) \in T$.

$(x, y), (y, z) \in T$

$\Rightarrow (x, z) \in T$

$\Rightarrow$ T is equivalence.

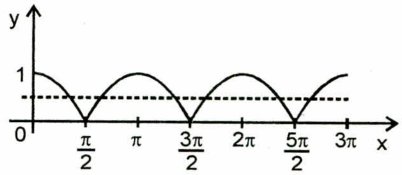
86. $2\cos^2\theta + 5|\cos\theta| - 3 = 0$

$\Rightarrow 2\cos^2\theta + 6|\cos\theta| - |\cos\theta| - 3 = 0$

$\Rightarrow 2|\cos\theta|(|\cos\theta| + 3) - 1(|\cos\theta| + 3) = 0$

$\Rightarrow |\cos\theta| = \dfrac{1}{2}, -3$

$\Rightarrow |\cos\theta| \neq -3$



87. $\dfrac{5x - x^2}{4} \geq 1$; $1 \leq \dfrac{5x - x^2}{4} \leq \dfrac{25}{16}$

$0 \leq \log_{10}\left(\dfrac{5x - x^2}{4}\right) \leq \log_{10}\left(\dfrac{25}{16}\right)$

Range is $\left[0, \sqrt{\log_{10}\left(\dfrac{25}{16}\right)}\right]$ or $\left[0, \sqrt{2\log_{10}\left(\dfrac{5}{4}\right)}\right]$.

88. $\dfrac{a + b + 8 + 5 + 10}{5} = 6$

$\Rightarrow a + b = 7$

$\Rightarrow \dfrac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 = 6.8$

$\Rightarrow a^2 + b^2 + 189 = 214$

$\Rightarrow a^2 + b^2 = 25$

$\Rightarrow a = 3, b = 4$ or $a = 4, b = 3$

$\Rightarrow$ Median is 5.

89. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow x = \dfrac{\pi}{2} - 2\tan^{-1}(\sqrt{\cos\alpha})$

$\Rightarrow \sin x = \cos\left(2\tan^{-1}\sqrt{\cos\alpha}\right)$

$\Rightarrow \sin x = \dfrac{1 - \cos\alpha}{1 + \cos\alpha}$

$\dfrac{1 - \sin x}{1 + \sin x} = \dfrac{2\cos\alpha}{2} = \cos\alpha$

90. $\vec{v}\cdot\vec{u} = \vec{w}\cdot\vec{u}$

$\vec{v}\cdot\vec{w} = 0$

$|\vec{u} + \vec{v} - \vec{w}|^2$

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2\vec{u}\cdot\vec{v} - 2\vec{u}\cdot\vec{w} - 2\vec{v}\cdot\vec{w}$

$= 1 + 4 + 4$

$= 9$

■■