FIITJEE FARIDABAD

MOCK PRACTICE PAPER FOR
JEE -Advance- 2020

MOCK PRACTICE PAPER-30

Time: 3 hours ***Maximum marks: 204***

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions
1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-1** is Physics, **Part-2** is Chemistry and **Part-3** is Mathematics.
4. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
5. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet
1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Sections.

(i) **Section-I (01 – 8)** contains **8** multiple choice questions which have **one or more than one** correct answers. Each question carries **+4 marks** for correct answer and **–2 marks** for incorrect answers.

(ii) **Section-II (09 – 12)** contains **4** multiple choice questions related to **2** paragraphs with **2** questions on each paragraph which has **one or more than one** correct answers. Each question carries **+3 marks** for correct answer and**–1 marks** for incorrect answer

(iii) **Section-III (01 – 08)** contains **8** integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9 both included. Each question carries **+3 marks** for correct answer and **No** negative marking in this section.

Name of the Candidate :____

Batch :____________________ **Date of Examination :**__________________________

Enrolment Number :____

PART-1 : PHYSICS

SECTION–I

1. A bucket of water in which a ball floats is kept in an elevator. We have two cases.

Case 1: The elevator accelerates upwards with an acceleration a.

Case 2: The elevator accelerates downwards with an acceleration a ($<g$).

Choose the **CORRECT** statement(s):

(A) In case 1, The depth of submergence increases, the buoyant force on ball increases and the pressure on the base increases.

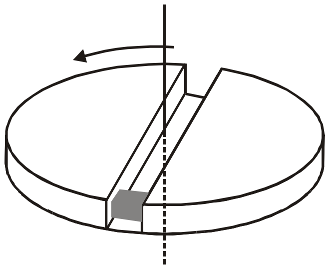
(B) In case 2, The depth of submergence decreases, the buoyant force on ball decreases and the pressure on the base decreases.

(C) In case 1, The depth of submergence remains same, the buoyant force on ball increases and the pressure on the base increases.

(D) In case 2, The depth of submergence remains same, the buoyant force on ball decreases and the pressure on the base decreases.

Space for Rough Work

2. A large, horizontal disk of radius R, shown below, starts to rotate from rest with an angular acceleration of α. The rotation is about a vertical axis through the centre of disk. The disk contains a narrow channel of length 2R and rectangular cross-section. Gravity acts in the vertical direction with an acceleration of g. There is a small rectangular puck that just fits easily in the aforementioned channel, as shown. The puck is situated a distance r from the axis of rotation.



(A) If the sides of the channel are frictionless but the bottom of the channel has a static coefficient of friction μ, then puck begins to slide at $t = \sqrt{\frac{\mu}{\alpha}}$

(B) In case mentioned in option (A), $t = \sqrt{\frac{\mu g}{r\alpha^2}}$

(C) Now, instead, the situation is that the bottom of the channel is frictionless but the walls have a static coefficient of friction μ. Now puck begins to slide at $t = \sqrt{\frac{\mu g}{r\alpha^2}}$

(D) In case mentioned in option (C), $t = \sqrt{\frac{\mu}{\alpha}}$

Space for Rough Work

3. Assume that a fireball of a detonating atomic bomb expands adiabatically. In this problem, the fireball is a sphere of hot monoatomic gas confined to a radius R that grows with time. Assume that the number of particles in the sphere stays constant. The pressure P changes in time but is independent of position inside the fireball. Temperature of gas is T :-

(A) $T \propto \frac{1}{R^2}$ (B) $P \propto \frac{1}{R^5}$ (C) $T \propto \frac{1}{R}$ (D) $P \propto \frac{1}{R^4}$

4. A star of radius R_s with surface temperature T_s radiates light according to the blackbody spectrum into the cold background of space (T ~ 0 K). At a distance d away from the star, a planet with radius R_p ($d >> R_p$) absorbs this radiation. Assume planet to be a black body. Planet's surface comes to a uniform temperature T_p :-

(A) $T_P = T_S\sqrt{\frac{R_S}{2d}}$

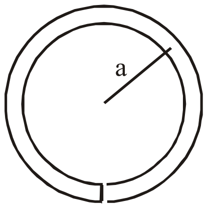
(B) $T_P = T_S\sqrt{\frac{R_S}{d}}$

(C) The energy/time radiated back to star by planet in steady state is $\frac{dE}{dt} = \frac{\pi R_S^2 R_P^2}{4d^2}\sigma T_S^4$

(D) The energy/time radiated back to star by planet in steady state is $\frac{dE}{dt} = \frac{\pi R_S^4 R_P^2 \sigma T_S^4}{4d^4}$

Space for Rough Work

5. A narrow cut is taken out of a narrow pipe, as shown, and one end is sealed. This pipe resonates with certain frequencies. The speed of sound is 340 m/s. Choose the **CORRECT** statement(s) :



(A) If the fundamental frequency of the pipe is 85 Hz then the radius a is $\frac{2}{\pi}$ m

(B) The pipe can oscillate only in odd harmonics

(C) If the pipe is oscillating in 5th harmonic, the pressure antinodes can be located at an angle of 0, $\frac{4\pi}{5}$ and $\frac{8\pi}{5}$ w.r.t. sealed end.

(D) If the pipe is oscillating in 5th harmonic, the pressure nodes can be located at an angle of 0, $\frac{4\pi}{5}$ and $\frac{8\pi}{5}$ w.r.t. sealed end.

6. Consider two speakers playing the same steady tone (in phase). Your ear is 3.0 m from one speaker, 2.7m from the other. Assume the speed of sound to be 330 m/s. (Audible range is 20 Hz to 20 kHz)

(A) In the audible range, we can hear 19 different frequencies strongly (constructive interference).

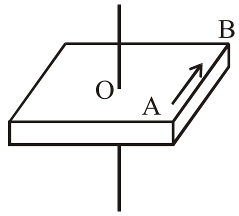
(B) In the audible range, we can hear 18 different frequencies strongly (constructive interference).

(C) In the audible range, we can hear 17 different frequencies weakly (destructive interference).

(D) In the audible range, we can hear 19 different frequencies weakly (destructive interference).

Space for Rough Work

7. A horizontal square platform of mass m and side a is free to rotate about a vertical axis passing through its centre O. The platform is stationary and a person of the same mass (m) as the platform is standing on it at point A. The person now starts walking along the edge from A to B (see figure). The speed v of the person with respect to the platform is constant. Taking v = 5 m/s and a = 1m,



(A) Angular velocity of platform when the person is at A is 6 rad/s.

(B) Angular velocity of platform increases first and then decreases again as it reaches B.

(C) Angular velocity of platform decreases first and then increases again as it reaches B.

(D) Angular velocity of platform remains constant as the person goes from A to B.

8. Sea waves travel with velocity C and are incident on a beach with a frequency of f_0. Wavefronts are parallel to the beach. Wtih what frequency f_1 waves strike a boat coming from deep sea with a speed v, directed at an angle α to the beach? What will be the frequency f_2 if the boat takes a U turn and starts moving in the opposite direction?

(A) $f_1 = \dfrac{C}{C - v\cos\alpha} f_0$ (B) $f_1 = \dfrac{C - v\sin\alpha}{C} f_0$ (C) $f_2 = \dfrac{C}{C - v\sin\alpha} f_0$ (D) $f_2 = \dfrac{C + v\sin\alpha}{C} f_0$

Space for Rough Work

SECTION–II

Paragraph for Questions 9 and 10

Melde's experiment has the string attached to a tuning fork. The string, driven by the tuning fork, vibrates at the fork's natural frequency. By suitably adjusting the length and tightness (or tension) of the string, we can make the string vibrate in one of its normal modes. Two different experiments can be performed with this setup. In the first, we keep the string tension constant and vary only the length of the string (remember the frequency is always that of the tuning fork). We find that for certain lengths, we get a normal mode vibration with a large amplitude. The length is held constant, and the string tension is varied in the second experiment. An easy way of doing this is to hang the end of the string over a pulley and suspend some weights from it. We again find large amplitude normal mode vibrations for certain values of the tension. The tuning fork is not located exactly at a node of the string's vibration. The reason, of course, is that the vibrating tuning fork (and attached string) has a small, but non-zero, amplitude.

Space for Rough Work

9. Without changing the tuning fork, which of the following action will make string oscillate in 4 loops if initially string was oscillating as shown in the diagram :-

(A) Submerging the hanging block in a liquid whose density is $\frac{7}{16}$ times density of block

(B) Increasing the length to $\frac{4}{3}$ times original length

(C) Decreasing the length to $\frac{9}{16}$ times original length

(D) Submerging the hanging block in a liquid whose density is $\frac{1}{4}$ times density of block.

10. If Melde's experiment is used to find frequency of tuning fork :-

(A) Due to vibration of fork, $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ gives a value which is less than true value.

(B) Due to vibration of fork, $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ gives a value which is more than true value.

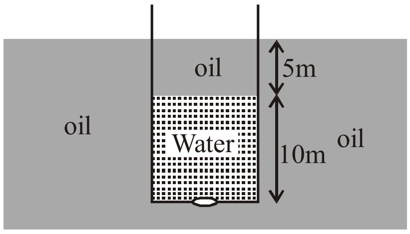
(C) Due to vibration of fork, $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ gives a value which is exactly equal to true value.

(D) If the tension is more than a certain value, the string can not oscillate in mode of vibration as shown.

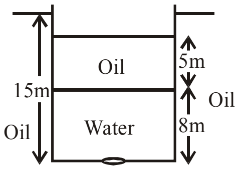
Space for Rough Work

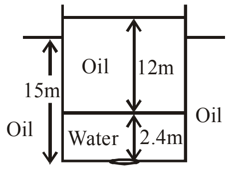
Paragraph for Questions 11 and 12

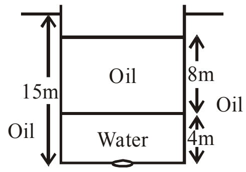
A vessel of uniform cross-section open at the top with an orifice at its bottom contains oil (relative density 0.8) on top of water. It is immersed vertically in a large open tank of same oil as shown in figure. (Take : area of orifice = 1 mm^2 and area of vessel as 100 cm^2)

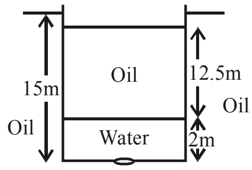


11. In which of following configuration will liquid level remain same. (Water does not come out or oil does not enter vessel) :-

(A) 

(B) 

(C) 

(D) 

12. In the figure shown in comprehension,

(A) Water comes out from vessel into surrounding oil.

(B) Oil from surrounding enters water.

(C) Speed of liquid entering or leaving orifice is $\sqrt{40}$ m/s

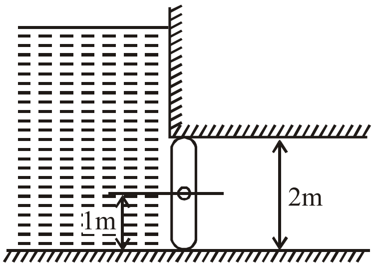
(D) Speed of surface of oil in vessel is $\sqrt{\frac{2}{5}}$ mm/s

---

Space for Rough Work

SECTION–III

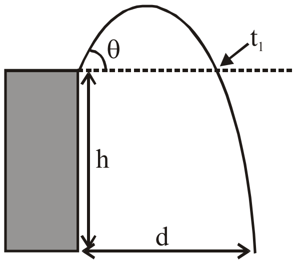
1. As shown in figure, a flat water gate, height 2m and width 6m, is free to rotate about a horizontal axis. What is the torque (in N-m) that we should exert on the gate about the axis to keep the water gate closed? Assume that the right side is exposed to atmosphere. Fill Torque/10^4 in OMR sheet.



2. The end of a glass capillary tube of radius r = 0.35 mm lowered into the water to a depth of h = 2.0 cm. What gauge pressure Δp (in Pa) is needed inside the capillary tube to blow a hemispherical air bubble at its lower end? Take Surface tension of water as 7×10^{-3} N/m. Wetting is complete. Fill $\Delta p/40$ in OMR sheet.

Space for Rough Work

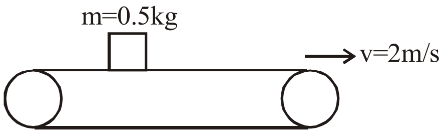
3. A projectile is launched from a cliff a height h = 10 m above the ground at an angle θ above the horizontal. After a time t_1 = 1 sec has elapsed since the launch, the projectile passes the level of the cliff top moving downward. It eventually lands on the ground a horizontal distance d = 10 m from its launch site. Find 2 tanθ and fill it in OMR sheet.



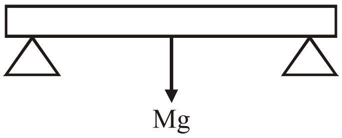
4. A ring of mass m can be hinged at a point on its periphery. There are two ways in which it can be made to oscillate: (a) It oscillates in a vertical plane and (b) it oscillates in a direction perpendicular to its plane. The time period of oscillation in case (b) is $\sqrt{3}$ sec. Find the time period of oscillation in case (a) (in sec).

Space for Rough Work

5. A block of mass 0.5 kg is gently kept on a conveyor belt moving at constant velocity of 2 m/s. After some time, the block comes to rest with respect to the belt due to friction. What is the work done by force (in joule) pulling the conveyor belt till then?

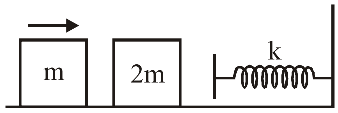


6. A uniform thin rod of mass M and length L is supported horizontally by two supports, one at each end. The acceleration of gravity g is constant and in the downward direction. At time t = 0, the left support is removed. Normal force on the right support immediately after the left support is removed is $\frac{Mg}{x}$. Fill x in OMR sheet.

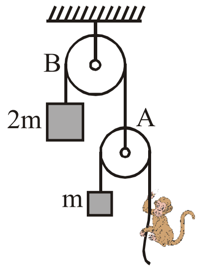


Space for Rough Work

7. A block of mass $m = 0.5$ kg is moving on a smooth table when it collides elastically with a second block of mass $2m$, which then elastically strikes a massless spring which compresses an amount $d = 10$ cm before the block comes to rest. Find the original velocity (in m/s) at which the first block was moving. Take Spring constant k = 400 N/m.



8. A monkey of mass m is balanced by a counterweight on the pulley A. Pulley A balances a load on the opposite side of pulley B (Figure). The system is stationary. With what speed V (in m/s) will the block of mass 2m move, if the monkey starts climbing with a speed of 8 m/s with respect to the rope? Rope and pulleys are ideal.



Space for Rough Work

PART-2 : CHEMISTRY

SECTION–I

1. A metallocene derivative (molecular weight = 282) has approximately 100/3 % sulfur by mass. Number of S atoms in 2.82 kg of metallocene derivative is - [S = 32] , [$N_A = 6.022 \times 10^{23}$]

(A) 10 N_A (B) 3 N_A (C) 30 N_A (D) 6.6 N_A

2. In order to prepare 25.92 gm of HBr in 20 litre container by following reaction what minimum mass of equimolar mixture of H_2 and Br_2 should be taken ?

Given : $H_2(g) + Br_2(g) \rightleftharpoons 2HBr(g)$, $K_{eq} = 64$ [H = 1, Br = 80]

(A) 64 gm (B) 32.4 gm (C) 80 gm (D) 80.4 gm

3. 5 mol gas are introduced in 1 litre container at 47ºC. Select the correct option (s) :

[R = 0.08 litre-atm/mol-K]

(A) Pressure would be 128 atm if it behaves ideally

(B) Pressure would be 28 atm if it follows vander wall equation, a = 4 atm -litre^2 /mol^2 & b = 0

(C) Pressure would be 33.33 atm if it follows vander wall equation, a = 4 atm -litre^2 /mol^2 & b = 0.04l/mole

(D) Pressure would be 160 atm if it follows vander wall equation, a = 0 atm -litre^2 /mol^2 & b = 0.04l/mole

Space for Rough Work

4. Barium permagnate solution (20ml, 0.1M) is mixed with 0.1N I^- , giving precipitate of IO_3^- and MnO_2. Resulting solution is filtered & titrated against Mo^{3+}, giving MoO_2^{2+} and Mn^{2+}, which required 0.5 M, 10ml acidified Mo^{3+}. Select the correct option(s)

(A) Volume of I^- solution taken is 30 ml

(B) Volume of I^- solution taken is 50 ml

(C) Per mole Mn^{2+} formed, 4 moles of H^+ are consumed

(D) Per mole IO_3^- formed, 2 moles of MnO_4^- are consumed

5. Which of the following atom has lowest negative electron gain enthalpy.

(A) O (B) S (C) Se (D) Te

6. Which of the following statements are **INCORRECT** ?

(A) The elements after thorium are called transuranium elements

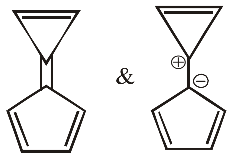
(B) P < Si < Be < Mg < Na, the order of increasing metallic character

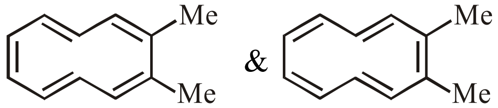
(C) The f-block elements are called transition elements

(D) Zn, Cd and Hg are called transition element

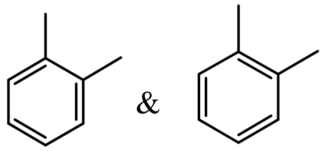
Space for Rough Work

7. Which of the following is not a pair of valid resonating structure

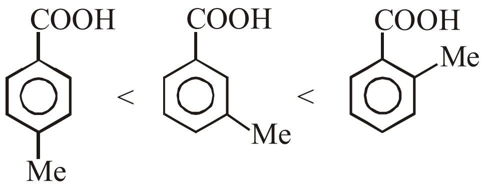
(A) 

(B) 

(C) $\mathrm{H{-}C({=}O){-}OH}$ & $\mathrm{H{-}C({=}\overset{\oplus}{O}{-}H){-}\overset{\ominus}{O}}$

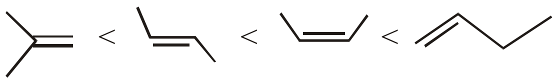
(D) 

8. Among the following, select the correct option(s)

(A) Order of acidic strength is 

(B) Order of basic strength is $NH_3 < MeNH_2 < Me_2NH < Me_3N$ in aqueous medium

(C) $CH_3CH_2CH_2-CONH_2$ and $CH_3-\underset{\displaystyle CONH_2}{CH}-CH_3$ are positional isomers.

(D) Order of heat of combustion is 

Space for Rough Work

SECTION–II

Paragraph for Questions 9 and 10

The state of equilibrium is in a dynamic balance between forward and backward reaction. This balance can be disturbed by changing concentration, temperature or pressure. If done so a certain net change occurs in the system. The direction of change can be predicted with the help of Le-Chatelier principle. It states that when a system in equilibrium is disturbed by a change in concentration or temperature, a 'net' change in it in a direction that tends to decrease the disturbing factor.

9. For the equilibrium

$Fe^{3+}(aq) + SCN^{-}(aq.) \rightleftharpoons [Fe(SCN)^{2+}(aq.)]$

(yellow) (deep red)

Select the correct option.

(A) Addition of $H_2C_2O_4$ which forms $[Fe(C_2O_4)_3]^{3-}$ deepens red colour.

(B) Addition of H_2O has no effect on the colour

(C) Addition of SCN^- intensifies red colour.

(D) Addition of Hg^{2+} which forms $[Hg(SCN)_4]^{2-}$ deepens red colour.

10. Consider the following exothermic heterogenous equilibrium.

$M_2O(s) + 2HNO_3\ (aq.) \rightleftharpoons 2MNO_3(aq.) + H_2O(l)$ with

$K_C = 3$ at 300 K. Select the **INCORRECT** option.

(A) Addition of $H_2O\ (l)$ to above equilibrium has no effect on equilibrium composition (%) of HNO_3 & MNO_3.

(B) On dilution concentration of both HNO_3 & MNO_3 decreases.

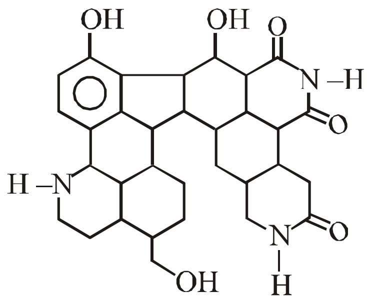
(C) At 310 K, $K_C < 3$.

(D) K_C is dependent on equilibrium concentration of HNO_3

Space for Rough Work

Paragraph for Questions 11 and 12

Consider the following structure and answer the following questions.



11. Double bond equivalent of the compound is ?

(A) 13 (B) 14 (C) 15 (D) 16

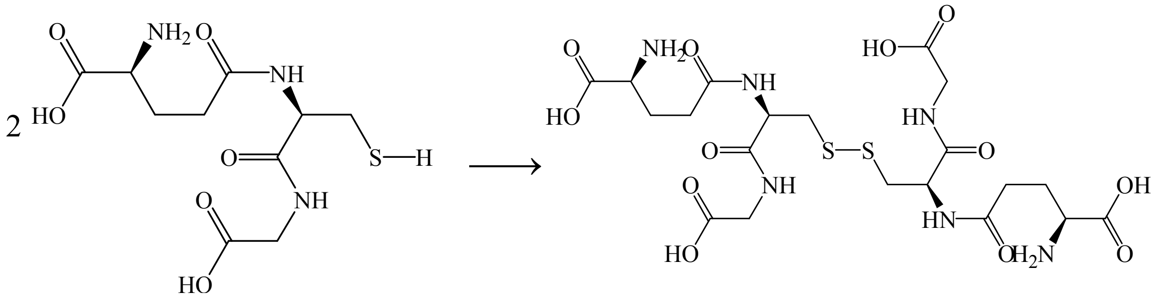
12. Number of types of functional groups present in this compound?

(A) 3 (B) 4 (C) 5 (D) 6

Space for Rough Work

SECTION–III

1. Glutathione (molecular weight = 307 gm/mol) is important antioxidant preventing damage to cellular components from free radicals, where it dimerises as



Calculate gm equivalent weight of glutathione.

Fill your answer as sum of digits (excluding decimal places) till you get the single digit answer.

2. 138 gm N_2O_4 is introduced into 8.21 litre container at 300 K. Temperature is increased to 600K where it dissociates into NO_2. If equilibrium partial pressure of N_2O_4 and NO_2 are equal then K_p (in atm) for $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ AT 600 K

Space for Rough Work

3. How many of following of statement(s) is/are correct

1. Schrodinger equation can not be solved exactly for multi electron atom.
2. Schrondinger equation when solved for H-atom wave equation gives quantised energy of the system.
3. With perfect instruments & technique, the uncertainty in position & momentum of electron will become zero.
4. When an electron in any energy state, Ψ contains all the measurable information about of the electron.
5. Electron and blue light of same wavelength have same speed.
6. Classical mechanics ignores dual behaviour of matter.
7. Splitting of spectral lines in H spectrum was sucessfully explained by Bohr.

4. For the reaction $A \xrightarrow{h\nu} B$, 9×10^{17} molecules of B were formed on absorption of 6.4 J at 310 nm. Calculate quantum efficiency (in percentage) of the process.

5. $Xe + 2O_2F_2 \rightarrow X + 2O_2$

If X is a planar compound of Xe then find the number of lone pair on central atom of X.

Space for Rough Work

6. Hypothetical scheme is given below for one molecule of acids.

$$H_2O_2 \xrightarrow{SO_3} \underset{\substack{\downarrow \\ \text{peroxomonosulphuric acid}}}{H_2SO_5} \xrightarrow{SO_3} (X)$$

$$H_2S \xrightarrow{SO_3} \underset{\substack{\downarrow \\ \text{thiosulphuric acid}}}{H_2S_2O_3} \xrightarrow{SO_3} (Y)$$

$$H_2S_n \xrightarrow{SO_3} H_2S_{(n+1)}O_3 \xrightarrow{SO_3} (Z)$$

find the sum of peroxylinkages in X, Y and Z.

7. Find the number of ions/molecules which are isoelectronic with O_2^{2+}.

N_2^{2-}, F_2^{2+}, NO^+, $\overset{\ominus}{C}N$, CO

Space for Rough Work

8. How many of the following statements are incorrect ?

(a) Cyclooctene shows geometrical isomerism

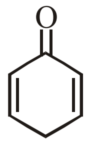
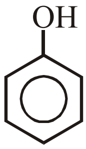
(b) Minimum 4 carbon atoms are required in a compound to show geometrical isomerism

(c) 1,6–epoxy–1–ethyl–6–methyl hexane is correct IUPAC name

(d) 3– cyclohexyl cyclopropene is correct IUPAC name

(e) 5–hydroxy pentan–2–sulphonic acid is incorrect IUPAC name

(f) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-OCH_3$ and $CH_3-CH_2-CH_2-OCH_3$ are chain isomer

(g)  is tautomer of 

(h) Enolic content $CH_3-\overset{O}{\overset{||}{C}}-H > CH_3-\underset{O}{\underset{||}{C}}-CH_2-\underset{O}{\underset{||}{C}}-CH_3$

Space for Rough Work

PART-3 : MATHEMATICS

SECTION–I

1. If cosx = tanx, then which of the following is true ?

(A) $\frac{1}{\sin x}+\cos^4 x=1$

(B) $\frac{1}{\sin x}+\cos^4 x=2$

(C) $\cos^4 x+\cos^2 x=1$

(D) $\cos^4 x+\cos^2 x=2$

2. If $f(n)=\sum_{n=1}^{n}\tan^{-1}\left(\frac{2.3^n}{3+9^n}\right)$, then-

(A) $f(2)=\tan^{-1}\frac{4}{5}$

(B) $f(3)=\tan^{-1}\frac{13}{14}$

(C) As n tends to infinity f(n) tends to $\frac{\pi}{4}$

(D) As 'n' tends to infinity f(n) tends to $\frac{\pi}{2}$

3. If $f(x)=\cos^{-1}\left(\frac{-2x-x^2}{x^2+2x+2}\right)$, then-

(A) $f\left(-\frac{1}{2}\right)=2\tan^{-1}\frac{1}{2}$

(B) $f(0)=2\tan^{-1}1$

(C) $f(1)=2\tan^{-1}2$

(D) $f\left(\frac{1}{2}\right)=\tan^{-1}3$

Space for Rough Work

4. Let $S_1 = x^2 + y^2 - 1 = 0$; $S_2 \equiv x^2 + y^2 - 2x - 2y = 0$, P & Q be the points on S_1 & S_2. Now which of the following is true ?

(A) Radical axis of $S_1 = 0$ & $S_2 = 0$ is $2x + 2y = 1$

(B) The acute angle of intersection of $S_1 = 0$ & $S_2 = 0$ is $\cos^{-1}\dfrac{1}{2\sqrt{2}}$.

(C) The maximum distance between P & Q is $1+2\sqrt{2}$

(D) The minimum distance between P & Q is 1.

5. Let $\{a_n\}$ be an arithmetic progression with 1st term 1 and common difference 1 and $\{g_n\}$ be a geometric progression with 1st term 1 and common ratio r. A new progression is defined as $r_n = \dfrac{a_n}{g_n}$ $(n \in N)$. Let $f(r) = \sum_{n=1}^{\infty} r_n$, then -

(A) $f(2) = 4$ (B) $f(3) = \dfrac{9}{4}$ (C) $f(4) = \dfrac{16}{9}$ (D) $f(5) = \dfrac{25}{16}$

6. For the fucntion $f : R \to [a, b]$; $f(x) = \dfrac{x^4 + x^2 + 1}{(x^2 + x + 1)^2}$, which of the following holds good.

(A) $f(x)$ is many one function

(B) If $f(x)$ is onto function, then $ab = 1$

(C) If $f(x)$ is onto function, then $a + b = \dfrac{10}{3}$

(D) $f(1)f(-1)f(10)f(-10) = 1$

7. Let a, b, c be the positive integers such that $a < b < c$. If the two curves $y = |x - a| + |x - b| + |x - c|$ and $2x + y = 2003$ have exactly one point in common, then -

(A) least possible value of c is 1002.

(B) greatest possible value of b is 1001

(C) least possible value of b is 1002

(D) greatest possible value of a is 1000

8. There are 5 boxes numbered from 1 to 5. There is 1 Red and 2k black balls in k^{th} box, k = 1,2,3,4,5. From each box either one red ball is taken or one or more than one black balls are taken. But from each box both coloured balls are never taken (balls of same coloure are all alike). Now which of the following holds good ?

(A) Total number of ways of selecting odd number of red balls is 4725

(B) Total number of ways of selecting even number of red balls is 5670

(C) Total number of ways of selecting odd number of red balls is 5670

(D) Total number of ways of selecting even number of red balls is 4725

Space for Rough Work

SECTION–II

Paragraph for Questions 9 and 10

Consider the number N = 1! × 2! × 3! ×........× 10! $=\prod_{r=1}^{10}(r!)$. Let P be the number of zeroe's at the end of N and Q be the number of perfect cubes that can divide N.

9. The value of 'P' is-

(A) 4 (B) 7 (C) 17 (D) 38

10. Which of the following holds good ?

(A) Total number of divisors of Q = 18
(B) Total number of odd divisors of Q = 6
(C) Total number of even divisors of Q = 6
(D) Total number of divisors of Q = 24

Space for Rough Work

Paragraph for Questions 11 and 12

Consider the function $f : R \to R$, $f(x) = \sqrt[3]{x + \sqrt{1+x^2}} + \sqrt[3]{x - \sqrt{1+x^2}}$, then

11. If $\sum_{n=1}^{n} f^{-1}(n) = \frac{n^2(n+1)^2}{\lambda} + \frac{3n(n+1)}{\mu}$, then

(A) $\lambda + \mu = 12$ (B) $3\lambda = \mu$ (C) $\lambda = 2\mu$ (D) $\lambda = \mu$

12. Range of $g(\theta) = f^{-1}(\sin\theta)$ is-

(A) [–1,1] (B) [–2,2] (C) [–3,3] (D) [–4,4]

Space for Rough Work

SECTION–III

1. Let f be a function satisfying the functional rule $2f(x) + f(1 - x) = x \ \forall \ x \in R$. Then the value of $f(1) + f(2) + f(3)$ is

2. Four horses compete in a race. Let N be the total number of different orders in which the horses can cross the finish line. Assume that all four horses finish the race and two or more horses can cross the finishing line together. The value of $\left[\frac{N}{10}\right]$ is (where [.] denotes greatest integer function)

3. If $S = \cos\left(\frac{2\pi}{28}\right)\text{cosec}\left(\frac{3\pi}{28}\right)+\cos\left(\frac{6\pi}{28}\right)\text{cosec}\left(\frac{9\pi}{28}\right)+\cos\left(\frac{18\pi}{28}\right)\text{cosec}\left(\frac{27\pi}{28}\right)$, then the value of

$\left|\log_{(2-\sqrt{3})}\left(1+S+S^2+S^3\right)\right|$ is

Space for Rough Work

4. Consider a triangle with sides 6,4 and $\sqrt{52}$. Let the area of this triangle is 'S' whereas area of triangle whose length of sides are equal to length of medians of given triangle be G. If $\frac{G}{S}$ is $\frac{a}{b}$ where a and b are relatively prime, then the value of a + b is

5. Let 'f' be a function such that $2f(x)+f\left(\frac{x+1}{x-1}\right)=x \forall x \in R-\{1\}$. If $3f(x)+\frac{x+1}{x-1}=\lambda x \ \forall x \in R-\{1\}$, then value of λ is

6. If the greatest value of $\sin^2\alpha \cos^6\alpha$ is $\frac{a}{256}$, then the value of $\left[\frac{a}{10}\right]$ is (where [.] denotes greatest integer function)

Space for Rough Work

7. Let AB be a variable chord of length 5 to the circle $x^2 + y^2 = \frac{25}{2}$. A triangle ABC is constructed such that BC = 4 & CA = 3. If locus of 'C' is $x^2 + y^2 = a$, then the only possible integral value of $\frac{1}{a}$ is

8. If the coefficient of x^{50} in the expansion $(1 + x)^{1000} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} + ... + 1001x^{1000}$ is N, then sum of the digits of the number $\frac{952!50!}{1001!}N$ is

---

Space for Rough Work

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1	CD	1	C	1	BC
2	BD	2	B	2	ABC
3	AB	3	ABD	3	ABC
4	AD	4	AD	4	ABC
5	BC	5	A	5	ABCD
6	BC	6	ACD	6	ABCD
7	AD	7	B	7	ABD
8	BD	8	AD	8	AB
9	AB	9	C	9	B
10	BD	10	D	10	AB
11	ABD	11	C	11	AC
12	ACD	12	C	12	B
1	4	1	1	1	5
2	6	2	6	2	7
3	2	3	4	3	0
4	2	4	9	4	7
5	2	5	2	5	2
6	4	6	1	6	2
7	3	7	3	7	2
8	2	8	5	8	3

PAPER-1

PART-1 : PHYSICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C,D	B,D	A,B	A,D	B,C	B,C	A,D	B,D	A,B	B,D
	Q.	11	12								
	A.	A,B,D	A,C,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	4	6	2	2	2	4	3	2		

SOLUTION

SECTION-I

1. Ans. (C,D)

Sol. $\frac{R}{L} = \frac{\rho}{\rho_w} \to$ same

$B = \rho V g_{eff}$

$p = p_0 + h\rho g_{eff}$

2. Ans. (B,D)

Sol. (A) $\mu mg = m\omega^2 r$

$= \alpha^2 r t^2$

$$t = \sqrt{\frac{\mu g}{\alpha^2 r}}$$

(C) $N = mr\alpha$

$\mu N = m\omega^2 r$

$\mu m r\alpha = m\omega^2 r$

$= \alpha^2 t^2$

$$t = \sqrt{\frac{\mu}{\alpha}}$$

3. Ans. (A,B)

Sol. $PV^{\gamma} = C \Rightarrow P \times \left(R^3\right)^{\frac{5}{3}} = C$

$TV^{2/3} = \text{constant}$

$T \times R^{3\times 2/3} = \text{constant}$

4. Ans. (A,D)

Sol. (A) $\frac{4\pi R_S^2 \sigma T_S^4}{4\pi d^2} \times \pi R_p^4 = 4\pi R_p^2 \sigma T_p^4$

$$T_p = T_s\sqrt{\frac{R_s}{2d}}$$

(C) $\frac{4\pi R_p^2 \times \sigma T_p^4}{4\pi d^2} \times \pi R_S^2 = \frac{dE}{dt}$

$$\frac{\pi R_S^2 \sigma T_S^4}{d^2} \times \frac{R_S^2}{4d^2}$$

5. Ans. (B,C)

Sol. (A) $2\pi a = \frac{\lambda}{4} \Rightarrow \lambda = \frac{340}{85} = 4\text{m}$

$\Rightarrow 2\pi a = 1$

$$a = \frac{1}{2\pi}\text{m}$$

(B) closed organ pipe

(C) Sealed end is PAN

$$2\pi a = \left(2\frac{1}{2}\right)\frac{\lambda}{2}$$

$$2\pi a = \frac{5\lambda}{4}$$

$$\lambda = \frac{8\pi a}{5}$$

$$\theta = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$$

$$\Rightarrow \theta = \frac{4\pi}{5}, \frac{8\pi}{5}$$

6. Ans. (B,C)

Sol. $Dx = 0.3 = n\lambda$

$$= n \times \frac{330}{f}$$

$$f = \frac{3300n}{0.3}$$

$$f_{max} = 20000$$

$$n = \frac{20000}{1100}$$

$$\Delta x = 0.3 = \left(n + \frac{1}{2}\right)\lambda$$

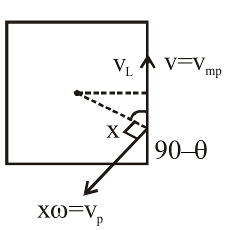
$$f = 1100\left(n + \frac{1}{2}\right)$$

$$n + \frac{1}{2} = 18.22$$

$\Rightarrow n = 0$ to 17

7. Ans. (A,D)

Sol.



$$v_m = v_{mp} + v_p$$

$$= \sqrt{v^2 + x^2\omega^2 + 2v\omega}\cos(90+\theta)$$

$$= \sqrt{v^2 + x^2\omega^2 - 2vx\omega\sin\theta}$$

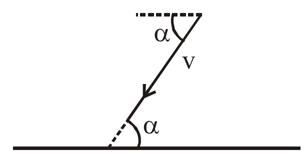
$$L_i = 0 = L_f$$

$$= -\frac{m}{12}\left(a^2 + a^2\right)\omega + m\left(v - x\omega\sin\theta\right)\frac{\alpha}{2}$$

$$\Rightarrow \frac{a^2}{6}\omega + \frac{a^2}{4}\omega = \frac{va}{2} \Rightarrow \omega = \frac{6v}{5a}$$

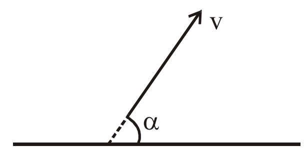
8. Ans. (B,D)

Sol.



Observer moving away from source.

$$\nu = \frac{C - v\sin\alpha}{C}\nu_0$$



Observer moving towards source.

$$\nu_1 = \frac{C - v\sin\alpha}{C}\nu_0$$

9. Ans. (A,B)

10. Ans. (B,D)

11. Ans. (A,B,D)

Sol. $p_0 + h_1\rho_1 g = p_0 + h_2\rho_1 g + h_3\rho_3 g$

12. Ans. (A,C,D)

Sol. $p_{in} = 10^5 + 5 \times 0.8 \times 10^3 \times 10 + 10 \times 10^3 \times 10$

$$= 10^5 + 4 \times 10^4 + 10^5$$

$$= 2.4 \times 10^5$$

$$p_{out} = 10^5 + 0.8 \times 15 \times 10^3 \times 10 = 2.2 \times 10^5$$

$$p_{out} + \frac{1}{2}\rho v^2 = p_{in}$$

$$\frac{1}{2} \times 10^3 \times v^2 = 0.2 \times 10^5$$

$v = \sqrt{40}$ m/s

$\sqrt{40} \times 10^{-6} = 100 \times 10^{-4}\, v_s$

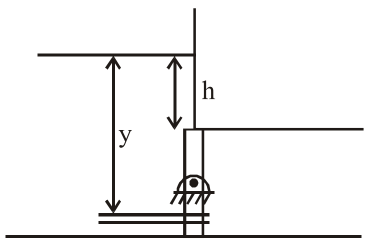
$v_s = \sqrt{40} \times 10^{-4}$ m/s

$= \sqrt{\frac{40}{10}}$ mm/s $= \sqrt{\frac{40}{100}}$ mm/s $= \sqrt{\frac{2}{5}}$ mm/s

SECTION-IV

1. Ans. 4

Sol.



$$\int_h^{h+2} \rho g y dy \times 6 \times \left(y - (h+1)\right)$$

$$= 10^4 \times 6\left[\frac{y^3}{3} - (h+1)\frac{y^2}{2}\Big|_h^{h+2}\right]$$

$$= 6\times10^4\left[\frac{1}{3}\left((h+2)^3 - h^3\right) - \frac{(h+1)}{2}\left((h+2)^2 - h^2\right)\right]$$

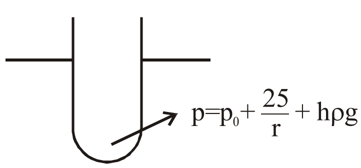
$$= 6\times10^4\left[\frac{1}{3}\left[6h^2 + \frac{6h}{12} + 8 - \frac{(h+1)}{2}\left[4^2h + 4^2\right]\right]\right]$$

$$= 6\times10^4\left[\frac{1}{3}\left(6h^2 + 12h + 8\right) - 2\left(h^2 + 2h + 1\right)\right]$$

$$= 6\times10^4\left[\frac{8}{3} - 2\right] = 4\times10^4\,\text{Nm}$$

2. Ans. 6

Sol.



$$\Delta p = \frac{25}{r} + h\rho g$$

$$= \frac{2\times7\times10^{-3}}{0.35\times10^{-3}} + 2\times10^{-2}\times10^4 = 240$$

$$\frac{\Delta p}{40} = 6$$

3. Ans. 2

Sol. $t_1 = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{10}{2\sin\theta}$

$$-h = d\tan\theta - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$$

$$-10 = 10\tan\theta - \frac{1}{2}\times\frac{10\times10}{25}\tan^2\theta$$

$$2\tan^2\theta - \tan\theta - 1 = 0$$

$$\tan\theta = \frac{1\pm\sqrt{1+8}}{4} = \frac{1+3}{4}, \frac{1-3}{4} = 1, -\frac{1}{2}$$

4. Ans. 2

Sol $T_a = 2\pi\sqrt{\frac{2mR^2}{mgR}}$

$$T_b = 2\pi\sqrt{\frac{\frac{3}{2}mR^2}{mgR}} = \sqrt{3}$$

$$\Rightarrow 2\pi\sqrt{\frac{mR^2}{2mgR}} = 1$$

$T_a = 2$ sec

5. Ans. 2

Sol. $W = \Delta KE + \Delta H_{friction}$

as seen from belt frame

$$\Delta H_{friction} = \frac{1}{2}m\left(0^2 - v^2\right)$$

$$\Rightarrow W = mv^2$$

$$= 0.5 \times 2^2 = 2J$$

6. Ans. 4

Sol. $mg - N = ma = m\frac{\ell}{2}\alpha$

$$mg\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$$

$$\alpha = \frac{3g}{2\ell}$$

$$mg - \frac{m\ell}{2}\times\frac{3g}{2\ell} = N$$

$$N = \frac{mg}{4}$$

7. Ans. 3

Sol. $mv_0 = 2mv_2 + mv_1$

$e = 1 = \dfrac{v_2 - v_1}{v_0}$

$v_2 - v_1 = v_0$

$2v_2 + v_1 = v_0$

$3v_2 = 2v_0$

$v_2 = \dfrac{2v_0}{3}$

$\dfrac{1}{2}2mv_2^2 = \dfrac{1}{2}kd^2$

$v_2 = \sqrt{\dfrac{k}{2m}}d$

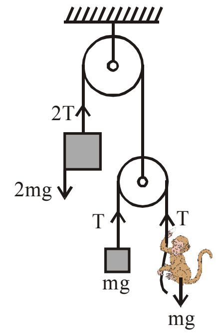
$v_0 = \dfrac{3}{2}d\sqrt{\dfrac{k}{2m}}$

$= \dfrac{3}{2} \times 0.1\sqrt{\dfrac{400}{0.5 \times 2}}$

$v_0 = 3$

8. Ans. 2

Sol.



$T - mg = ma_b = ma_m$

$2T - 2mg = 2ma_B$

$\Rightarrow a_B = a_b = a_m \Rightarrow v_B = v_b = v_m = v$

$v_p = -v$

$v_R - v_b = 2v_p$

$v_R = -2v - v = -3v$

$v_{mR} = v - (-3v) = 4v = 8$

PART-2 : CHEMISTRY — ANSWER KEY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	B	A,B,D	A,D	A	A,C,D	B	A,D	C	D
	Q.	11	12								
	A.	C	C								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	1	6	4	9	2	1	3	5		

SOLUTION

SECTION - I

1. Ans. (C)

$282 \times \dfrac{33.33}{100} = 96 \Rightarrow 3$ 'S' atoms per molecule.

2. Ans. (B)

H_2	+	$Br_2 \rightleftharpoons$	$2HBr$
aM		aM	–
a -x		a-x	2x

$= \dfrac{25.92/81}{20} = \dfrac{0.32}{20} = 0.016\,M$

$64 = \dfrac{(2x)^2}{(a-x)^2}$

$8a - 0.064 = 0.016$

$8a = 0.08$

$a = 0.01$ M

$n_{H_2} = 0.01 \times 20 = 0.2 \text{ mol} \equiv 0.4 \text{ gm}$

$n_{Br_2} = 0.01 \times 20 = 0.2 \text{ mol} \equiv 32 \text{ gm}$

3. Ans. (A, B, D)

(A) $P \times 1 = 5 \times 0.08 \times 320 \Rightarrow P = 128$ atm

(B) $\left(P + \dfrac{4 \times 5^2}{1^2}\right)(1 - 0) = 5 \times 0.08 \times 320$

$\Rightarrow P = 28$ atm

(C) $\left(P + \dfrac{4 \times 5^2}{12}\right)(1 - 0.04 \times 5) = 128 \Rightarrow P = 60$ atm

(D) $P\,(1 - 0.04 \times 5) = 128 \Rightarrow P = 160$ atm

4. Ans.(A, D)

$$4H^+ + 3MnO_4^- + 5Mo^{3+} \rightarrow 3Mn^{2+} + 5\overset{+6}{M}o\ O_2^{2+}$$

20 ml $\quad n_f = 3$

0.2M $\quad$ 0.5 M

(nf = 5) $\quad$ 10 ml

$$H_2O + 2\overset{+7}{M}nO_4^- + \overset{-1}{I^-} \longrightarrow \overset{+5}{IO_3^-} + 2\overset{+4}{M}nO_2 + 2OH^-$$

0.1N

1 mmol $\quad$ 0.5 mmol

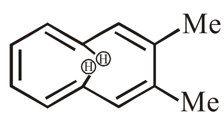
$(n_f = 3)$ $\quad n_f = 6$

V = 30 ml

5. Ans. (A)

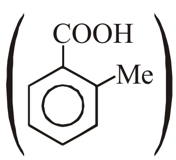
6. Ans. (A,C,D)

7. Ans. (B)



Non-planar, therefore conjugation over complete ring isn't possible.

8. Ans. (A,D)

Ortho substituted benzoic acid  is most acidic

$\overset{4}{C}H_3–\overset{3}{C}H_2–\overset{2}{C}H_2–\overset{1}{C}ONH_2$ & $\overset{3}{C}H_3–\overset{2}{C}H(–{}_1CONH_2)–CH_3$: Chain isomer

9. Ans.(C)

$$Fe^{3+}(aq) + 3H_2C_2O_4 \rightleftharpoons [Fe(C_2O_4)_3]^{3-} + 6H^+$$

$$Hg^{2+} + 4SCN^- \rightleftharpoons [Hg(SCN)_4]^{2-}$$

10. Ans.(D)

For exothermic equilibrium on increasing temperature Keq decreases.

11. Ans. (C)

12. Ans. (C)

Phenol, Alcohol , 2º-amine, Ketone, Cyclic amide

SECTION - IV

1. Ans. (307)

OMR ANS (1)

$$2C_{10}H_{17}N_3O_6\overset{-2}{S} \longrightarrow C_{20}H_{32}N_6O_{12}\overset{-1}{S} + 2H^+ + 2e^-$$

$(n_f = 1)$

2. Ans. (6)

3. Ans. (4)

Except 3, 5 & 7 all are correct

4. Ans. (9)

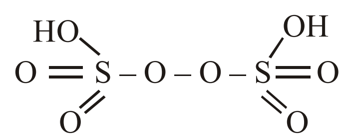
$$\frac{6.4}{1.6\times10^{-19}} = \frac{1240}{310}\times N \Rightarrow N = 10^{19}.$$

$$Q.\ E. = \frac{9\times10^{17}}{10^{19}}\times100 = 9\%$$

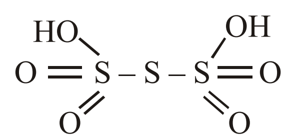
5. Ans. (2)

6. Ans. (1)

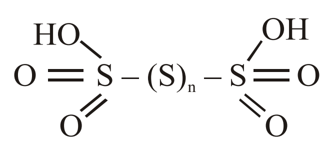
$X = H_2S_2O_8$ peroxodisulphuric acid



$Y = H_2S_3O_6$ trithionic acid



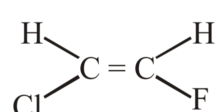
$Z = H_2S_{n+2}O_6$



7. Ans. (3)

8. Ans. (5)

b , c , e , f , h



(2-carbon)

Show G.I.

$\overset{3}{C}H_3–\overset{2}{C}H(–{}_1CH_3)–OCH_3$ & $\overset{3}{C}H_3–\overset{2}{C}H_2–\overset{1}{C}H_2–OCH_3$:

Position isomer

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	A,B,C	A,B,C	A,B,C	A,B,C,D	A,B,C,D	A,B,D	A,B	B	A,B
	Q.	11	12								
	A.	A,C	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	7	0	7	2	2	2	3		

SOLUTION

SECTION-I

1. Ans. (B,C)

$\cos^2 x = \sin x$

$\cos^4 x = 1 - \cos^2 x$

$\cos^4 x + \cos^2 x - 1 = 0$

$$\cos^2 x = \frac{-1+\sqrt{5}}{2}$$

$$\frac{1}{\sin x} + \cos^4 x = \frac{1}{\cos^2 x} + \cos^4 x$$

$$= \frac{2}{\sqrt{5}-1} + \frac{3-\sqrt{5}}{2} = \frac{\sqrt{5}+1}{2} + \frac{3-\sqrt{5}}{2} = 2$$

(B) & (C) are correct.

2. Ans. (A,B,C)

$$f(n) = \sum_{n=1}^{n} \tan^{-1}\left(\frac{2.3^{n-1}}{1+3^n.3^{n-1}}\right)$$

$$= \sum_{n=1}^{n} \tan^{-1}(3^n) - \tan^{-1}(3^{n-1})$$

$$f(n) = \tan^{-1} 3^n - \frac{\pi}{4}$$

Now check option.

3. Ans. (A,B,C)

$$f(x) = \cos^{-1}\left(\frac{1-(x+1)^2}{1+(x+1)^2}\right)$$

$$x + 1 = \tan\theta \quad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

$$y = \cos^{-1}\cos 2\theta$$

$$= \begin{cases} -2\tan^{-1}(x+1), & x < -1 \\ 2\tan^{-1}(x+1), & x \ge -1 \end{cases}$$

4. Ans. (A,B,C)

5. Ans. (A,B,C,D)

$$r_n = \frac{n}{r^{n-1}}$$

$$f(r) = 1 + \frac{2}{r} + \frac{3}{r^2} + \frac{4}{r^3} + ...\infty \quad .....(1)$$

$$\frac{f(r)}{r} = \frac{1}{r} + \frac{2}{r^2} + \frac{3}{r^3} + ...\infty \quad .....(2)$$

$$\Rightarrow f(r) = \left(\frac{r}{1-r}\right)^2$$

Now check options.

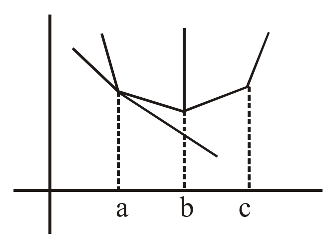
6. Ans. (A,B,C,D)

$$f(x) = \frac{(x^2-x+1)(x^2+x+1)}{(x^2+x+1)^2}$$

$$f(x) = \frac{x^2-x+1}{x^2+x+1}$$

Now check options.

7. Ans. (A,B,D)



By compairing the slopes, condition in problem is satisfied for $(a, b + c - 2a)$ to lie on the line. Putting in line we get

$b + c = 2003$

$b_{max} = 1004$

$c_{min} = 1002$

8. Ans. (A,B)

Let P denotes total number of ways of selecting even number of red balls and Q denotes total number ob ways of selecting odd number of red balls.

(2 – 1)(4 – 1)(6 – 1)(8 – 1)(10 – 1) = P – Q

(2 + 1)(4 + 1)(6 + 1)(8 + 1)(10 + 1) = P + Q

Now solve to get P & Q.

Paragraph for Question 9 to 10

$N = 2^a 3^b 5^c 7^d$

$$a = \sum_{m=2}^{10}\sum_{n=1}^{\infty}\left[\frac{m}{2^n}\right] = 38$$

similarly b = 17, c = 7, d = 4

$N = 2^{38}\ 3^{17}\ 5^7\ 7^4$

obviously P = 7

$Q = 13 \times 6 \times 3 \times 2$

$\quad = 2^2.3^2 \times 13'$

9. Ans. (B)

10. Ans. (A,B)

Paragraph for Question 11 to 12

$$y = \sqrt[3]{x+\sqrt{x^2+1}} + \sqrt[3]{x-\sqrt{x^2+1}}$$

Image in y = x & then cubing

$$x^3 = \left(\sqrt[3]{y+\sqrt{y^2+1}} + \sqrt[3]{y-\sqrt{y^2+1}}\right)^3$$

$x^3 = 2y + 3(-1)x$

$$f^{-1}(x) = \frac{x^3+3x}{2}$$

11. Ans. (A,C)

12. Ans. (B)

SECTION-IV

1. Ans. 5

Replace x by 1 – x f solve to get $f(x)$

2. Ans. 7

$H_1 - H_2 - H_3 - H_4 = \underline{|4} = 24$

$\boxed{H_1H_2} - H_3 - H_4 = {}^4C_2\underline{|3} = 36$

$\boxed{H_1H_2} - \boxed{H_3H_4} = \dfrac{{}^4C_2}{2}\underline{|2} = 6$

$\boxed{H_1H_2H_3} - H_4 = {}^4C_3.\underline{|2} = 8$

$H_1H_2H_3H_4 = 1 \Rightarrow 75$

3. Ans. 0

$$s = \sum_{r=1}^{3}\frac{\cos(2.3^{r-1}\theta)}{\sin 3^r\theta} \qquad \left(\theta = \frac{\pi}{28}\right)$$

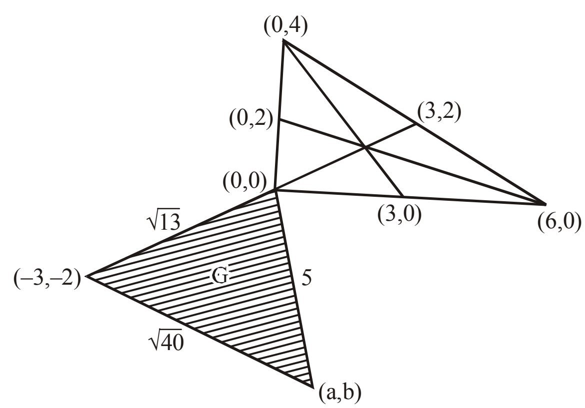
$$\Rightarrow \quad s = \sum_{r=1}^{3}\frac{\sin(3^{r-1}\theta)\cos(2.3^{r-1}\theta)}{\sin(3^r\theta)\sin(3^{r-1}\theta)}$$

$$= \frac{1}{2}\sum_{r=1}^{3}\frac{\sin 3^r\theta - \sin 3^{r-1}\theta}{\sin 3^r\theta \sin 3^{r-1}\theta}$$

$$S = \sum_{r=1}^{3}\mathrm{cosec}(3^{r-1}\theta) - \mathrm{cosec}(3^r\theta)$$

$$= \mathrm{cosec}\theta - \mathrm{cosec}27\theta = 0 \ \ \{\because\ \theta + 27\theta = \pi\}$$

4. Ans. 7



$a^2 + b^2 = 25$ ....(1)

$(a + 3)^2 + (b + 2)^2 = 40$

$6a + 4b = 2$ ....(2)

Solving (1) & (2)

possible value of a is 3

$\Rightarrow \quad b = 4$

$$\frac{G}{S} = \frac{3}{4}$$

5. Ans. 2

$$\left.\begin{array}{l} 2f(x) + f\left(\dfrac{x+1}{x-1}\right) = x \\ x \to \dfrac{x+1}{x-1} \\ \Rightarrow f(x) + 2f\left(\dfrac{x+1}{x-1}\right) = \dfrac{x+1}{x-1} \end{array}\right\} \begin{array}{l} \text{Solving} \\ 3f(x) = 2x - \dfrac{x+1}{x-1} \\ 3f(x) + \dfrac{x+1}{x-1} = 2x \\ \qquad \Rightarrow \lambda = 2 \end{array}$$

6. Ans. 2

$$y = \sin^2\alpha(1-\sin^2\alpha)^3$$

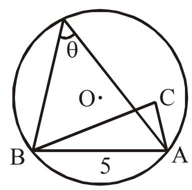
$$y = t(1-t)^3 \qquad 0 < t \le 1$$

$$\frac{t + \frac{3(1-t)}{3}}{4} \ge \sqrt[4]{\frac{t(1-t)^3}{27}}$$

$$\frac{t(1-t)^3}{27} \le \frac{1}{4^4}$$

$$t(1-t)^3 \le \frac{3^3}{256}$$

7. Ans. 2

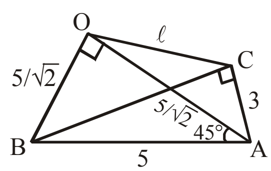


$$\frac{5}{2\sin\theta} = \frac{5}{\sqrt{2}}$$

$$\theta = 45°$$

$\Rightarrow$ BOA = 90°

ACO = 135°



$$-\frac{1}{\sqrt{2}} = \frac{9+\ell^2-\frac{25}{2}}{6\ell}$$

$$-3\sqrt{2}\ell = \ell^2 - \frac{7}{2}$$

$$\ell^2 + 3\sqrt{2}\ell - \frac{7}{2} = 0$$

$$\ell = \frac{-3\sqrt{2}+\sqrt{18+14}}{2} = \frac{1}{\sqrt{2}}$$

8. Ans. 3

$$s = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots\ldots + 1001x^{1000}(1+x)^0$$

$$\frac{xs}{(1+x)} = x(1+x)^{999} + 2x^2(1+x)^{998} + \ldots\ldots \ldots\ldots + 1001x^{1000} + \frac{1001+x^{1001}}{(1+x)}$$

$$\frac{s.1}{(1+x)} = (1+x)^{1000} + x(1+x)^{999} + \ldots\ldots \ldots + x^{1000}(1+x)^0 - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = (1+x)^{1000}\left\{\frac{\left(\frac{x}{1+x}\right)^{1001}-1}{\frac{x}{1+x}-1}\right\} - \frac{1001x^{1001}}{(1+x)}$$

$$\frac{s}{(1+x)} = \frac{x^{1001}-(1+x)^{1001}}{-1} - \frac{1001x^{1001}}{(1+x)}$$

$$s = (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$$

coeff. of $x^{50} = {}^{1002}C_{50}$