

# FIITJEE FARIDABAD

## MOCK PRACTICE PAPER FOR JEE -Advance- 2020

### MOCK PRACTICE PAPER-31

**Time: 3 hours**

**PAPER - II**

**Maximum marks: 240**

### INSTRUCTIONS

**Caution:** Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

#### A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Parts.
3. **Part-1** is Physics, **Part-2** is Chemistry and **Part-3** is Mathematics.
4. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
5. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

#### B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

#### C. Marking Scheme For All Sections.

- (i) **Section-I (01 – 10)** contains **10** multiple choice questions which has **one or more than one** correct answers. Each question carries **+4 marks** for correct answer and **-2** for incorrect in this section.
- (ii) **Section-II (01 – 02)** contains **2 Matrix Match Type questions** containing statements given in 2 columns. Statements in the first column have to be matched with statements in the second column. Each question carries **+8 marks** for all correct answer. For each correct row **+2 marks** will be awarded and **- 1 mark** for wrong answer. There may be **one or more than one correct** choice. No marks will be given for any wrong match in any question
- (iii) **Section-III (01 – 08)** contains **8** integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9 both included. Each question carries **+3 marks** for correct answer and **No** negative marking in this section.

**Name of the Candidate :** \_\_\_\_\_

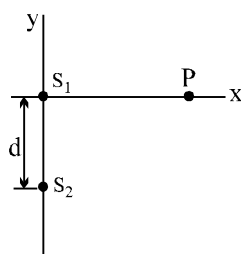
**Batch :** \_\_\_\_\_ **Date of Examination :** \_\_\_\_\_

**Enrolment Number :** \_\_\_\_\_

## PART-1 : PHYSICS

### SECTION-I

1. Figure shows two point sources  $S_1$  and  $S_2$  that emit sound of wavelength  $\lambda = 2.0$  m. The emissions are isotropic and inphase and the separation between the sources is  $d = 16$  m. At any point P on the axis, the wave from  $S_1$  and the wave from  $S_2$  interfere. When P is very far away ( $x \approx \infty$ ), mark the **CORRECT** statement(s):-



- (A) The phase difference between the arriving waves from  $S_1$  and  $S_2$  is zero.  
(B) Interference they produce is approximately fully constructive.  
(C) As we then move P along the x axis towards  $S_1$ , the phase difference between the waves from  $S_1$  and  $S_2$  increases  
(D) As we then move P along the x axis towards  $S_1$ , the phase difference between the waves from  $S_1$  and  $S_2$  decreases

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Space for Rough Work

2. A small sphere of radius  $R$  is arranged to pulsate so that its radius varies in simple harmonic motion between a minimum of  $R - \Delta R$  and a maximum of  $R + \Delta R$  with frequency  $f$ . This produces sound waves in the surrounding air of density  $\rho$ , take atmospheric pressure to be  $p_{\text{atm}}$  and ratio of specific heats constant pressure ( $C_p$ ) and constant volume ( $C_v$ ) to be  $\gamma$ . (Assume the amplitude of oscillation of the sphere is the same as that of the air at the surface of the sphere.) Mark the **CORRECT** statement(s):-

(A) The intensity of sound waves at the surface of sphere is  $I = 2\pi^2 f^2 \sqrt{\rho \gamma p_{\text{atm}}} (\Delta R)^2$

(B) The total acoustic power radiated by the sphere is  $P = 8\pi^3 R^2 f^2 \sqrt{\rho \gamma p_{\text{atm}}} (\Delta R)^2$

(C) At a distance  $d \gg R$  from the center of the sphere, the amplitude is  $A = \left(\frac{R}{d}\right) \Delta R$

(D) At a distance  $d \gg R$  from the center of the sphere, the pressure amplitude is

$$p_{\text{max}} = 2\pi \left(\frac{Rf}{d}\right) \sqrt{\rho \gamma p_{\text{atm}}} \Delta R$$

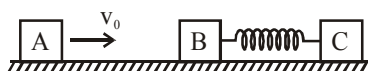
3. Two blocks B and C each of mass  $m$  are connected by a light spring of force constant  $k$  and natural length  $L$ . The whole system rests on a frictionless table such that  $x_B = 0$  and  $x_C = L$ , where  $x_B$  and  $x_C$  are coordinates of the block B and C respectively. Another block of mass  $M$ , which is travelling at speed

$V_0$  collides head on elastically with the block B at  $t = 0$ . Mass ratio is  $\frac{m}{M} = \gamma$ . For  $t > 0$  the positions of the blocks are given by

$$x_B = \alpha t + \beta \sin \omega t$$

$$x_C = L + \alpha t - \beta \sin \omega t$$

Mark the **CORRECT** option(s) :



(A)  $\omega = \sqrt{\frac{2k}{m}}$

(B)  $\alpha = \frac{V_0}{1 + \gamma}$

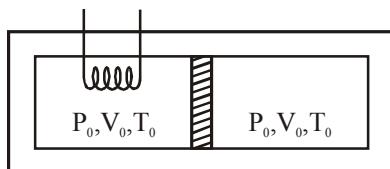
(C)  $\beta = \frac{V_0}{\omega(1 + \gamma)}$

(D) Maximum speed of C is  $\frac{2V_0}{1 + \gamma}$

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Space for Rough Work

4. Figure shows a double chambered vessel with thermally insulated walls and partitions. On each side there are  $n$  moles of an ideal monoatomic gas. Initially the pressure, volume and temperature in each side is  $P_0, V_0, T_0$ . The heater in first chamber supplies heat very slowly till the gas in the first chamber expands such that the pressure, volume and temperature of the gas on the left side is  $P_1, V_1, T_1$  respectively and on right chamber is  $P_2 = \frac{27P_0}{8}, V_2$  and  $T_2$  respectively :-

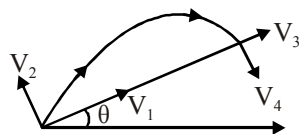


- (A) Volume of first chamber is  $\left[ 2 - \left( \frac{8}{27} \right)^{3/5} \right] V_0$
- (B) Temperature in second chamber is  $\left( \frac{27}{8} \right)^{2/5} T_0$
- (C) Work done on the gas in second chamber in terms of molar heat capacity at constant volume and  $T_0$  is  $nC_v T_0 \left[ \left( \frac{27}{8} \right)^{2/5} - 1 \right]$
- (D) Work done on the gas in first chamber in terms of molar heat capacity at constant volume and  $T_0$  is  $nC_v T_0 \left[ \left( \frac{27}{8} \right)^{2/5} - 1 \right]$

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Space for Rough Work

5. A projectile is projected on the inclined plane as shown.  $V_1$  &  $V_2$  are components of its initial velocity along the incline and perpendicular to incline and  $V_3$  &  $V_4$  are components of its final velocity along the incline and perpendicular to incline. {Here we are comparing the magnitudes only}



- (A)  $V_1 > V_3$       (B)  $V_1 = V_3$       (C)  $V_2 = V_4$       (D)  $V_2 > V_4$
6. Let  $n_1$  and  $n_2$  moles of two different ideal gases be mixed. If ratio of specific heats of the two gases are  $\gamma_1$  and  $\gamma_2$  respectively, then the ratio of specific heats  $\gamma$  of the mixture is given through the relation :

(A)  $\gamma = \frac{\frac{n_1\gamma_1}{\gamma_1-1} + \frac{n_2\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}}$

(B)  $\frac{(n_1 + n_2)}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

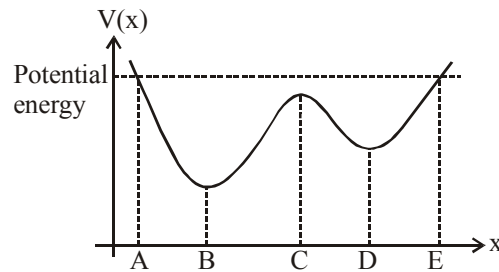
(C)  $(n_1 + n_2) \frac{\gamma}{\gamma - 1} = n_1 \frac{\gamma_1}{\gamma_1 - 1} + n_2 \frac{\gamma_2}{\gamma_2 - 1}$

(D)  $(n_1 + n_2) (\gamma - 1) = n_1(\gamma_1 - 1) + n_2(\gamma_2 - 1)$

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Space for Rough Work

7. The rate of change of angular momentum of a system of particle about the centre of mass is equal to the sum of external torques about the centre of mass when the centre of mass is :  
 (A) fixed with respect to an inertial frame (B) in linear acceleration  
 (C) in rotational motion (D) is in a translational motion
8. A particle moves in one dimension in a conservative force field. The potential energy is depicted in the graph below. If the particle starts to move from rest from the point A, then :



- (A) The speed is zero at the points A and E  
 (B) The acceleration vanishes at the points A,B,C,D,E  
 (C) The acceleration vanishes at the points B,C,D  
 (D) The speed is maximum at the point D

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**Space for Rough Work**

9. Consider a sinusoidal wave moving in the +x direction.

$$y_1(x, t) = A \sin \frac{2\pi}{T} \left( t - \frac{x}{v} \right)$$

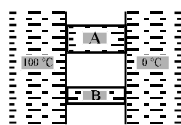
A wall located at  $x = L$  reflects this wave. Mark the **CORRECT** statement(s) :

- (A) If the wall is a fixed end the expression for reflected wave is  $-A \sin \left\{ \frac{2\pi}{T} \left( t + \frac{x - 2L}{v} \right) \right\}$
- (B) If the wall is a free end the expression for reflected waves is  $A \sin \left\{ \frac{2\pi}{T} \left( t + \frac{x - 2L}{v} \right) \right\}$
- (C) If the wall is a fixed end the resultant wave is  $2A \sin \left( \frac{2\pi}{T} \cdot \frac{L - x}{v} \right) \cdot \cos \left\{ \frac{2\pi}{T} \left( t - \frac{L}{v} \right) \right\}$
- (D) If the wall is a free end the resultant wave is  $2A \cos \left( \frac{2\pi}{T} \cdot \frac{L - x}{v} \right) \cdot \sin \left\{ \frac{2\pi}{T} \left( t - \frac{L}{v} \right) \right\}$

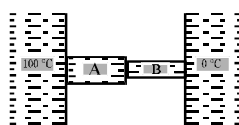
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Space for Rough Work

10. A "hot reservoir" at  $100.0^\circ\text{C}$  is connected to a "cold reservoir" at  $0^\circ\text{C}$  via two separate rods each of length  $10\text{ cm}$  which are insulated perfectly on their sides as shown in the figure(i). Rod A is made of steel, has a cross sectional area of  $4.0\text{ cm}^2$ . Rod B is made of copper. Take thermal conductivities of steel and copper respectively as  $50\text{ W m}^{-1}\text{ K}^{-1}$  and  $400\text{ W m}^{-1}\text{ K}^{-1}$ . The areas are such that in figure (i) rate of heat flow through both rods are equal. Mark the **CORRECT** statement(s) :-



Figure(i)



Figure(ii)

- (A) The cross-sectional area of the copper rod is  $0.5\text{ cm}^2$   
 (B) In figure (i) the time it takes for a total of  $1.0 \times 10^3\text{ J}$  of heat to flow from the hot to the cold reservoir via the steel rod is  $50\text{ sec}$   
 (C) In figure (ii) suppose the two rods of above specifications were welded end to end and the free ends connected to the reservoirs. The rate of flow of heat from hot to cold reservoirs in this situation is  $15\text{ J/sec}$ .  
 (D) In figure (ii) suppose the two rods of above specifications were welded end to end and the free ends connected to the reservoirs. The rate of flow of heat from hot to cold reservoirs in this situation is  $10\text{ J/sec}$ .

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Space for Rough Work



## SECTION-II

1. A calorimeter of water equivalent 1 kg contains 10 kg of ice & 10 kg of water in thermal equilibrium. The atmospheric temperature is  $15^\circ$  below freezing point due to which the calorimeter loses heat. As a result ice is formed inside the calorimeter at a rate of 10.8 gm per second. To try to compensate for this heat loss, steam at  $100^\circ\text{C}$  is supplied to the calorimeter at a rate of  $r$ . ( $L_v = 540 \text{ cal/gm}$ ,  $L_f = 80 \text{ cal/gm}$ , sp heat of water  $1 \text{ cal/gm } ^\circ\text{C}$ .) Column-I gives the value of  $r$  and column-II gives the situation just after the introduction of steam.

### Column-I

- (A)  $r = 1.6 \text{ gm/sec}$
- (B)  $r = 1.35 \text{ gm/sec}$
- (C)  $r = 1.2 \text{ gm/sec}$
- (D)  $r = 1 \text{ gm/sec}$

### Column-II

- (P) Amount of ice in calorimeter increases.
- (Q) Amount of water in calorimeter increases.
- (R) Amount of ice remains constant at 10 kg
- (S) Amount of water remains constant at 10 kg
- (T) Amount of ice in calorimeter decreases.

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Space for Rough Work

2. Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems for which phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is  $x(t)$  vs.  $p(t)$  curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. Similarly we may also plot momentum of a pendulum versus  $\theta$  (with sign convention shown in figure (b)).

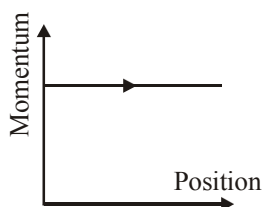


Figure (a)

Figure (b) shows phase diagram of motion of simple pendulum (momentum  $P$  versus angle  $\theta$ ). Choose potential energy level at the lowest point of the pendulum.  $E$  represents total energy of simple pendulum. Pendulum has a point mass connected with light rod.

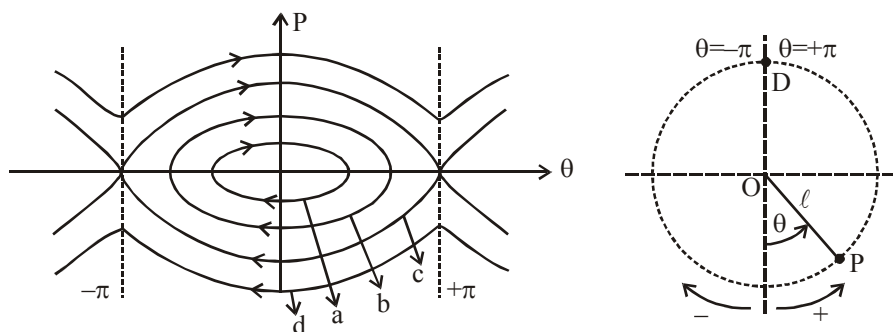


Figure (b)

#### Column-I

- (A) Phase diagram a
- (B) Phase diagram b
- (C) Phase diagram c
- (D) Phase diagram d

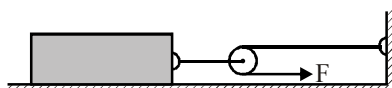
#### Column-II

- (P)  $E < 2mg\ell$
- (Q)  $E \geq 2mg\ell$
- (R) May perform periodic and oscillatory motion
- (S) May represent SHM
- (T) May represent angular velocity  $\omega$  versus  $\theta$  for a pendulum bob

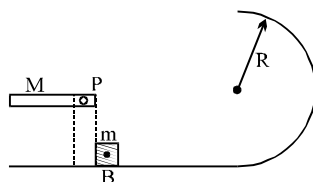
Space for Rough Work

### SECTION-III

1. The block has mass  $M$  and rests on a surface for which the coefficient of friction  $\mu$ . If a force  $F = kt$  is applied to the cable (see figure). Find the power developed by the force at  $t = t_2$  in watt. If your answer is  $N$  fill value of  $\frac{N}{320}$ . (Given :  $M = 20 \text{ kg}$ ,  $\mu = 0.4$ ,  $k = 40 \text{ N/s}$ ,  $t_2 = 3 \text{ sec.}$ )

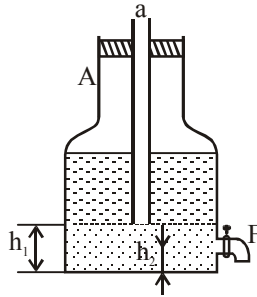


2. A rod of length  $R$  and mass  $M$  is free to rotate about a horizontal axis passing through hinge  $P$  as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the block  $B$  of mass  $m$  and stops. Find the ratio of masses  $\left(\frac{M}{m}\right)$  such that the block  $B$  completes the circle. Neglect any friction. If your answer is  $N$  fill value of  $\frac{N^2}{3}$ .



Space for Rough Work

3. A vessel A filled with water (Mariotte vessel) communicates with the atmosphere through a glass tube passing through the throat of the vessel as shown in figure. A faucet F is  $h_2 = 2\text{cm}$  from the bottom of the vessel. Find the velocity (in m/s) with which the water flows out of the faucet F when the distance between the end of the tube and the bottom of the vessel is  $h_1 = 10\text{ cm}$ . If your answer is  $N\sqrt{\frac{1}{10}}$  fill value of N.

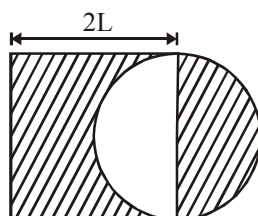


4. A sphere made of polymer is floating on the surface of water which is at a temperature of  $20^\circ\text{C}$  and 1% of the volume of the sphere is above the water. To what final temperature (in  $^\circ\text{C}$ ) must the water be heated in order to submerge the polymer sphere completely. Take coefficient of linear expansion of the polymer  $2.30 \times 10^{-5}/^\circ\text{C}$ , and coefficient of volume expansion of water  $3.19 \times 10^{-4}/^\circ\text{C}$ . If your answer is N fill value of  $\frac{N}{20}$ .

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Space for Rough Work

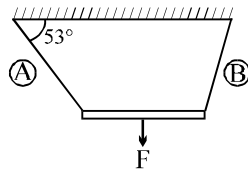
5. A pan with a set of weights is attached to a spring. The period of vertical oscillations is 0.3 s. After additional weights are placed on the pan, the period of vertical oscillations becomes 0.4 s. By how much does the spring stretch (in cm) owing to the additional weight? If your answer is  $n$  fill value of  $4n$ . ( $g = \pi^2$ )
6. Consider a uniform square plate of side  $2\ell = 6\text{m}$  made of wood. A semicircular portion is cut and attached to the right as shown. Determine the displacement of centre of mass of the redesigned plate.



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Space for Rough Work

7. Shin Chan and his mother have a tin whistle each. The pipe length of Shin chan's tin whistle is 52 cm long while the pipe length of mother's tin whistle is 50 cm long. They both play at the same time, sounding the whistles at their fundamental resonant frequencies. They note that they are not in tune with each other. The velocity of sound in air is 325 m/s. Assume the whistle is a pipe with one end closed find the beat frequency (in Hz) that is heard when both whistles are playing simultaneously. If  $n$  beats are heard in 4 sec fill value of  $\sqrt{n}$ .
8. Two light wires A and B of breaking stress  $8 \times 10^8$  Pa and  $3 \times 10^8$  Pa are used to support a light bar horizontally as shown. The area of cross-section of A & B are  $1 \text{ mm}^2$  and  $2 \text{ mm}^2$  respectively. An increasing external force directed vertically downward is applied as shown. If the angle made by wire B with horizontal is such that both wires break simultaneously. What is the value of external force (in N) at which wires break? If your answer is  $X$  fill value of  $\frac{X}{250}$ .




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**Space for Rough Work**

## PART-2 : CHEMISTRY

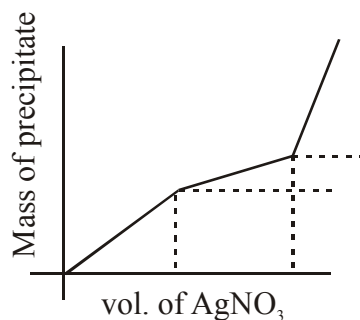
### SECTION-I

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1. Comment about the fraction of molecules moving between 400 to 500 m/sec for a gas (molecular mass = 20 gm/mol) if its temperature increases from 300 K to 400 K [ $R = 25/3$  J/mol/K]
- (A) Fraction of molecules increases      (B) Fraction of molecules decreases  
(C) Fraction of molecules remains constant      (D) Fraction of molecules may increase or decrease
2. 0.1M  $\text{AgNO}_3$  solution was slowly added to a solution containing salt  $\text{NaX(aq.)}$ ,  $\text{MgY}_2\text{(aq.)}$ ,  $\text{AlZ}_3\text{(aq.)}$ . If it is known that silver forms precipitate with X, Y, Z one by one (order may be different). Find the element which was precipitated first using following information. (Consider complete dissociation of salt).

Given : Atomic mass of X = 120

Atomic mass of Y = 80

Atomic mass of Z = 60



(A) X

(B) Y

(C) Z

(D) Incomplete information

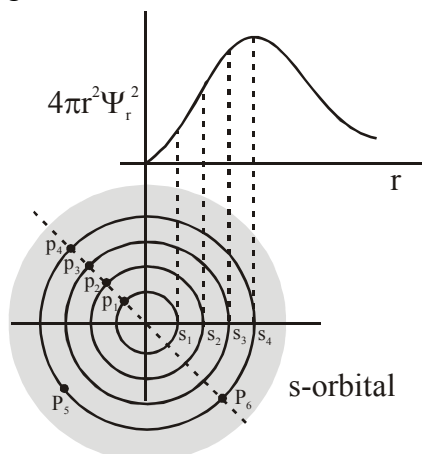
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Space for Rough Work

3. For s-orbital,  $4\pi r^2 \Psi_r^2$  vs  $r$  is plotted. Select correct statements (s)

**Note :** S = spherical surface at given distance

P = point (very small volume) at given distance



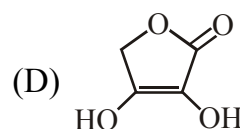
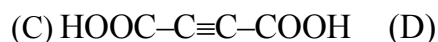
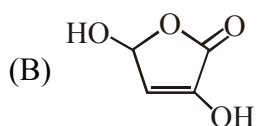
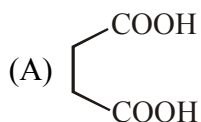
- (A) Order of probability of finding an electron at points is  $P_4 > P_3 > P_2 > P_1$   
 (B) Order of probability of finding an electron at spherical surface is  $S_4 > S_3 > S_2 > S_1$   
 (C) Order of probability of finding an electron at points is  $P_6 = P_5 = P_4$   
 (D) Order of probability of finding an electron at points is  $P_4 = P_3 = P_2 = P_1$
4. A sample containing 1 mol  $\text{KHC}_2\text{O}_4 \cdot \text{H}_2\text{C}_2\text{O}_4$  is titrated with different reagent select correct statement-
- (A) 1 mol of KOH are used  
 (B)  $3/2$  moles of  $\text{Ba}(\text{OH})_2$  are used  
 (C)  $4/5$  mol of  $\text{KMnO}_4$  are used in alkaline medium  
 (D)  $2/3$  mol of  $\text{K}_2\text{Cr}_2\text{O}_7$  are used in acidic medium
5. Which of the following pair of orbital has electron density along the axes.
- (A)  $d_{xy}, d_{yz}$                       (B)  $d_{x^2-y^2}, d_{xy}$                       (C)  $d_{xz}, d_{z^2}$                       (D)  $d_{x^2-y^2}, d_{z^2}$
6. Which of the following pair of elements are not of same group of periodic table ?
- (A) Li, Na                      (B) Be, B                      (C) N, As                      (D) O, At

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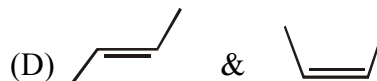
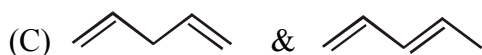
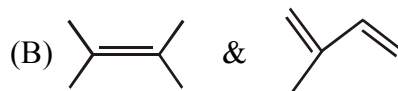
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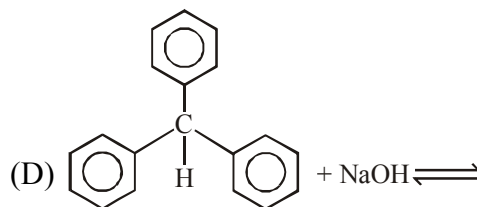
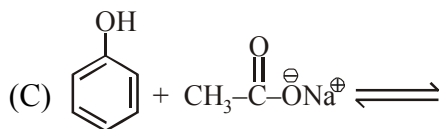
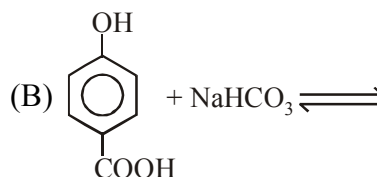
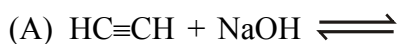
7. Among the following most acidic compound of molecular formula  $C_4H_4O_4$  is :



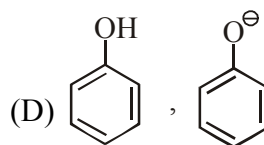
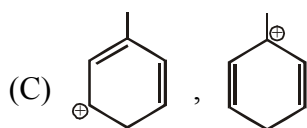
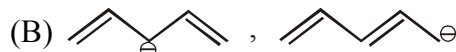
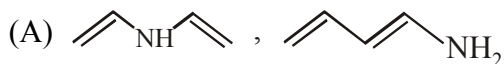
8. In which of the following pair(s), (II) has more heat of hydrogenation as well as heat of combustion than (I) ?



9. Which of the following reactions will not undergo in forward direction ?



10. In which of the following pair(s), (II) has more resonance energy than (I) ?



Space for Rough Work

## SECTION-II

1.	Column-I	Column-II
(A)	$\text{H}_2\text{O}(l) \rightarrow \text{H}_2\text{O}(g)$ Addition of $\text{H}_2\text{O}(l)$ at equilibrium	(P) Favours forward reaction
(B)	$\text{I}_3^-(aq) \rightarrow \text{I}_2(aq.) + \text{I}^-(aq.)$ Addition of $\text{H}_2\text{O}(l)$ at equilibrium	(Q) Does not shift equilibrium state
(C)	$2\text{AB}(g) \rightarrow \text{A}_2(g) + \text{B}_2(g)$ on increasing the volume at equilibrium	(R) Concentration of product decrease
(D)	$\text{A}_2(g) \rightarrow 2\text{A}(g)$ addition of catalyst at equilibrium	(S) Concentration of product remain constant
		(T) Concentration of reactant decreases
2.	Column-I	Column-II
(A)	$\text{SO}_3$	(P) Paramagnetic
(B)	$\text{SO}_2$	(Q) $d_\pi$ - $p_\pi$ bond
(C)	$\text{CO}_2$	(R) linear
(D)	$\text{NO}_2$	(S) planar
		(T) angular

Space for Rough Work

## SECTION-III

- 
1. Calculate the wavelength of a rested electron (in Å) after it absorbs a photon of wavelength 9 nm. [Given  $h = 6 \times 10^{-34}$  J-sec ,  $m = 9 \times 10^{-31}$  kg].
  2. An ideal gas occupy 2 litre volume at 300K & 1atm. Calculate the volume occupied by equal moles of real gas at same temperature and pressure.  
Given :  $b = 0.05$  litre/mol  
 $R = 0.08$  atm  
 $Z = 1.5$  at given condition
- 

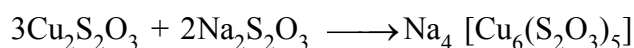
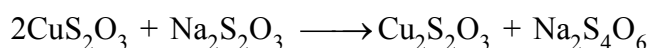
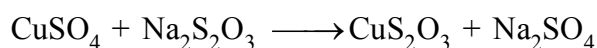
Space for Rough Work

3.  $\alpha$ -D-glucopyranose reacts with periodate ion as follows :



In a typical experiment, a 1 ml solution of  $\alpha$ -D-Glucopyranose required 80 ml 0.25 M periodate solution to reach the equivalence point. The solution is made free from formic acid and iodate ion by extraction and then treated with  $\text{H}_2\text{O}_2$ , an oxidizing agent, oxidizing all formaldehyde into formic acid and finally titrated against 0.1M NaOH solution. Titration required 40 mL of alkali to reach the equivalence point. Determine molarity of  $\alpha$ -D-glucopyranose solution.

4. Maximum moles of  $\text{Na}_4[\text{Cu}_6(\text{S}_2\text{O}_3)_5]$  which can be produced by 6 moles of  $\text{CuSO}_4$  and 10 moles of  $\text{Na}_2\text{S}_2\text{O}_3$  using following series of reaction -

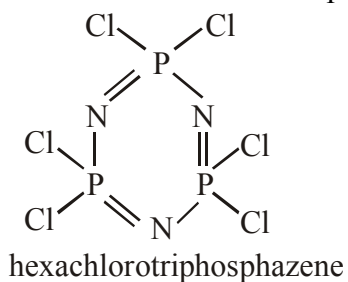


**Fill your answer to nearest integer.**

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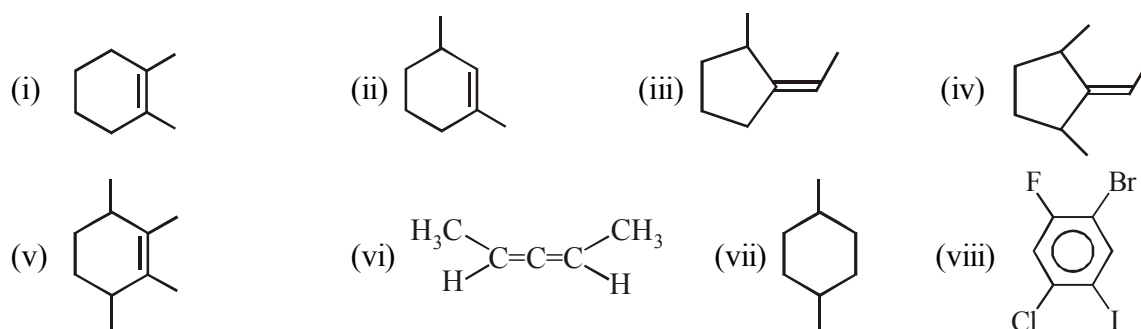
5.  $\text{XO}_n(\text{OH})_m$  is a formula of oxyacid then find the value of  $(n + m)$  if oxyacid has basicity 5 and central atom has covalency seven.
6. Find the ratio of  $\text{sp}^3$  and  $\text{sp}^2$  hybridised atoms which are present in ring in following molecule.



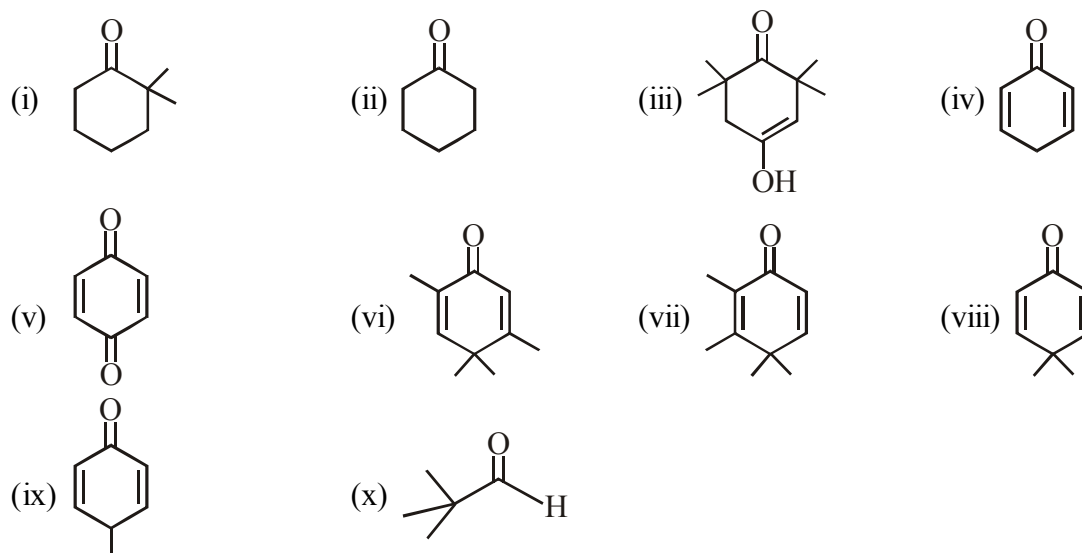
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Space for Rough Work

7. How many of the following compound(s) will show geometrical isomerism ?



8. How many of the following compounds will have tautomerism phenomenon ?



Space for Rough Work

## PART-3 : MATHEMATICS

### SECTION-I

1. The terms in the expansion of  $\left(\sqrt{x} + \frac{1}{2\sqrt[4]{x}}\right)^n$  ( $n > 1, n \in \mathbb{N}$ ), are arranged in descending powers of  $x$ . If the coefficients of first three terms form an arithmetic progression, then-
  - (A) Number of terms with integer powers of  $x$  is 3
  - (B) Total number of terms in above expansion is 10
  - (C) Coefficient of term independent of  $x$  is 256
  - (D) Coefficient of middle term in above expansion is  $\frac{35}{8}$
  
2. Consider the equation of circles
 
$$S_1 : x^2 + y^2 + 24x - 10y + a = 0$$

$$S_2 : x^2 + y^2 = 36$$
  - (A) Number of non-negative integral values of  $a$  such that  $S_1 = 0$  represents a real circle 170
  - (B) If  $S_1 = 0$  and  $S_2 = 0$  has no point in common, then number of integral values of  $a$  is 49
  - (C) If  $S_1 = 0$  and  $S_2 = 0$  intersect orthogonally, then  $a = 36$
  - (D) If  $a = 0$ , then number of common tangents to the circles  $S_1 = 0$  and  $S_2 = 0$  are 3.

Space for Rough Work

3. If  $\sin x + \sin y = \frac{96}{65}$  and  $\cos x + \cos y = \frac{72}{65}$ , then-

(A)  $\sin(x + y) = \frac{24}{25}$

(B)  $\cos\left(\frac{x - y}{2}\right) = \pm \frac{12}{13}$

(C)  $\tan\left(\frac{x + y}{2}\right) = \frac{4}{3}$

(D)  $\cos(x + y) = -\frac{7}{25}$

4. Let  $S = \sum_{k=0}^{2014} {}^{2014}C_k \cdot k$  and E denotes the quotient when S is divided by 19, then-

(A) Highest exponent of 2 in E is 2014

(B) Number of divisors of E is 4030

(C)  $(E + 1)$  is an even number

(D) Sum of coefficients in the expansion of  $(a - 2b + 3c)^{2016}$  is equal to E

5. For the equation  $\frac{40}{x-1} - \frac{160}{x-4} - \frac{200}{x-5} + \frac{320}{x-8} = 6x^2 - 27x$

(A) Number of real solutions of above equation is 3

(B) If E denotes the product of non-zero real or complex roots of the equation, then sum of divisors of E is 2904

(C) If S denotes the set of all real roots of the equation then, sum of elements of S taken two at a time is 81

(D) If  $\alpha_1, \alpha_2 \in \mathbb{R}$  be two roots of the equation such that  $\log_{\alpha_2}(2\alpha_1)$  is defined then it must be 1.

Space for Rough Work



6. Consider the trigonometric equation  $\frac{1}{\cot^6 x + 2\sqrt{2}|\cos^3 x|} + \frac{|\cos^3 x|(\operatorname{cosec} x)^6}{|\sec^3 x| + 2\sqrt{2}} + \frac{2\sqrt{2}|\sin^3 x|}{|\tan^3 x| + |\cot^3 x|} = \frac{3}{2}$ , which of the following is correct ?
- (A) Number of solutions of the equation in  $[0, 4\pi]$  is 8
- (B) Number of solutions of the equation in  $[0, 4\pi]$  is 16
- (C) If  $f : A \rightarrow B$  be a function where A is set of solutions of above equation in  $[0, 3\pi]$  and B is set of solutions of above equation in  $[0, 5\pi]$ , then number of such function(s) is/are  $10^6$
- (D) Sum of first four positive solution of equation is  $13\pi$ .
7. Given  $f(x) = \frac{1}{\sqrt[3]{x^2 + 2x + 1} + \sqrt[3]{x^2 - 1} + \sqrt[3]{x^2 - 2x + 1}}$  and  $E = f(1) + f(3) + f(5) + \dots + f(999)$ , then-
- (A) Value of E is an irrational number
- (B) Value of E is 5
- (C) In the expansion of  $\left(1 + \frac{E}{15}\right)^7$ , greatest term are  $T_2$  and  $T_3$
- (D) The value of  ${}^E C_k$  is maximum for  $k = 3$  only.
8. Consider the function  $f : (0, \infty) \rightarrow \mathbb{R}$  defined as  $f(x) = \begin{cases} e^{\lfloor \ln x \rfloor} & , \quad 0 < x \leq 3 \\ |x - 6| & , \quad x > 3 \end{cases}$ , then which of the following is correct ?
- (A) If a, b, c and d are four distinct positive number such that  $f(a) = f(b) = f(c) = f(d)$  then number of possible integral values of abcd is 7
- (B)  $f(x)$  is a surjective function
- (C) Area bounded by the line  $x = 1$ ,  $x = 3$ ,  $y = f(x)$  and x-axis is 4 sq. units
- (D) There exists only one circle touching  $f(x)$  at  $(7, 1)$  and radius  $\sqrt{2}$  which is  $x^2 + y^2 - 16x + 62 = 0$

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Space for Rough Work

9. Which of the following statements is/are correct ?
- (A) If  $f(x) = \log((x^2 + 1)^{1/2} + x) + \sin x$  and  $f(a + 2) = 5$  and  $f(b - 7) = -5$ , then possible value of  $(a + b)$  is equal to 5.
- (B) If  $f(x) = (\sin x) \left[ x^2 \right] + \frac{1}{\sqrt{1-x^2}}$  (where  $[.]$  denotes greatest integer function), then  $f(x)$  is an even function.
- (C) If  $f(x^{2015} + 1) = x^{4030} + x^{2015} + 1$ , then the sum of the coefficients of  $f(x^{2015} - 1)$  is 1.
- (D) The number of solution of  $\sin^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(2x)$  is 2.
10. Let A,B,C be angles of  $\triangle ABC$  and  $\angle D = \frac{5\pi + A}{32}$ ,  $\angle E = \frac{5\pi + B}{32}$  &  $\angle F = \frac{5\pi + C}{32}$ , then
- (where  $D, E, F \neq \frac{n\pi}{2}$  &  $n \in \mathbb{I}$ )
- (A)  $\cot D \cot E + \cot E \cot F + \cot D \cot F = 1$
- (B)  $\cot D + \cot E + \cot F = \cot D \cdot \cot E \cdot \cot F$
- (C)  $\tan D \tan E + \tan E \tan F + \tan F \tan D = 1$
- (D)  $\tan D + \tan E + \tan F = \tan D \tan E \tan F$

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Space for Rough Work

## SECTION-II

1.	Column-I	Column-II
(A)	The value of $\sum_{n=0}^{\infty} 2^{-n+(-1)^n}$ is	(P) 1
(B)	Number of solutions of equation $16 \sin^3 x = 14 + \sqrt[3]{\sin x + 7}$ in $[0, 4\pi]$ is	(Q) 2
(C)	Number of solutions of equation $\tan^{-1}(2 \sin x) = \cot^{-1}(\cos x)$ in $[0, 10\pi]$ is	(R) 3
(D)	If the sum of roots of the quadratic equation $(-a + 1)x^2 + (a^2 - a + 4)x - 2a + 3 = 0$ is minimum, then the value of $a$ ( $a > 1$ ) is	(S) 4 (T) 5

Space for Rough Work

**Column-I**

**Column-II**

(A) The value of  $\left[ \sum_{n=0}^{\infty} \tan^{-1} \left( \frac{2}{\sqrt{n+2} + \sqrt{n} + (n+2)\sqrt{n} + n(\sqrt{n+2})} \right) \right]$

(P) 1

(where  $[.]$  denotes greatest integer function) is

(B) If a,b,c are the sides of a triangle, then  $\frac{2(a^2 + b^2 + c^2)}{ab + bc + ca}$

(Q) 2

is always less than

(C) If a,b,c are distinct real numbers such that  $a^2(b+c) = b^2(a+c) = 2$ , then the value of  $c^2(a+b)$  is

(R) 3

(D) If  $P = \sin A \sin B$ ,  $Q = \sin C \cos A$ ,  $R = \sin A \cos B$ ,  $S = \cos A \cos C$ , then the value of  $5(P^2 + Q^2 + R^2 + S^2)$  is

(S) 4

(T) 5

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**Space for Rough Work**

### SECTION-III

- 
1. If the value of  $[\sin(\sin^{-1}(0.5) + \sin^{-1}(0.4)) \times \sin(\sin^{-1}(0.5) - \sin^{-1}(0.4))]$  can be expressed as  $\frac{p}{q}$  (where  $p, q \in \mathbb{N}$  are relatively prime), then  $\left(\frac{q-p}{13}\right)$  is
  
  2. For positive integer  $n$ , let  $S_n$  denotes the minimum value of the sum  $\sum_{k=1}^n \sqrt{(2k-1)^2 + a_k^2}$  where  $a_1, a_2, \dots, a_n$  are positive real numbers whose sum is 17. If there exist a unique positive integer  $n$  for which  $S_n$  is also an integer, then  $\left(\frac{n}{2}\right)$  is
- 

Space for Rough Work

3. Triangle ABC has right angle at B and contains a point P for which  $PA = 10$ ,  $PB = 6$  and  $\angle APB = \angle BPC = \angle CPA$ , then  $\left(\frac{PC}{11}\right)$  is

4. If  $\sum_{n=1}^{2015} (-1)^n \left(\frac{n^2 + n + 1}{(n)!}\right) = -a - \frac{b}{c!}$  (where  $a, b, c \in \mathbb{N}$ ), then the minimum value of  $\left(\frac{a + b + c}{576}\right)$  is

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**Space for Rough Work**

5. 6 lecturers A,B,C,D,E,F want to deliver lecture in a particular batch (one by one), such that A wants to take class before B and B wants to take class before C (not necessarily consecutive), if total number of ways are  $\frac{|2n|}{|n|}$ , then n is equal to (where  $n \in \mathbb{N}$ )

6. Let x and y be positive real numbers and  $\theta$  an angle such that  $\theta \neq \frac{n\pi}{2}$  for any integer n. Suppose

$$\frac{\sin \theta}{x} = \frac{\cos \theta}{y} \text{ and } \frac{\cos^4 \theta}{x^4} + \frac{\sin^4 \theta}{y^4} = \frac{97 \sin 2\theta}{x^3 y + x y^3}, \text{ then the value of } \frac{x}{y} + \frac{y}{x} \text{ is}$$

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Space for Rough Work

7. Let  $C_1$  and  $C_2$  be externally tangent circles with radius 2 and 3 respectively. Let  $C_1$  and  $C_2$  both touch circle  $C_3$  internally at points A and B respectively. The tangents to  $C_3$  at A and B meet at T and  $TA = 4$ , then radius of circle  $C_3$  is
8. If  $\sum_{p=1}^n \sum_{m=p}^n {}^nC_m \cdot {}^mC_p = 19$ , then the value of n is

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**Space for Rough Work**



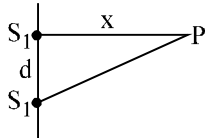
<b>PHYSICS</b>		<b>CHEMISTRY</b>		<b>MATHEMATICS</b>	
1	ABC	1	B	1	AD
2	ABCD	2	B	2	AC
3	ABCD	3	BC	3	ABCD
4	ABC	4	BD	4	AB
5	AC	5	D	5	AB
6	ABC	6	BD	6	AC
7	ABCD	7	D	7	BC
8	AC	8	D	8	AC
9	ABCD	9	ACD	9	ABCD
10	ABD	10	AD	10	BC
1	A – q,t B – q,r C – p,s D – p	1	A – q,s B – p,r,t C – q,r,t D – q,s	1	A – r B – q C – t D – t
2	A – p,r,s,t B – p,r,s,t C – q,r,t D – q,t	2	A – q,s B – q,s,t C – r,s D – p,s,t	2	A – q B – s,t C – q D – t
1	6	1	1	1	7
2	5	2	3	2	6
3	4	3	4	3	3
4	3	4	1	4	7
5	7	5	6	5	3
6	1	6	1	6	4
7	5	7	4	7	8
8	4	8	7	8	3

### SECTION-I

1. **Ans. (A,B,C)**

**Sol.** Path difference

$$\Delta x = S_2P - S_1P = \sqrt{x^2 + d^2} - x$$



(A) when  $x \approx \infty$

$\Delta x = 0$ , so phase difference is zero

(B) In some phase so constructive

(C) when path difference  $\Delta x = n\lambda$ , then constructive interference

when  $\Delta x = (2n + 1)\lambda/2$  where  $n = 0, 1, 2, \dots$  destructive

2. **Ans. (A,B,C,D)**

**Sol.** We know that  $I = \frac{2\pi^2 B}{v} s_0^2 v^2$  and  $P = I \times \text{area}$

also  $I = \frac{p_0^2}{2\rho v}$  and here  $s_0 = \Delta R$ ,  $v = \sqrt{\frac{B}{\rho}}$  and  $v = f$

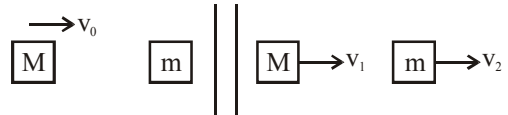
3. **Ans. (A,B,C,D)**

**Sol.** Block B & C will perform SHM about COM

$$\therefore \omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{m/2}} = \sqrt{\frac{2k}{m}}$$

For collision between M & m

$$Mv_0 = Mv_1 + mv_2 \text{ \& }$$



$$v_2 - v_1 = v_0$$

$$\therefore v_0 = v_1 + \gamma v_2$$

$$v_0 = v_2 - v_1$$

$$2v_0 = v_2 (1 + \gamma)$$

$$\therefore v_2 = \frac{2v_0}{1 + \gamma}$$

$$\therefore v_{CM} = \frac{mv_2 + m \times 0}{m + m} = \frac{v_0}{1 + \gamma}$$

$$\beta = \frac{v_{\max/\text{COM}}}{\omega} = \frac{v_0}{\omega(1 + \gamma)}$$

$$v_{\max} = \frac{2v_0}{1 + \gamma}$$

4. **Ans. (A,B,C)**

**Sol.** Since the process in chamber 2 is adiabatic

$$\therefore P_0 V_0^\gamma = P_2 V_2^\gamma$$

$$\therefore P_0 V_0^{5/3} = \frac{27}{8} P_0 V_2^{5/3}$$

$$\therefore V_2 = \left(\frac{8}{27}\right)^{3/5} V_0$$

$\therefore$  Volume of chamber

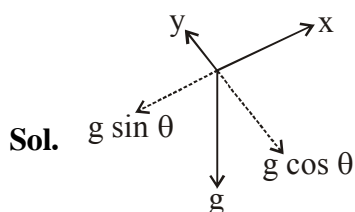
$$1 = 2V_0 - V_2 = \left[2 - \left(\frac{8}{27}\right)^{3/5}\right] V_0$$

$$P_0^{1-\gamma} T_0^\gamma = C$$

$$\therefore T_2 = \left(\frac{27}{8}\right)^{2/5} T_0$$

$$\text{Work by the gas} = \frac{P_0 V_0 - P_2 V_2}{\gamma - 1}$$

5. **Ans. (A,C)**



$$V_3^2 = V_1^2 - 2g \sin \theta x \text{ so } V_3 < V_1$$

$$V_4^2 = V_2^2 - 2g \cos \theta (\Delta y) = V_2^2 - 2g \cos \theta (0) = V_2^2$$

$$V_4 = V_2$$

6. **Ans. (A,B,C)**

**Sol.** 
$$\gamma = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}} = \frac{\frac{n_1 \gamma_1}{\gamma_1 - 1} + \frac{n_2 \gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$$

$$(n_1 + n_2) C_v = n_1 C_{v1} + n_2 C_{v2}$$

$$(n_1 + n_2) \frac{R}{\gamma - 1} = \frac{n_1 R}{\gamma_1 - 1} + \frac{n_2 R}{\gamma_2 - 1}$$

7. **Ans. (A,B,C,D)**

8. **Ans. (A,C)**

**Sol.** 
$$F = \frac{dV}{dx}$$

$$\frac{dV}{dx} = 0 \text{ at B,C,D}$$

$$\therefore F = 0 \text{ at B,C,D}$$

$$\text{so } a = 0 \text{ at B,C,D}$$

speed is maximum at B as potential energy at B is maximum

9. **Ans. (A,B,C,D)**

**Sol.** For reflection from denser medium, the reflected wave experience a phase difference of  $\pi$  & for reflection from rarer medium, there is no phase difference.

10. **Ans. (A,B,D)**

**Sol.** For same heat flow rate,

$$\left. \frac{d\theta}{dt} \right|_A = \left. \frac{d\theta}{dt} \right|_B$$

$$\Rightarrow \frac{k_A A_A (100 - 0)}{\ell_A} = \frac{k_B A_B (100 - 0)}{\ell_B}$$

In figure (ii), both rods are in series

$$\therefore R_{eq} = R_1 + R_2$$

## SECTION-II

1. **Ans. (A)-(Q, T); (B)-(Q, R); (C)-(P, S); (D)-(P)**

**Sol.** Rate of heat loss =  $80 \times 10.8 = 54 \times 16$  cal/sec.

(A)  $r = 1.6$

$\Rightarrow$  rate of heat supplies by forming steam to

water at  $0^\circ = 1.6 \times 640 > 54 \times 16$

$\therefore$  additional ice will melt

(B) Rate of heat loss

$$= 54 \times 16 = 64 \times 13.5 \text{ cal/sec.}$$

$$r = 1.35$$

= rate of heat supplied for converting steam to

water at  $0^\circ \text{C} = 1.35 \times 640 = 13.5 \times 64$ .

no additional ice will melt or water will fuse.

(C) Rate of heat loss =  $54 \times 16 = 72 \times 12$  cal/sec.

Rate of heat supplied by converting steam to ice

at  $0^\circ \text{C} = 1.20 \times 720 = 12 \times 72$  cal/sec

no additional ice will melt or water will fuse.

(D) Additional water will fuse to ice.

2. **Ans. (A)-(P, R, S, T); (B)-(P, R, S, T);**

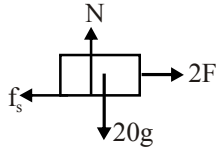
**(C)-(Q, R, T); (D)-(Q, T)**

**Sol.** From phase diagram, in case 'C' & 'D' particle are able to reach at highest point, while in 'a' & 'b' they don't.

#### SECTION-IV

1. **Ans. 6**

**Sol.**  $t_{\max} = \mu N = 0.4 \times 20g = 80$



$$F = 40t$$

Sliding starts when

$$F = f_{\max}$$

$$80t = 80$$

$$\therefore t = 1s$$

$$\int 2F dt - \int f_k dt = mv \quad 0$$

$$\Rightarrow \int_1^3 80t dt - 80 \int_1^3 dt = 20v$$

$$\therefore v = 8m/s$$

$$P = \vec{F} \cdot \vec{v} = 40t \times 2v = 40 \times 3 \times 16 = 1920$$

2. **Ans. 5**

**Sol.** Since 'B' completes the circle  $\Rightarrow v_B$  at lowest point  
 $= \sqrt{5gR}$

From work energy theorem for rod,

$$Mg \frac{\ell}{2} = \frac{1}{2} \times \frac{M\ell^2}{3} \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{\ell}}$$

For angular momentum conservation before & after collision,

$$\frac{M\ell^2}{3} \cdot \omega = mv_B \ell$$

$$\Rightarrow \left( \frac{M}{m} \right) = \frac{3v_B}{\ell\omega} = \frac{3}{\ell} \frac{\sqrt{5gR}}{\sqrt{\frac{3g}{\ell}}} = N$$

$$\therefore N^2 = \frac{9 \times 5gR}{\ell^2 \cdot \frac{3g}{\ell}} \quad \text{since } \ell = R(\text{given})$$

$$\therefore N^2 = 15$$

$$\therefore \frac{N^2}{3} = 5$$

3. **Ans. 4**

**Sol.** From Bernoulli's equation,

$$P_0 + \rho g (h_1 - h_2) = \frac{1}{2} \rho v^2 + P_0$$

[Since cross section Area of beaker  $\gg$  cross section area of faucet]

$$\therefore v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times \frac{8}{100}} = \frac{4}{\sqrt{10}}$$

4. **Ans. 3**

**Sol.** For floating

$mg = \text{Buoyant force}$

Since mass doesnot change with temperature

$$F_{B1} = F_{B2} \Rightarrow \rho_{w1} V_1 g = \rho_{w2} V_2 g$$

$$\rho_{w1} (0.99 V_0) = \left( \frac{\rho_{w1}}{1 + \gamma_w \Delta T} \right) (V_0 (1 + 3\alpha \Delta T))$$

$$\Rightarrow \left( 1 - \frac{1}{100} \right) = \left( \frac{1 + 3\alpha \Delta T}{1 + \gamma_w \Delta T} \right)$$

By Binomial approx.

$$1 - \frac{1}{100} = (1 + (3\alpha - \gamma_w) \Delta T)$$

$$\frac{1}{100} = (\gamma_w - 3\alpha) \Delta T$$

$$\Rightarrow \Delta T = \frac{1}{100(\gamma_w - 3\alpha)} = 40^\circ\text{C}$$

$$\Rightarrow T_f = 60^\circ\text{C}$$

5. Ans. 7

Sol. We have  $T_1 = 2\pi\sqrt{\frac{m}{k}}$

or

$$T_1^2 = 4\pi^2 \frac{m}{k} \quad \dots (1)$$

After more weights  $\Delta m$  are added, we have

$$T_2 = 2\pi\sqrt{\frac{m + \Delta m}{k}}, \text{ or } T_2^2 = 4\pi^2 \frac{m + \Delta m}{k} \dots (2)$$

By subtracting Eq. (1) from Eq. (2), we get

$$T_2^2 - T_1^2 = 4\pi^2 \frac{\Delta \ell}{g}, \text{ or } \Delta \ell = \frac{g}{4\pi^2} (T_2^2 - T_1^2).$$

Upon inserting the numerical data we obtain  $\Delta \ell = 1.75 \text{ cm}$ .

6. Ans. 1

Sol. For the semicircular plate of radius  $\ell$ , the centre

of mass lies at a distance of  $\frac{4\ell}{3\pi}$  from the centre.

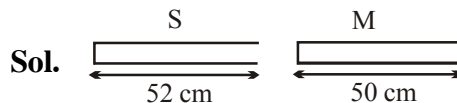
Taking  $\sigma$  to be the mass per unit area, the position of centre of mass of the remaining piece of the

square would be at a distance of  $\frac{\ell(3\pi - 4)}{3(8 - \pi)}$  from

the centre of the original square plate. Now, taking the centre of the original square to the origin, the centre of mass of the new structure can be determined. This turns out to be at a distance of

$\frac{\ell}{3}$  to the right of the origin.

7. Ans. 5



$$v = 325 \text{ m/s}$$

$$v_0 = \frac{V}{4L} \therefore v_{10} = \frac{325}{4 \times 0.52}$$

$$v_{20} = \frac{325}{4 \times 0.50}$$

$$v_{10} - v_{20} = \frac{325}{4} \left( 1 - \frac{1}{0.264} \right)$$

$$= \frac{325}{4} \left( \frac{0.004}{0.260 \times 0.264} \right)$$

$$= \frac{0.325}{0.260 \times 0.264} = 6.25 \text{ Hz}$$

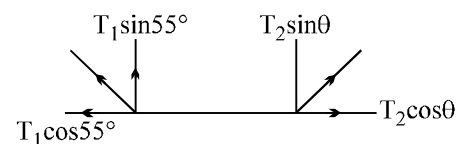
8. Ans. 4

Sol.  $T_1 = 8 \times 10^8 \times 10^{-6} = 800 \text{ N}$

$$T_2 = 3 \times 10^8 \times 2 \times 10^{-6} = 600 \text{ N}$$

$$800 \times \frac{3}{5} = 600 \times \cos \theta$$

$$\Rightarrow \cos \theta = 4/5 \Rightarrow \theta = 37^\circ$$



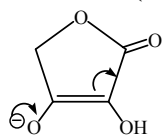
$$\Rightarrow 800 \sin 53^\circ + 600 \sin 37^\circ = F$$

$$\Rightarrow 800 \times \frac{4}{5} + 600 \times \frac{3}{5} = F$$

$$\Rightarrow F = 1000 \text{ N}$$

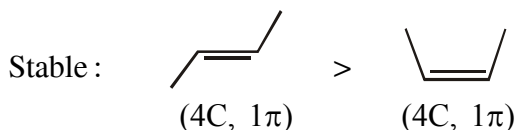
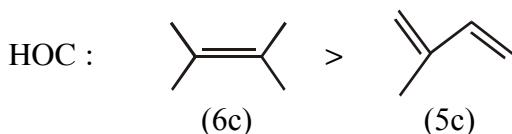
### SECTION-I

1. **Ans. (B)**
2. **Ans. (B)**
3. **Ans. (B, C)**
4. **Ans. (B, D)**
5. **Ans. (D)**
6. **Ans. (B, D)**
7. **Ans. (D)**  
DOU = 3 (So, option A & C can not be answer)



(Resonance stabilisation by extended conjugation)

8. **Ans. (D)**



In option (D) both compound has same number of carbon &  $\pi$ -bond. So, stability of compound is deciding factor for heat of hydrogenation as well as for heat of combustion.

9. **Ans. (A,C,D)**  
Acid-base reaction favours in formation of weak acid & weak base. Acidic strength order is :  $\text{CH}_3\text{COOH} > \text{Ph-OH} > \text{H}_2\text{O} > \text{HC}\equiv\text{CH}$
10. **Ans. (A,D)**  
In option (B) & (C), given pairs are not two different compounds they represent same ion whose resonance energy is fixed.

### SECTION-II

1. **Ans (A)-(Q,S); (B)-(P,R,T); (C)-(Q, R, T); (D)-(Q, S)**
2. **Ans (A) - (Q,S); (B) - (Q,S,T); (C) - (R,S); (D) - (P,S,T)**

### SECTION-IV

1. **Ans. (1)**

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{\sqrt{2m \cdot KE}} = \frac{\sqrt{h\lambda}}{\sqrt{2mC}}$$

$$= \sqrt{\frac{6 \times 10^{-3} \times 9 \times 10^{-9}}{2 \times 9 \times 10^{-31} \times 3 \times 10^8}} = 10^{-10} = 1\text{\AA}$$

2. **Ans. (3)**

$$Z = \frac{V_{\text{real}}}{V_{\text{ideal}}}$$

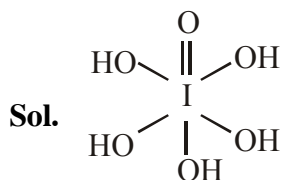
$$1.5 = \frac{V_{\text{real}}}{2}$$

$$V_{\text{real}} = 3$$

3. **Ans. (4)**

4. **Ans. (1)**

5. **Ans. (6)**



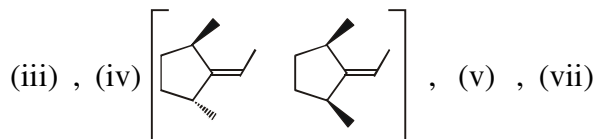
6. **Ans. (1)**

**Sol.**  $\text{sp}^3$  hybridised P atom = 3

$\text{sp}^2$  hybridised N atom = 3

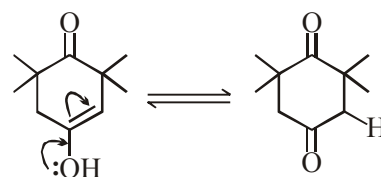
$$\text{ratio} = \frac{3}{3} = 1$$

7. **Ans. (4)**



8. **Ans. (7)**

(i) , (ii) , (iii) , (iv) , (vi) , (vii) , (ix)



## SECTION-I

1. Ans. (A,D)

$$\left(\sqrt{x} + \frac{1}{2\sqrt[4]{x}}\right)^n$$

$$T_{r+1} = {}^nC_r \cdot (\sqrt{x})^{n-r} \left(\frac{1}{2\sqrt[4]{x}}\right)^r$$

$$= \frac{{}^nC_r}{2^r} \cdot x^{\frac{n-r}{2}} \cdot \frac{1}{x^{r/4}} = \frac{{}^nC_r}{2^r} \cdot x^{\frac{2n-3r}{4}}$$

$$\frac{{}^nC_0}{2^0}, \frac{{}^nC_1}{2^1}, \frac{{}^nC_2}{2^2} \text{ are in A.P}$$

$${}^nC_1 = 1 + \frac{{}^nC_2}{4} \Rightarrow n = \frac{1+n(n-1)}{8}$$

$$n^2 - 9n + 8 = 0$$

$$\Rightarrow n = 1, 8$$

$$\therefore n = 8$$

$$\therefore T_{r+1} = \frac{{}^8C_r}{2^r} \cdot x^{\frac{16-3r}{4}}$$

$$\Rightarrow r = 0, 4, 8$$

2. Ans. (A,C)

$$(A) S_1 \equiv x^2 + y^2 + 24x - 10y + a = 0$$

$$\text{for real circle, } g^2 + f^2 - c \geq 0$$

$$144 + 25 - a \geq 0$$

$$a \leq 169$$

$$\text{Also } a \geq 0$$

$$\therefore \text{Total non-negative integral values of}$$

$$a = 170$$

$$(B) \text{ for no point in common } c_1 c_2 > r_1 + r_2$$

$$\& c_1 c_2 < |r_1 - r_2|$$

$$c_1 c_2 = 13$$

$$13 > \sqrt{169 - a} + 6$$

$$\Rightarrow 169 - a < 49$$

$$a > 120 \& a \leq 169$$

So in this condition we have 49 integral values of a

$$\text{But from } c_1 c_2 < |r_1 - r_2|,$$

we will get additional values of a.

for orthogonal cut

$$2 \cdot 12 \cdot 0 + 2(-5) \cdot 0 = -36 + a \Rightarrow a = 36$$

$$\text{If } a = 0, c_1 c_2 = 13 \& r_1 + r_2 = 19$$

$$c_1 c_2 < r_1 + r_2$$

$$\text{No. of common tangent} = 2$$

3. Ans. (A,B,C,D)

$$\sin x + \sin y = \frac{96}{65}$$

$$\Rightarrow 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) = \frac{96}{65} \quad \dots(1)$$

$$\text{Also } \cos x + \cos y = \frac{72}{65}$$

$$\Rightarrow 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) = \frac{72}{65} \quad \dots(2)$$

(1) ÷ (2), we get

$$\tan\left(\frac{x+y}{2}\right) = \frac{96}{72} = \frac{8}{6} = \frac{4}{3}$$

$$\text{Now, } \sin(x+y) = \frac{2 \cdot \frac{4}{3}}{1 + \frac{16}{9}} = \frac{24}{25}$$

$$\cos(x+y) = -\frac{7}{25}$$

Square and add (1) & (2)

$$\cos^2\left(\frac{x-y}{2}\right) \cdot 4 = \frac{8^2(144+81)}{65^2}$$

$$\cos^2\left(\frac{x-y}{2}\right) = \frac{16 \cdot 3^2 \cdot 5^2}{13^2 \cdot 5^2}$$

$$\therefore \cos\left(\frac{x-y}{2}\right) = \pm \frac{12}{13}$$

4. **Ans. (A,B)**

$$S = \sum_{k=0}^{2014} {}^{2014}C_k \cdot k = 2014 \cdot 2^{2013}$$

$$= 1007 \cdot 2^{2014} = 19 \times 53 \times 2^{2014}$$

$$\& E = 53 \times 2^{2014}$$

$\therefore$  Highest exponent of 2 = 2014

Number of divisors of E =  $2015 \times 2 = 4030$

5. **Ans. (A,B)**

$$\frac{1}{x-1} - \frac{4}{x-4} - \frac{5}{x-5} + \frac{8}{x-8} = \frac{6x^2 - 27x}{40}$$

Now,

$$\left( \frac{1}{x-1} + 1 \right) - \left( \frac{4}{x-4} + 1 \right) - \left( \frac{5}{x-5} + 1 \right)$$

$$+ \left( \frac{8}{x-8} + 1 \right) = \frac{6x^2 - 27x}{40}$$

$x = 0$

Clubbing 1<sup>st</sup> & last & 2<sup>nd</sup> & 3<sup>rd</sup>, we get

$$\frac{2x-9}{(x-1)(x-8)} - \frac{2x-9}{(x-4)(x-5)} = \frac{3}{40}(2x-9)$$

$$x = \frac{9}{2} \Rightarrow \frac{1}{(x-1)(x-8)} - \frac{1}{(x-4)(x-5)} = \frac{3}{40}$$

Solving, we get  $x = 9$

Now, we can verify the options.

6. **Ans. (A,C)**

Put  $2\sqrt{2} |\sin^3 x| = a, |\tan^3 x| = b, |\cot^3 x| = c$

$$\text{we get } \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{3}{2}$$

$$\Rightarrow \frac{a+b+c}{b+c} + \frac{b+c+a}{c+a} + \frac{c+a+b}{a+b} = \frac{9}{2}$$

$$(a+b+c) \left( \frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) = \frac{9}{2}$$

Using  $AM \geq HM$ ,

$$\frac{(a+b) + (b+c) + (c+a)}{3} \geq \frac{3}{\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a}}$$

$$(a+b+c) \left( \frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b} \right) \geq \frac{9}{2}$$

$$a = b = c$$

$$2\sqrt{2} |\sin^3 x| = |\tan^3 x| = |\cot^3 x|$$

$$\therefore x = 2n\pi \pm \frac{\pi}{4} \quad (n \in \mathbb{I})$$

$\therefore$  Number of solution in  $[0, 4\pi]$  is 8

(C)  $n(A) = 6$  ;  $n(B) = 10$

$\therefore$  Number of functions =  $10^6$

(D) Sum of 4 (+ve) solutions

$$\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4} = 4\pi$$

7. **Ans. (B,C)**

Rationalise

$$f(x) = \frac{1}{2} \left[ (x+1)^{1/3} - (x-1)^{1/3} \right]$$

$$f(1) = \frac{1}{2} \left[ 2^{1/3} - 0^{1/3} \right]$$

$$f(3) = \frac{1}{2} \left[ 4^{1/3} - 2^{1/3} \right]$$

$$f(5) = \frac{1}{2} \left[ 6^{1/3} - 4^{1/3} \right]$$

$$f(999) = \frac{1}{2} \left[ (1000)^{1/3} - (999)^{1/3} \right]$$

$$\therefore E = \frac{1}{2} \times 10 = 5$$

Now, for (c) part

$$T_{r+1} \geq T_r$$

$${}^7C_r \cdot \left( \frac{1}{3} \right)^r \geq {}^7C_{r-1} \left( \frac{1}{3} \right)^{r-1}$$

$$\frac{1}{r} \cdot \frac{1}{3} \geq \frac{1}{8-r}$$

$$8-r \geq 3r$$

$$2 \geq r$$

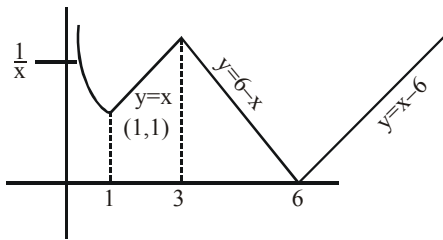
$\therefore$  Greatest terms are  $T_2$  &  $T_3$ .

8. **Ans. (A,C)**

$$f(x) = \begin{cases} \frac{1}{x} & 0 < x \leq 1 \\ x & 1 < x \leq 3 \\ 6-x & 3 < x \leq 6 \\ x-6 & x > 6 \end{cases}$$

Plotting  $f(x)$  we get





Clearly, if  $a, b, c, d$  are positive distinct numbers such that  $f(a) = f(b) = f(c) = f(d)$ , then  $y = t \in (1, 3)$  must intersect the graph of  $y = f(x)$  at four points

$$\therefore a = \frac{1}{t}, b = t, c = 6 - t, d = t + 6$$

$$\therefore abcd = 36 - t^2$$

$$t^2 \in (1, 9)$$

$$\therefore \text{Range of } abcd \text{ is } (27, 35)$$

$$\therefore \text{Number of integral values in the range } 7.$$

9. **Ans. (A, B, C, D)**

(A)  $f(x)$  is odd  $\therefore a + 2 = -b + 7 \Rightarrow a + b = 5$

(B) Clearly  $x^2 \in (0, 1) \Rightarrow [x^2] = 0$

$$\Rightarrow f(x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \text{even}$$

(C) for sum of coefficients; put  $x = 1 \therefore f(0) = 1$

10. **Ans. (B, C)**

$$\angle D + \angle E + \angle F = \frac{15\pi + \pi}{32} = \frac{\pi}{2}$$

$$\therefore \tan(D + E + F) = \frac{s_1 - s_3}{1 - s_2}$$

$$\therefore s_2 = 1$$

$$\therefore \tan D \tan E + \tan E \tan F + \tan F \tan D = 1$$

$$\& \cot D + \cot E + \cot F = \cot D \cot E \cot F$$

### SECTION - II

1. **Ans. (A)  $\rightarrow$  (R); (B)  $\rightarrow$  (Q); (C)  $\rightarrow$  (T); (D)  $\rightarrow$  (T)**

$$(A) \sum_{n=0}^{\infty} 2^{-n+(-1)^n} = 2 + 2^{-2} + 2^{-1} + 2^{-4} + \dots \infty$$

$$= \left( 2 + \frac{1}{2} + \frac{1}{2^3} + \dots \infty \right) + \left( \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots \infty \right)$$

$$= \frac{2}{1 - \frac{1}{2^2}} + \frac{\frac{1}{2^2}}{1 - \frac{1}{2^2}} = \frac{2.4}{3} + \frac{1}{3} = 3$$

(B) Put  $\sin x = t$ , we get

$$16t^3 = 14 + \sqrt[3]{t+7}$$

$$\therefore f(t) = f^{-1}(t)$$

$$\sin x = 1 \Rightarrow x = 2n\pi + \frac{\pi}{2} (n \in \mathbb{I})$$

$$\therefore 2 \text{ solutions.}$$

(C)  $\tan^{-1}(2 \sin x) = \cot^{-1}(\cos x)$

$$\tan^{-1}(2 \sin x) + \tan^{-1}(\cos x) = \frac{\pi}{2}$$

$$\therefore \sin x > 0, \cos x > 0 \& 2 \sin x \cos x = 1$$

$$\therefore x = 2n\pi + \frac{\pi}{4}$$

$$\Rightarrow 5 \text{ solutions}$$

(D) Sum of roots

$$= \frac{a^2 - a + 4}{a - 1} = a + \frac{4}{a - 1} = a - 1 + \frac{4}{a - 1} + 1 \geq 5$$

Minimum value is 5

2. **Ans. (A)  $\rightarrow$  (Q); (B)  $\rightarrow$  (S, T); (C)  $\rightarrow$  (Q); (D)  $\rightarrow$  (T)**

$$(A) \sum_{n=0}^{\infty} \tan^{-1} \left( \frac{2}{(\sqrt{n+2} + \sqrt{n})(1 + \sqrt{n+2}\sqrt{n})} \right)$$

$$= \sum_{n=0}^{\infty} \tan^{-1} \left( \frac{\sqrt{n+2} - \sqrt{n}}{1 + \sqrt{n+2}\sqrt{n}} \right)$$

$$= \sum_{n=0}^{\infty} (\tan^{-1} \sqrt{n+2} - \tan^{-1} \sqrt{n})$$

$$= \frac{3\pi}{4} \quad \therefore \left[ \frac{3\pi}{4} \right] = 2$$

(B)  $c > |a - b| \Rightarrow c^2 > a^2 + b^2 - 2ab$

Similarly,  $a^2 > b^2 + c^2 - 2bc$

$$b^2 > a^2 + c^2 - 2ac$$

$$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca)$$

$$2 \frac{(a^2 + b^2 + c^2)}{ab + bc + ca} < 4$$

(C)  $a^2(b + c) = b^2(a + c)$

$$a^2b - b^2a + a^2c - b^2c = 0$$

$$ab(a - b) + c(a - b)(a + b) = 0$$

$$a \neq b$$

$$\therefore ab + ac + bc = 0$$

Multiply both sides by  $(a - c)$ ,

$$\text{we get } (a - c)(ab + ac + bc) = 0$$

$$a^2b + a^2c - ac^2 - bc^2 = 0$$

$$a^2(b + c) = c^2(a + b) = 2$$

$$\begin{aligned}
 \text{(D)} \quad P^2 + Q^2 + R^2 + S^2 \\
 = \sin^2 A \sin^2 B + \sin^2 C \cos^2 A + \sin^2 A \cos^2 B \\
 + \cos^2 A \cos^2 C \\
 = \sin^2 A + \cos^2 A = 1
 \end{aligned}$$

#### SECTION-IV

1. **Ans. 7**

$$\begin{aligned}
 \sin(x-y) \sin(x+y) &= \sin^2 x - \sin^2 y \\
 \therefore E &= \sin^2(\sin^{-1}(0.5)) - \sin^2(\sin^{-1}(0.4)) = 0.09
 \end{aligned}$$

2. **Ans. 6**

Interpret the problem geometrically consider  $n$  right triangle joined at their vertices with bases  $a_1, a_2, a_3, \dots, a_n$  and heights  $1, 3, \dots, 2n-1$ . The sum of their hypotenuses is the value of  $S_n$ . The minimum value of  $S_n$ , then is the length of the straight line. Connecting the bottom vertex of first right triangle & the top vertex of the last right triangle, so

$$S_n \geq \sqrt{\left(\sum_{k=1}^n (2k-1)^2\right) + \left(\sum_{k=1}^n a_k\right)^2}$$

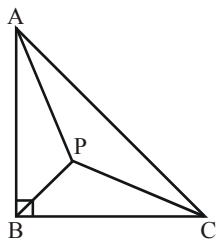
$$\therefore S_n \geq \sqrt{17^2 + n^4}$$

$$\text{Now } 17^2 + n^4 = m^2 \quad (m \in \mathbb{I})$$

$$(m - n^2)(m + n) = 289.1$$

$$\therefore n^2 = 144 \therefore n = 12$$

3. **Ans. 3**



using pythagoreas theorem

$$AB^2 + BC^2 = CA^2$$

$$10^2 + 6^2 + 10.6^2 + 6^2 + PC^2 + 6PC$$

$$= 10^2 + PC^2 + 10PC$$

$$\Rightarrow 4PC = 132 \Rightarrow PC = 33$$

4. **Ans. 7**

$$\begin{aligned}
 \sum_{n=1}^{2015} (-1)^n \left( \frac{n}{(n-1)!} + \frac{n+1}{n!} \right) \\
 = \left( -1 - \frac{2}{1!} \right) + \left( \frac{2}{1!} + \frac{3}{2!} \right) - \left( \frac{3}{2!} + \frac{4}{3!} \right) + \dots \\
 - \left( \frac{2015}{(2014)!} + \frac{2016}{(2015)!} \right)
 \end{aligned}$$

$$= -1 - \frac{2016}{(2015)!} \quad \therefore a+b+c = 4032$$

5. **Ans. 3**

Order at A,B,C is one

$${}^6C_3 \cdot 3! = \frac{6!}{3!} \quad \therefore n = 3$$

6. **Ans. 4**

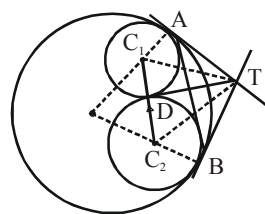
Let  $x = k \sin \theta$  &  $y = k \cos \theta$

$$\therefore \frac{\cos^4 \theta}{\sin^4 \theta} + \frac{\sin^4 \theta}{\cos^4 \theta} = \frac{194 \sin \theta \cos \theta}{\sin \theta \cos \theta (\cos^2 \theta + \sin^2 \theta)} = 194$$

$$\therefore t = \frac{x}{y} + \frac{y}{x}, \text{ then } (t^2 - 2)^2 - 2 = 194$$

$$\therefore t = 4$$

7. **Ans. 8**



Let D be the point of tangency of  $C_1$  and  $C_2$ . T will be radical center of 3 circles.

$$\therefore TD = 4$$

$$\text{Now, } \tan\left(\frac{\angle ATD}{2}\right) = \frac{1}{2} \quad \& \quad \tan\left(\frac{\angle BTD}{2}\right) = \frac{3}{4}$$

$$\therefore \text{Radius of } C_3 = TA \tan\left(\frac{\angle ATB}{2}\right) = 8$$

8. **Ans. 3**

$$\begin{aligned}
 {}^nC_m \cdot {}^mC_p &= \frac{n!}{m!(n-m)!} \times \frac{m!}{(m-p)!p!} \times \frac{(n-p)!}{(n-p)!} \\
 &= {}^nC_p \cdot {}^{n-p}C_{m-p}
 \end{aligned}$$

$$\therefore \sum_{p=1}^n \sum_{m=p}^n {}^nC_p \cdot {}^{n-p}C_{m-p} = \sum_{p=1}^n {}^nC_p \left( {}^{n-p}C_0 + {}^{n-p}C_1 + \dots + {}^{n-p}C_{n-p} \right)$$

$$= \sum_{p=1}^n {}^nC_p \cdot 2^{n-p} = \sum_{p=0}^n {}^nC_p \cdot 2^{n-p} - 2^n$$

$$= 3^n - 2^n = 19$$

$$\therefore n = 3$$