HINTS & SOLUTIONS

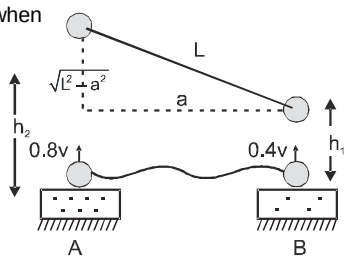
PAPER-1

PART-I (Physics)

1. Two masses m_1 and m_2 .....

Sol. (A)

String will become tight when



$h_2 - h_1 = \sqrt{L^2 - a^2}$

$h_1 = 0.4vt - \frac{1}{2}gt^2$

and $h_2 = 0.8vt - \frac{1}{2}gt^2$

$h_2 - h_1 = 0.4\ vt$

$\therefore$ $0.4vt = \sqrt{L^2 - a^2}$, where $v = \sqrt{2gH}$

2. In the system shown, the.....

Sol. (C) $4a = 4g - T$

$4a = T$

$T = 20$ N

$a = 5$

$$50 = \frac{n}{2\times 0.6}\sqrt{\frac{20}{\frac{1}{20}}} \quad \Rightarrow \quad n = 3$$

$$0.2 = \frac{1}{2}5t^2 \quad \Rightarrow \quad t = \sqrt{0.08}$$

$t_1 = 0$ $\quad t_2 = \sqrt{0.08}$ $\quad t_3 = \sqrt{0.16}$

$\Delta t_1 = \sqrt{0.08}$ $\quad \Delta t_2 = \sqrt{0.16} - \sqrt{0.08}$

$$\frac{\Delta t_1}{\Delta t_2} = \frac{1}{\sqrt{2}-1}$$

3. For two thermodynamic process.....

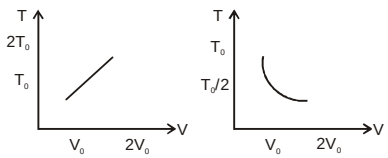
Sol. (B)

First process is constant pressure

Hence , $W_1 = nR(2T_0 - T_0) = nRT_0$

Equation of second process is $T = \frac{c}{V}$

Hence, $P = \frac{nRT}{V} = \frac{nRc}{V^2}$



$$\therefore \quad W_2 = \int_{V_0}^{2V_0} PdV = \frac{nRT_0}{2}$$

$$\frac{W_1}{W_2} = 2:1$$

4. A block of mass m is.....

Sol. (B)

Force exerted by block along Incline plane

$F = ma = m\omega^2 x = m\omega^2 A \sin \omega t$

Horizontal component of this force, so

$f = F\cos 60° \quad \Rightarrow \quad f = \frac{1}{2} m\omega^2 A \sin \omega t$

5. Ultraviolet light of wavelength .....

Sol. (C) $E_1 = \frac{hC}{\lambda_1} - 13.6eV$; $E_2 = \frac{hC}{\lambda_2} - 13.6eV$

$$E_1 - E_2 = hC\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) ; E_1 - E_2 = hC\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)$$

$$h = \frac{(E_1 - E_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

6. The ratio of total acceleration.....

Sol. (B) $a = \frac{V^2}{r}$

$$a \propto \frac{Z^2}{1/Z} \quad \text{or} \quad a \propto Z^3 \quad \Rightarrow \quad \frac{a_1}{a_2} = \left(\frac{2}{1}\right)^3 = 8$$

7. In the circuit shown in the.....

Sol. (D) By Q = CV

Effective capacitance across combination of C_3 and C_4 is 7C.

8. **S_1:** When an electron in a hydrogen.....

Sol. (A) **S_2:** $\frac{V_2}{V_1} = \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3$.

S_3: Initially when sphere is rolling up, friction force is directed upwards along the inclined. If suddenly friction becomes zero only gravitational force will act on the sphere.

9. **S_1 :** A man is standing on a.....

Sol. (B) **S_1:** On the whole system there is an external resultant force applied by fixed vertical axis, so centre of mass of the whole system will move with respect to ground.

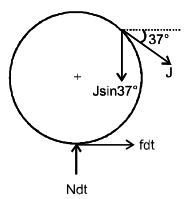
S_2: Due to induced electric field

S_3: If the angular momentum of a system is constant in magnitude then the torque (if any) acting on the system must be perpendicular to the angular momentum.

10. **S_1:** The COM of a rod is.....

Ans. (C)

11. Find the velocity of centre .....



Sol. (D) $Ndt = \frac{3}{5}mV_0$

$mV_{CM} = J\cos 37° + \mu Ndt$

$mV_{CM} = \frac{4}{5}mV_0 + \frac{1}{3}.\frac{3}{5}mV_0$

$V_{CM} = V_0$

12. Find the angular velocity .....

Sol. (A)

$I\omega = mV_0R - \mu NdtR$

$$\frac{2}{5}mR^2\omega = mV_0R - \frac{1}{3}\cdot\frac{3}{5}mV_0R \quad \Rightarrow \quad \omega = \frac{2V_0}{R}$$

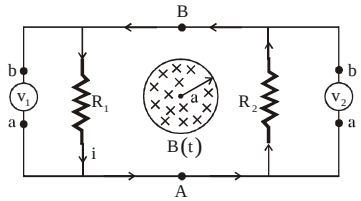
13. The emf induced .....

Ans. (B)

14. The reading of .....

Ans. (C)

Sol.(13 to 14)



$$\phi = \pi a^2 B_0 t$$

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\Rightarrow \varepsilon = \pi a^2 B_0 \quad \text{(induced emf)}$$

$$i = \frac{\pi a^2 B_0}{R_1 + R_2} \quad \text{(induced current)}$$

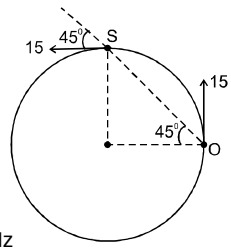
$$\Delta V_1 = -\int_a^b \vec{E}.\vec{d\ell} = -(-iR_1) = \frac{\pi a^2 B_0 R_1}{R_1 + R_2}$$

$$\Delta V_2 = -\int_a^b \vec{E}.\vec{d\ell} = -(iR_2) = -\frac{\pi a^2 B_0 R_2}{R_1 + R_2}$$

15. A train is moving with a constant.....

Ans. 15

Sol. $$f' = f\left(\frac{V - V_O}{V - V_S}\right)$$



$$= \left(\frac{V + 15\cos 45}{V + 15\cos 45}\right) f$$

$$f' = f = 1.5\text{kHz} = 3/2\text{kHz}$$

16. Figure shows the variation of.....

Ans. 14

Sol. For rectangular hyperbola

XY = constant

$U\rho$ = constant

$n\,C_V T\,\rho$ = constant

$$n\frac{5}{2}R\;T\frac{M}{V} = \text{constant}$$

$$\frac{5}{2}P\frac{VM}{V} = \text{constant} \Rightarrow P = \text{constant (isobaric process)}$$

$\Delta U_{B\to A} = 37 - 2 = 35$ J $= \frac{5}{2}nR\Delta T$

14 J = $nR\Delta T$ $\quad\therefore\quad$ W = 14 J

17. In Young's Double slit experiment.....

Ans. 15

Sol. $(\mu - 1)t = n\lambda$

n – number of fringes shifted

$(1.5 - 1)15\lambda = n\lambda$

n = 7.5

Total number of fringes shifted = 15.

18. Initial charges (with proper sign) .....

Ans. 40

Sol. Let final potential difference be V

Total charge on left hand side plates (of both upper & lower capacitor)

= 1 V + 1 V = 2 V

from the conservation of charge

2 V = 2 + 6

V = 4 volt

19. To reduce the light reflected .....

Ans. 30

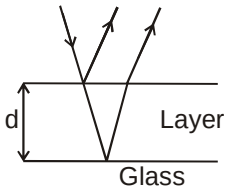
Sol. $2\mu d = (2n-1)\frac{\lambda}{2}$ (n = 1, 2, 3,.....)

$$\Rightarrow d = \frac{(2n-1)\lambda}{4\mu}$$

For light of wavelength

$\lambda_1 = 700\text{nm}$

$$d_1 = \frac{700}{4\mu}, \frac{2100}{4\mu}, \frac{3500}{4\mu}$$



for light of wavelength $\lambda_2 = 420\text{nm}$

$$d_2 = \frac{420}{4\mu}, \frac{1260}{4\mu}, \frac{2100}{4\mu} \Rightarrow d_{min} = \frac{2100}{4\mu} = \frac{2100}{4\times\frac{7}{4}}$$

$d_{min} = 300\text{nm} = 3\times10^{-7}\text{ m}$ = 30 × 10^{-8} m

20. Consider a huge charge reservoir .....

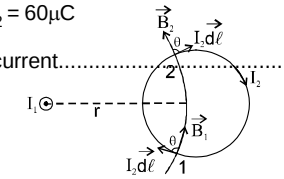
Ans. 60

Sol. After a large no. of time, the potential of each capacitor will be equal to the potential of the reservior.

$\Rightarrow Q_2 = C_2V \quad \Rightarrow \quad Q_2 = 60\mu C$

21. A long straight wire is carrying current.....

Ans. 00



Sol. The force on current elements 1 and 2 is equal in magnitude and opposite in direction

$$\Rightarrow F_{net} = 0$$

PART-II (Chemistry)

22. In which case van't Hoff factor is .....

Ans. (C)

Sol.

	Solute	y	x	i = [1 + (y – 1) x]
(A)	KCl	2	0.5	1.5
(B)	K_2SO_4	3	0.4	1.8
(C)	$FeCl_3$	4	0.3	1.9
(D)	$SnCl_4$	5	0.2	1.8

23. Following is the graphical presentation of .....

Ans. (D)

Sol. From the graph

$mCH_4 = 4g \Rightarrow \frac{4}{16}$ mol $\Rightarrow \frac{4}{16}$ × 22.4 Ltr. $\Rightarrow$ 5.6 Ltr.

$mH_2 = 1g \Rightarrow \frac{1}{2}$ mol $\Rightarrow \frac{1}{2}$ × 22.4 Ltr. $\Rightarrow$ 11.2 Ltr.

24. Which of the following statements is true .....

Ans. (C)

Sol. (A) $\overset{-}{N}=\overset{+}{N}=\overset{-}{N}$ Four membered ring is not stable.

M

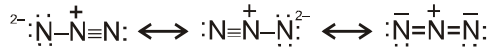
(B) Both N–N bond lengths are identical and that is 1.15 Å

(C) N_3^- and CO_2 both have same number of electrons i.e. 22 ; so isoelectronic.

$\overset{-}{N}=\underset{sp}{\overset{+}{N}}=\overset{-}{N}$ (linear) $\qquad$ $O=\underset{sp}{C}=O$ (linear)

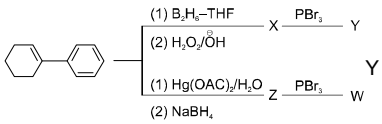
So both are also isostructural

(D) There are two σ and two π bonds.



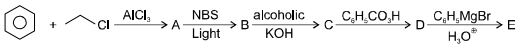
25. Which of the following is not correctly matched .....?
Ans. **(A)**
Sol. $Na_2[Cr\ (edta)]$ is correct representation.

26. In which of the following option, second compound .....
Ans. **(C)**

27. 

Y & W are .....

Ans. **(D)**

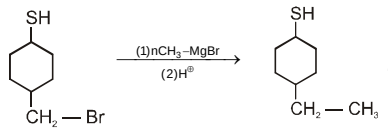
28. 

Identify E .....
Ans. **(B)**

29. S_1 **:** All adiabatic processes are isoentropic .....
S_2 **:** When $(\Delta G_{system})_{T,\,P} < 0$; the reaction .....
Ans. **(B)**
Sol. Only reversible adiabatic are isoentropic.
Endothermic reaction may be spontaneous if ΔS is positive.

30. S_1 **:** Addition of inert gas to the equilibrium .....
S_2 **:** Equilibrium $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is .....
S_3 **:** Formation of diamond is favourable at very .....
Ans. **(B)**

31. S_1 **:** Secondary-butyl magnesium bromide .....

S_2 **:** 

.....

Ans. **(D)**

32. The bottles (A), (B) and (C) contain the solutions .....
Ans. **(A)**
Sol. (A) Weak acid and strong base mixture is froming buffer of pH = 4.7.
(B) Strong acid and strong base mixture can from neutral salt.
(C) Strong acid and weak acid mixture will from acidic solution.
So 'A', 'B' and 'C' must be weak acid, strong base and strong acid respectively.

33. What will be degree of ionisation when .....
Ans. **(C)**

Sol. $pH = PK_a + \log \dfrac{[salt]}{[acid]}$

$4.7 = PK_a + \log \dfrac{50 \times 0.1}{100 \times 0.1}$

$PK_a = 5 \qquad k_a = 10^{-5}$

$HA + BOH \rightleftharpoons BA + H_2O$

150 ´ 0.1 50 ´ 0.1

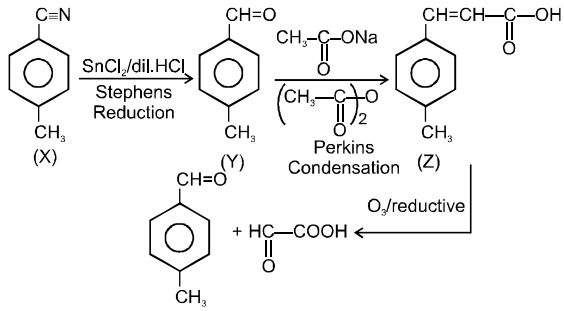
$\dfrac{100 \times 0.1}{\text{weak acid}}$

Bottle (A) = Weak acid Bottle
(B) = Strong base
Bottle (C) = Strong acid

34. The reductive ozonolysis products of (Z) are .....
Ans. **(D)**

35. What is not true about (Y).....
Ans. **(B)**

Sol. (34 to 35)



36. The percentage of copper in a copper(II) .....
Ans. **51**
Sol. From given reactions
mmoles of hypo = mmoles of iodine × 2
= mmoles of Cu^{2+} ions
= 24.5 × 0.1 mmoles
so mass of copper = $24.5 \times 0.1 \times 10^{-3} \times 63.5$ gm

so % of copper = $\dfrac{24.5 \times 0.1 \times 10^{-3} \times 63.5}{0.305} \times 100\% \simeq 51.0\ \%$

37. If for the equilibrium :.....
Ans. **21**

Sol. $NH_2COONH_4(s) \rightleftharpoons N_2(g) + 3H_2(g) + CO(g) + \dfrac{1}{2}O_2(g)$

$\qquad p \quad 3p \quad p \quad \dfrac{p}{2}$

Total pressure at equilibrium = 22 atm

$p + 3p + p + \dfrac{p}{2} = 22 \quad \Rightarrow \dfrac{11}{2} = 22 \quad \Rightarrow p = 4$ atm

$Kp = p(3p)^3\, p\sqrt{\dfrac{p}{2}} = 27 \times p^{11/2} \qquad 2^{-1/2}$

$\Rightarrow 27 \times 2^{\lambda/2} = 27\,(4)^{11/2}\, 2^{-1/2} \qquad \Rightarrow 2^{\lambda/2} = 2^{21/2}$

$\lambda = 21$

38. 30 litre gas at 400 K and .....
Ans. **10** litre

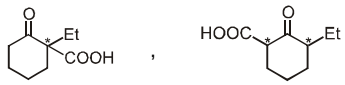
Sol. $Z_1 = \dfrac{P_1V_1}{RT_1}$ and $Z_2 = \dfrac{P_2V_2}{RT_2}$

$\dfrac{Z_1}{Z_2} = \dfrac{P_1}{P_2} \times \dfrac{T_2}{T_1} \times \dfrac{V_1}{V_2} \qquad \Rightarrow V_2 = 10$ litre

39. For a given reaction, energy of activation for forward .....
Ans. **02**
Sol. $\Delta H = E_f - E_b \qquad -40 = 80 - E_b \qquad E_b = 120$ kJ/mole,
catalyst lower the E_f To 20 kJ/ mole for forward Rxn then $E_f' = 20$ kJ/mol
we know catalyst decreases the Activation energy equal amount in both direction
$E_b' = (120 - 60) = 60$ kj/mol

$\dfrac{E_b}{E'_b} = \dfrac{120}{60} = 2.0$

40. Total number of β-Keto monocarboxylic acids .....
Ans. **06**

Sol. 

41. 'M' is smallest optically active branched alkane.....

Ans. 13

Sol. $CH_2=CH-\underset{(\pm)}{\overset{CH_3}{\overset{|}{C}H}}-CH_2-CH_2-CH_3$ $\quad$ $CH_3-CH=\underset{cis/trans}{\overset{CH_3}{\overset{|}{C}}}-CH_2-CH_2-CH_3$

$CH_3-CH_2-\underset{cis/trans}{\overset{CH_3}{\overset{|}{C}}}=CH-CH_2-CH_3$ $\quad$ $CH_2-CH_2-\underset{(1)}{\overset{CH_2}{\overset{||}{C}}}-CH_2-CH_2-CH_3$

$CH_3-CH_2-\underset{Four}{\overset{CH_3}{\overset{|}{\underset{*}{C}H}}}-CH=CH-CH_3$ $\quad$ $CH_3-CH_2-\underset{(\pm)}{\overset{CH_3}{\overset{|}{\underset{*}{C}H}}}-CH_2-CH=CH_2$

42. $CH_3-CH_2-\overset{CH_3}{\underset{CH_3}{C}}-\overset{}{\underset{OH}{CH}}-CD_3 \xrightarrow[\Delta]{H_3PO_4}$ Total no.....

Ans. 06

Sol. X can be $CH_3-CH_2-\overset{CH_3}{\underset{CH_3}{C}}-CH=CD_2$

$CH_3-CH_2-\underset{CH_3}{C}=\underset{CH_3}{C}-CD_3$ (cis/trans)

$CH_3-CH=\underset{CH_3}{C}-\overset{*}{\underset{CH_3}{C}H}-CD_3$ cis (d,ℓ) & trans (d,ℓ)

$CH_3-CH_2-\underset{CH_2}{C}-\overset{*}{\underset{CH_3}{C}H}-CD_3$ (d + ℓ)

Total 9 alkenes are formed and after fractional distilation 6 are separated.

PART-III (Mathematics)

43. Consider the sequence .....

Sol. (A) Since $45^2 = 2025$ and $46^2 = 2116$, there are precisely 45 perfect squares ≤ 2056 which are left out from the sequence of positive integers. Since 2056 – 45 = 2011, we conclude that the 2011th element is 2056.

44. Suppose that x and y are real numbers .....

Sol. (B) Note that

$$\frac{5}{3} = 4^x - 4^y = (2^x - 2^y)(2^x + 2^y) = 1.(2^x + 2^y).$$

Therefore, $2^x = \dfrac{(2^x+2^y)+(2^x-2^y)}{2} = \dfrac{\frac{5}{3}+1}{2} = \dfrac{4}{3}$

and $2^y = \dfrac{(2^x+2^y)-(2^x-2^y)}{2} = \dfrac{\frac{5}{3}-1}{2} = \dfrac{1}{3}$,

which implies $2^{x-y} = \dfrac{2^x}{2^y} = \dfrac{4/3}{1/3} = 4$

and thus $x - y = 2$.

45. An ordinary die is rolled .....

Sol. (D) Among the number 1, ..., 6 on the faces of the die, there are three odd numbers, three even numbers, three prime numbers (2, 3, and 5), five factors of 12 (all except 5), and four factors of 18 (all except 4 and 5). Therefore the number rolled is most likely to be a factor of 12 (with probability $\frac{5}{6}$).

46. The largest prime factor .....

Sol. (D) $7^{99} + 7^{100} + 7^{101} = 7^{99}(1 + 7 + 7^2) = 7^{99} \times 57 = 7^{99} \times 3 \times 19$. The largest prime factor is 19.

47. The solution of the equation .....

Sol. (B) $2x + y = t \Rightarrow \dfrac{dy}{dx} + 2 = \dfrac{dt}{dx}$

$\dfrac{dt}{dx} + xt = x^3t^3 \Rightarrow \dfrac{1}{t^3}\dfrac{dt}{dx} + \dfrac{1}{t^2}x = x^3$

$\dfrac{1}{t^2} = u \Rightarrow \dfrac{-2}{t^3}\dfrac{dt}{dx} = \dfrac{du}{dx}$

$\dfrac{du}{dx} + (-2x)u = -2x^3$

I.F. $= e^{-\int 2x\,dx} = e^{-x^2} \Rightarrow u.\,e^{-x^2} = \int e^{-x^2}(-2x^3)\,dx$

$\dfrac{e^{-x^2}}{(2x+y)^2} = -2\int e^{-x^2}.x^3\,dx$

$\dfrac{e^{-x^2}}{(2x+y)^2} = \int e^{-x^2}.x^2(-2x)\,dx \Rightarrow -x^2 = v$

$-2xdx = dv \Rightarrow \dfrac{e^{-x^2}}{(2x+y)^2} = -\int e^v v\,dv$

$\dfrac{e^{-x^2}}{(2x+y)^2} + v.e^v - e^v = C \Rightarrow \dfrac{e^{-x^2}}{(2x+y)^2} - x^2e^{-x^2} - e^{-x^2} = C$

$\dfrac{1}{(2x+y)^2} = (x^2+1) + Ce^{x^2}$

48. If f : [–1, 5] → [–10, 2] .....

Sol. (B) Range of g(x) = [–1, 5]

$(g(x))^2 + 2g(x) + 3 = (g(x) + 1)^2 + 2$

Range of $(g(x))^2 + 2g(x) + 3 = [2, 38]$

Range of $\log_{\frac{1}{2}}\left((g(x))^2 + 2g(x) + 5\right)$ is

$\left[\log_{\frac{1}{2}} 38, \log_{\frac{1}{2}} 2\right] = \left[\log_{\frac{1}{2}} 38, -1\right]$

Ans. (B)

49. If $\sin^{-1}\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{4} - .....\right)$ .....

Sol. (B) $\sin^{-1}\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{4}.....\right) + \cos^{-1}$

$\left(x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{4} + ...\right) = \dfrac{\pi}{2}$

$\sin^{-1}\left(\dfrac{x}{1+(x/2)}\right) + \cos^{-1}\left(\dfrac{x^2}{1+(x^2/2)}\right) = \dfrac{\pi}{2}$

$\dfrac{2x}{2+x} = \dfrac{2x^2}{2+x^2}$

$2x + x^3 = 2x^2 + x^3$

$x = 0, 1$ $\quad$ But $\therefore$ $|x| > 0$

so x = 1 is the only answer.

50. S_1 **:** It is possible that for a .....

Sol. (D)

51. S_1 **:** A straight line can .....

Sol. (B) S_1 **:** A straight line always have two set of direction cosines.

S_2 : f(x) is even and increasing in [0, 1]

so range of f(x) is $\left[0, \frac{\pi}{2}\sin 1\right]$

S_3 :

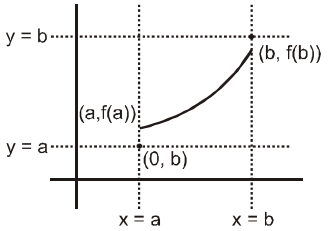
52. S_1 : Equation of director circle .....

Sol. (B) S_1 :

S_2 : Let g(x) = f(x) – x

g(a) = f(a) – a ≥ 0

g(b) = f(b) – b ≤ 0



⇒ g(a) g(b) ≤ 0 (By IVP theorem)

⇒ g(c) = 0 for some c ∈ [a, b]

⇒ f(c) – c = 0 ⇒ f(c) = c

S_3 : The graph of f(x) = sin x is concave downwards for x ∈ (0, π)

$$\Rightarrow \quad \frac{\sin A + \sin B + \sin C}{3} \le \sin\left(\frac{A+B+C}{3}\right)$$

$$\Rightarrow \quad \sin A + \sin B + \sin C \le \frac{3\sqrt{3}}{2}$$

53. The number of ways in which '4' pairs .....

Sol. (B) Since n = 4

$$\therefore \quad \text{Number of matched arrangements} = \frac{8!}{4!\ 5!} = \frac{{}^8C_4}{5}$$

54. If a stamp vendor sells tickets .....

Sol. (A) n = 3

$$\therefore \quad \text{Number of matched arrangements} = \frac{6!}{3!\,4!}$$

$$\text{Total number of arrangements} = \frac{6!}{3!\,3!}$$

$$\therefore \quad \text{probability} = \frac{1}{4}$$

55. If $2m\ell + \ell^2 - 3m^2 + 1 = 0$ .....

Sol. (D) Substituting $\ell = \frac{1-my}{x}$ in the given condition, we get

$(y^2 - 3x^2 - 2xy)\, m^2 + 2(x - y)\, m + x^2 + 1 = 0$

D = 0 ⇒ $3x^2 + 2xy - y^2 + 4 = 0$ which is a hyperbola.

56. The locus of the points from which .....

Sol. (C) Equation of the pair of the tangents to the curve $3x^2 + 2xy - y^2 + 4 = 0$ is

$(3x_1x + xy_1 + x_1y - y_1y + 4)^2 = (3x_1^2 + 2x_1y_1 - y_1^2 + 4)$
$(3x^2 + 2xy - y^2 + 4)$

Since the two tangents are perpendicular to each other,

$\therefore \quad x_1^2 + y_1^2 = 2 \quad \therefore$ the locus is $x^2 + y^2 = 2$

57. Consider a family of circles passing .....

Sol. (25) Family of circles is (x – 3) (x – 6) + (y – 7) (y – 5) + λ (2x + 3y – 27) = 0

i.e. $x^2 + y^2 + x(2\lambda - 9) + y(3\lambda - 12) + (53 - 27\lambda) = 0$.

Common chord of family of circles and the given circle is

$(-5x - 6y + 56) + \lambda(2x + 3y - 27) = 0$

which represents family of line passing through point of intersection of the lines

$-5x - 6y + 56 = 0$ and $2x + 3y - 27 = 0$

The point of intersection is $\left(2, \frac{23}{3}\right)$

a = 2

$b = \frac{23}{3}$

a + 3b = 2 + 23 = 25

58. If eccentricity of ellipse .....

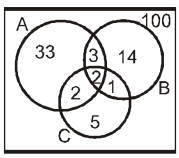
Sol. (4) case I a > 0 , then $e = \sqrt{\frac{4a}{a^2+4a}} < \frac{1}{\sqrt{2}} \Rightarrow a > 4$

case II a < 0, then $e = \sqrt{\frac{-4a}{a^2}} = \sqrt{\frac{-4}{a}} < \frac{1}{\sqrt{2}} \Rightarrow a < -8$

$\therefore \quad a \in (-\infty, -8) \cup (4, \infty) \qquad \therefore \quad |\lambda + \mu| = 4$

59. In a town of 10,000 families .....

Sol. (14)



$$\text{Probability of families to buy newspaper B only} = \frac{14}{100}$$

60. If $C_n C_{n-2} + C_{n-1}C_{n-3} + C_{n-2}C_{n-4} + \ldots\ldots\ldots\ldots$

Sol. (2) $C_0 C_2 + C_1C_3 + C_2C_4 + \ldots\ldots\ldots + C_{n-2}C_n = {}^{2n}C_{n+2}$

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2 x^2 + {}^nC_3x^3 + \ldots\ldots + {}^nC_nx^n$ .....(1)

$(x + 1)^n = {}^nC_0 x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \ldots\ldots + {}^nC_n$ .....(2)

multiplying equation (1) & (2)

$(1 + x)^{2n} = ({}^nC_0 + {}^nC_1x + {}^nC_2 x^2 + \ldots + {}^nC_nx^n)\,({}^nC_0 x^n + {}^nC_1x^{n-1} + {}^nC_2x^{n-2} + \ldots\ldots + {}^nC_n)$

Now required expression is coefficient of x^{n-2} in $(1 + x)^{2n}$

$= {}^{2n}C_{n-2} = {}^{2n}C_{n+2}$

61. If $f(x) = \frac{a\sin x - bx + cx^2 + x^3}{2x^2\ell n(1+x) - 2x^3 + x^4}$ .....

Sol. (15) $$\lim_{x\to 0} \frac{a\left(x - \frac{x^3}{6} + \frac{x^5}{120} - \ldots\ldots\right) - bx + cx^2 + x^3}{2x^2\left(x - \frac{x^2}{2} + \frac{x^3}{3}\ldots\ldots\right) - 2x^3 + x^4}$$

$$= \lim_{x\to 0} \frac{(a-b)x + cx^2 + \left(1 - \frac{a}{6}\right)x^3 + \frac{ax^5}{120} + \ldots}{\frac{2x^5}{3} - \frac{x^6}{2} + \ldots\ldots}$$

for this limit to be exist, a – b = 0, c = 0, & $1 - \frac{a}{6} = 0$

⇒ a = b = 6 & c = 0.

then $f(0) = \frac{6}{120}\cdot\frac{3}{2} = \frac{3}{40} \Rightarrow 200\ f(0) = 15$

62. If $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$ .....

Sol. (37) Let $x = u^6$, $dx = 6u^5\,du$

$$\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = \int \frac{6u^5 du}{u^3 + u^2} = 6\int \frac{u^3}{u+1}\,du$$

$$= 6\int\left(u^2 - u + 1 - \frac{1}{u+1}\right)du$$

$= 2u^3 - 3u^2 + 6u - 6\,\ell n\,(u + 1) + C$

$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\,\ell n(\sqrt[6]{x} + 1) + C$

$\therefore \quad a = 2, b = -3, c = 6, d = -6 \quad \therefore \quad 20a + b + c + d = 37$

63. If equation of the plane through .....

Sol. (14) Let equation of a plane containing the line be $\ell(x-1) + m(y+2) + nz = 0$

then $2\ell - 3m + 5n = 0$ and $\ell - m + n = 0$

$$\therefore \quad \frac{\ell}{2} = \frac{m}{3} = \frac{n}{1}$$

$\therefore$ the plane is $2(x-1) + 3(y+2) + z = 0$

i.e. $2x + 3y + z + 4 = 0 \quad \therefore \quad a = 2, b = -3, c = 1$

$\therefore \quad a^2 + b^2 + c = 14$

PAPER-2

PART-I (Physics)

1. A particle is executing simple.....

Sol. (D) $\frac{1}{2}KA^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$ and $\frac{1}{2}Kx^2 + \frac{1}{2}m(2V)^2 = \frac{1}{2}KA'^2$

$$\therefore \quad A' = \sqrt{4A^2 - 3X^2}$$

2. Two particles are moving towards.....

Sol. (C) As $\vec{V}_{cm} = 0 \quad \therefore \quad \vec{V}_1 = -\vec{u}_1$ and $\vec{V}_2 = -\vec{u}_2$

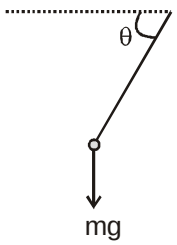
3. A particle of mass m oscillates.....

Sol. (C) $mg\,R\sin\theta = \frac{1}{2}mv^2$

$$2\,mg\sin\theta = \frac{mv^2}{R}$$

$mg\,R\sin\theta = K$

$$mg\sin\theta = \frac{K}{R}$$



$$N = \frac{mv^2}{R} + mg\sin\theta$$

$N = 2\,mg\sin\theta + mg\sin\theta$

$N = 3\,mg\sin\theta$

$$N = \frac{3K}{R}$$

4. Which of the following graph is .....

Ans. (A)

5. The work done by gas .....

Sol. (C) $W = \text{Area} = 9P_0V_0$.

6. The following figure shows.....

Sol. (C) Let the focal length of each piece be f

Then $\frac{1}{f_1} = \frac{1}{f} + \frac{1}{f} \Rightarrow \frac{1}{f_2} = \frac{1}{f} + \frac{1}{f} \Rightarrow f_1 = f_2$

For the third arrangerment the liquid forms a concave lens which has a diverging effect. So $f_3 > f_1 = f_2$

7. An equilateral triangular loop .....

Sol. (C) If ℓ is the side of the triangle, the distance of circumcenter from each of the sides of the triangle is $r = \frac{\ell\sqrt{3}}{6}$. The magnetic induction due to each of the sides of the triangle carrying a current i is $\frac{\mu_0}{4\pi}\frac{i}{r}(\sin 60° + \sin 60°) = \frac{\mu_0}{4\pi}\frac{6i}{\ell}$. Since the direction of magnetic field in each case is the same, three times this would be the total magnetic induction.

8. Two radioactive materials .....

Sol. (B) Using the law of radioactive decay, one can write

$$\frac{N_A(t)}{N_B(t)} = \frac{N_0\exp(-5\lambda t)}{N_0\exp(-\lambda t)} = \frac{1}{e}.$$

Solving this one gets the results.

9. In a situation, a board is moving.....

Sol. (A, C, D)

Velocity of A w.r.t plank = V

Velocity of B w.r.t plank = 3V

$$T = \frac{L}{V+3V} = \frac{L}{4V}$$

Their relative displacements w.r.t plank are $\frac{3L}{4}$ & $\frac{L}{4}$

10. A particle of mass m is .....

Sol. (B,D)

$U(x) = U_0(1 - \cos a\,x)$

$$\frac{dU}{dx} = U_0\,a\sin a\,x$$

$F = -\frac{dU}{dx}$; $F = -U_0\,a\sin a\,x$

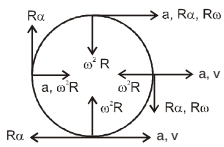
For small x $\quad F = -U_0a^2x$

acceleration $= -\frac{U_0a^2}{m}x$; $T = 2\pi\sqrt{\frac{m}{U_0a^2}}$

At x = 0, F = 0 hence mean position and speed of particle is maximum.

11. A circular disc of radius R .....

Sol. (A, B, C, D)



12. A wire shaped as a semicircle .....

Sol. (B,C,D) Let at time t the angle between magnetic field and area vector(semicircle) be θ, then $\theta = wt$

$$\phi = \vec{B}\cdot\vec{S} = \frac{\pi a^2 B}{2}\cos\omega t.$$

$$\varepsilon = -\frac{d\phi}{dt} = \frac{\pi Ba^2\omega}{2}\sin\omega t \Rightarrow \varepsilon_0 = \frac{\pi Ba^2}{2\sqrt{LC}} \text{ peak emf emf}$$

Since the circuit is in resoanance,

$$|z| = R \quad \Rightarrow \quad i_0 = \frac{\pi Ba^2}{2R\sqrt{LC}} \text{ peak current}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} \quad \Rightarrow \quad i_{rms} = \frac{\pi Ba^2}{2R\sqrt{2LC}}$$

$$V_C = \frac{1}{2}CV_0^2 \rightarrow \text{max. energy}, V_0 \rightarrow \text{peak voltage}$$

$$V_0 = i_0X_c = \frac{i_0}{C\omega} = \frac{i_0\sqrt{LC}}{C}$$

$$V_C = \frac{1}{2}C \times \frac{\pi^2B^2a^4}{4R^2C^2} = \frac{\pi^2B^2a^4}{8R^2C}$$

$$P_{Ext.} = P_{Dissipated} = \varepsilon_0 i_0 = \frac{\pi B a^2}{2\sqrt{LC}} \times \frac{\pi B a^2}{2R\sqrt{LC}}$$

$$P_{Ext.} = \frac{\pi^2 B^2 a^4}{4LCR}$$

13. Two refracting media are separated.....

Sol. (A,C) Use

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

14. Two isotropic blocks A and B.....

Sol. (A,B) Because in both the situation liquid displaced is equal to the weight of blocks

15. Choose the correct statements.....

Sol. (C,D) (A) r + e = 1 these means if r is large (good reflector) then e will be small (bad emitter)
(D) efficiency is equal to 1 - t_1/t_2 (t_1 cannot be zero so, efficiency can not be 100%)

16. A particle of mass *m* and charge.....

Sol. (B,C) Work done by tension = 0

Work done by Electric field $= qE \times \ell\left(1 - \cos 60^0\right)$

(B) $\frac{qE\ell}{2} = \frac{1}{2}mV^2 \Rightarrow V = \sqrt{\frac{qE\ell}{m}}$

(C) $T - qE = \frac{mV^2}{\ell} \Rightarrow T = 2qE$

17. **STATEMENT-1 :** The current density $\vec{J}$.....

Sol. (C) From relation $\vec{J} = \sigma\vec{E}$, the current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point. In space having non-uniform electric field, charges released from rest may not move along ELOF. Hence statement 1 is true while statement 2 is false.

18. **STATEMENT-1 :** Two particles undergo .....

Sol. (D) In statement-1, nothing is said about acceleration of both particles. Hence angle between velocity and acceleration of centre of mass may not be zero. Consequently centre of mass may not move along a straight line. Hence statement-1 is false.

19. **STATEMENT–1 :** Work done by a force.....

Sol. (C) Work done depends on displacement of point of application of force and not on displacement of centre of mass. Hence statement-2 is false.

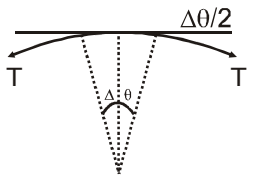
20. An object is kept infront of.....

Ans. **7**

21. A ring of radius r made .....

Ans. **2**

Sol. $2T \sin\frac{\Delta\theta}{2} = d\, m\omega^2 r$



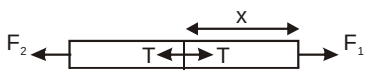
$2T\left(\frac{\Delta\theta}{2}\right) = \rho \times A \times r\Delta\theta \times \omega^2 \times r$

$\sigma = \frac{T}{A} = \rho r^2 \omega^2 \qquad \therefore \quad \omega = \sqrt{\frac{\sigma}{\rho}} = 2$ rad/s

22. Two opposite forces F_1 = 120N.....

Ans. **1**

Sol. $dL = \frac{T}{A}\frac{dx}{y}$



$T = F_1 - (F_1 - F_2)\frac{x}{L}$

$\int_0^{\Delta L} dl = \frac{(F_1 + F_2)L}{2Ay} = 1\times10^{-9}$ m

23. The difference between (n + 2)th .....

Ans. **8**

Sol. $r_n \;\alpha\; n^2 \quad ; \quad r_{n+2} = k(n+2)^2$

$r_n = kn^2 \quad ; \quad r_{n-2} = k(n-2)^2$

$(n+2)^2 - n^2 = (n-2)^2 \quad \Rightarrow$ n = 8

24. In the arrangement shown, the pendulum.....

Ans. **1**

PART-II (Chemistry)

25. In a gaseous mixture, if an alkane (C_xH_{2x+2}).....

Ans. **(B)**

Sol. $M_{mix.} = \frac{2.(14x+2) + 1.(14y)}{3} = 20$

28x + 14y = 56 ...(1)

$M_{mix.} = \frac{1.(14x+2) + 2.(14y)}{3} = 24$

14x + 28y = 70 ...(2)

$\Rightarrow$ x = 1, y = 2

26. A 1000 gm sample of water is reacted with .....

Ans. **(B)**

Sol. Total heat released = $65.2 \times 10^3 \times \frac{1000}{18}$

Mass of $Ca(OH)_2$ produced = $\frac{1000}{18} \times 74$ gm

$\frac{1000}{18} \times 74 \times 1.2\,(T_f - 25) = \frac{1000}{18} \times 65.2 \times 10^3$

27. Select the **INCORRECT** statement :
(A) N_2O with sodium metal in liquid ammonia forms .....

Ans. **(B)**

Sol. $3\,N_2O + 4\,Na + NH_3 \xrightarrow{NH_3(\ell)} NaN_3 + 3\,NaOH + 2\,N_2$

$\underbrace{2NH_3 + NaOCl}_{\text{dilute solutions}} \xrightarrow{\text{glue}} N_2H_4 + NaCl + H_2O$

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4\,H_2O + Cr_2O_3\downarrow$ (green)

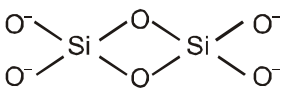
$CaCN_2 + 3\,H_2O \longrightarrow CaCO_3\downarrow + 2\,NH_3$

$Cu^{2+} + 4\,NH_3 \longrightarrow [Cu(NH_3)_4]^{2+}$ (deep blue)

28. Usually a disilicate share only one oxygen of silicate .....

Ans. **(B)**

Sol.



$= Si_2O_6^{4-} \Rightarrow K_4Si_2O_6$

29. An aqueous solution contains Al^{3+} & Zn^{2+} both. .....

Ans. **(A)**

Sol. In excess of NH_4OH ppt of $Zn(OH)_2$ will get dissolved.

30. Diethenylpentadiene is :.....

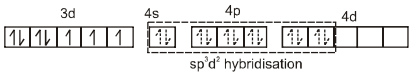
Ans. **(C)**

31. The reaction that gives the following molecule .....
Ans. **(B)**

32. CCl_3–CH=O reacts with chlorobenzene in .....
Ans. **(C)**

33. The 'brown ring' formed at the junction .....
Ans. **(ABC)**

Sol. $[\overset{I}{Fe}(H_2O)_5\overset{+}{N}O]$



Number of unpaired electrons = 3 ; So, $\mu = \sqrt{3(2+3)}$ = 3.87 B.M.

34. Make out the right combination of cell and condition .....
Ans. **(ABD)**
Sol. For spontaneity, $E_{cell} > 0$
$E^o_{cell} = 0$ for concentration cell.

(A) Anode : $H_2 \longrightarrow Zn^+ + 2e^-$
Cathode : $Zn^+ + 2e^- \longrightarrow H_2$

H_2 / anode $\longrightarrow$ H_2 / cathode

$$E_{cell} = -\frac{0.0591}{2}\log\frac{H_2/\text{cathode}}{H_2/\text{anode}} = -\frac{0.0591}{2}\log\frac{P_2}{P_1}$$
= +ve

(B) Anode : $Zn\ (s) \longrightarrow Zn^{2+} + 2e^-$
Cathode : $Zn^{2+} + 2e^- \longrightarrow Zn\ (s)$

Zn^{2+} / cathode $\longrightarrow$ Zn^{2+} / anode

$$E_{cell} = -\frac{0.0591}{2}\log\frac{Zn^{2+}/\text{anode}}{Zn^{2+}/\text{cathode}} = -\frac{0.0591}{2}\log\frac{C_1}{C_2} = +\text{ve}.$$

35. Which of the following statements is/are true for .....
Ans. **(AB)**

Sol. $M = \dfrac{(\%w/w)\times d\times 10}{\text{Mol. mass}_{solute}} = \dfrac{8\times 1.125\times 10}{60} = 1.5\ M$

(% w/v) = (% w/w/) × d = 8 × 1.125 = 9 %

$$\text{Molality} = \frac{1000\times M}{1000\times d - M\times \text{Mol. mass}_{solute}}$$

$$= \frac{1000\times 1.5}{1000\times 1.125 - 1.5\times 60}\ (> 1.5\ m)$$

Since, volume of solution is not given, so number of moles of solute cannot be calculated.

36. An energy of 40.8 eV is required to excite a Hydrogen-like species .....
Ans. **(ACD)**
Sol. $E_2 - E_1 = \Delta E$

$$40.8 = 13.6\ Z^2\left(\frac{1}{1}-\frac{1}{4}\right) = 13.6\times\frac{3}{4}\ Z^2$$

$Z^2 = 4 \Rightarrow Z = 2$
$IE = 13.6\ Z^2 = 13.6\times 2^2 = 54.4$ ev
$KE_1 = 13.6\ Z^2 = 13.6\times 2^2 = 54.4$ ev

$$E_2 = -\frac{13.6\times 2^2}{2^2} = -13.6\ \text{ev}$$

37. Which of the following statement(s) is are correct when a mixture of NaCl .....
Ans. **(ABD)**
Sol. Chromyl chloride confirmatory test for ionic chlorides which forms CrO_2Cl_2 (deepred)

38. Choose the correct options .....:
Ans. **(ABCD)**

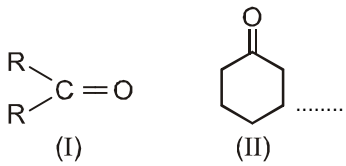
39. Out of the following which reactions give polar .....
Ans. **(ACD)**

40. Choose the correct options..... :
(A) Compound $CH_3(CH_2)NH_2$ react with KOH and $CHCl_3$ produces bad smell.....
Ans. **(ABD)**

41. **Statement - 1 :** For a polytropic process .....
Ans. **(D)**

42. **Statement - 1 :** $PbCl_2$ and AgCl precipitates can be .....
Ans. **(B)**
Sol. **Statement-1 :** $AgCl + 2NH_3 \longrightarrow [Ag(NH_3)_2]Cl$ (soluble complex)
$Pb^{2+} + 2NH_3 + 2H_2O \longrightarrow Pb(OH)_2\downarrow$ (white) $+ 2NH_4^+$
Statement - 2 : $PbCl_2\downarrow + 2Cl^- \longrightarrow [PbCl_4]^{2-}$ [soluble complex]

43. **Statement - 1 :**

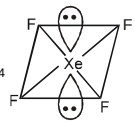


Compound II is more reactive towards .....
Ans. **(A)**

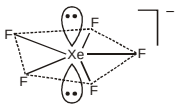
44. Calculate the osmotic pressure (in atm) of a solution .....
Ans. **6**
Sol. $p = (i_1C_1 + i_2C_2 + i_3C_3)\ RT$
= (2 × 0.065 + 3 × 0.02 + 1.2 × 0.05) × 0.08 × 300
= (0.13 + 0.06 + 0.06) × 0.08 × 300 = 6 atm.

45. In how many of the following species the central .....
Ans. **5**

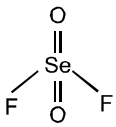
Sol. XeF_4



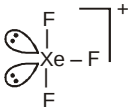
XeF_5^-



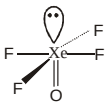
F_2SeO_2



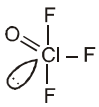
XeF_3^+



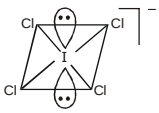
$XeOF_4$



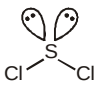
$ClOF_3$



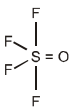
ICl_4^-



SCl_2



OSF_4



46. How many oxides are soluble in moderately .....
Ans. **7**
Sol. SO_3 , Cl_2O_7, N_2O_5 , GeO_2 Acidic
Cr_2O_3 Amphoteric
K_2O , BaO Basic
CO , Neutral
Reactions :

$SO_3 + 2NaOH \longrightarrow Na_2SO_4 + H_2O$

$Cl_2O_7 + 2NaOH \longrightarrow 2NaClO_4 + H_2O$

$N_2O_5 + 2NaOH \longrightarrow 2NaClO_4 + H_2O$

$CO + NaOH \longrightarrow$ No reaction

$K_2O + NaOH \longrightarrow$ No reaction but $K_2O + H_2O \longrightarrow 2KOH$

$BaO + NaOH \longrightarrow$ No reaction but $BaO + H_2O \longrightarrow Ba(OH)_2$

$Cr_2O_3 + 2NaOH + 3H_2O \longrightarrow 2Na[Cr(OH)_4]$

$GeO_2 + 2NaOH \longrightarrow Na_2GeO_3 + H_2O$

47. Observe the following substituted pyridines.....
Ans. **2**

48. Observe the following reaction and .....
Ans. **4**

PART-III (Mathematics)

49. A circle of radius 1 and a square .....
Sol. (A)

50. The largest positive root .....
Sol. (B)

$$x^2 + \frac{1}{x^2} + 2\left(x + \frac{1}{x}\right) - 22 = 0 \Rightarrow \left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) - 24 = 0$$

where we have complete the square. Now put $t = x + x^{-1}$, then $t^2 + 2t - 24 = (t - 4)(t + 6) = 0$ which implies $t = 4$ or $t = -6$. Putting $4 = x + x^{-1}$ leads to $x^2 - 4x + 1 = 0$ as above, and putting $-6 = x + x^{-1}$ leads to $x^2 + 6x + 1 = 0$.

51. Let ABCD be a rectangle and E .....
Sol. (C) Coordinate geometry leads to the correct result as well. Take A = (0, 0), B = (a, 0), C = (a, b) and D = (0, b). Then, the coordinates of E are found to be $\left(\frac{2ab^2}{a^2+b^2}, \frac{2a^2b}{a^2+b^2}\right)$. Then condition EB = EC yields the equation $\frac{2a^2b}{a^2+b^2} = \frac{b}{2}$, which can be reduced to $3a^2 = b^2$ or $\frac{b}{a} = \sqrt{3}$.

52. Shortest distance between .....

Sol. (A) Equation of normal to $y^2 = x^3$ at any point (t^2, t^3) is $y - t^3 = \frac{-2}{3t}(x - t^2)$ will passes through $\left(0, \frac{15}{9}\right)$ i.e., centre of the circle $\frac{5}{3} - t^3 = -\frac{2}{3t}(-t^2)$

$3t^3 + 2t - 5 = 0 \quad \Rightarrow \quad t = 1$

$\Rightarrow$ point corresponding to shortest distance (1, 1)

shortest distance = distance from (1, 1) to centre of circle – radius

$= \sqrt{1 + \frac{4}{9}} - 1 = \frac{\sqrt{13}}{3} - 1$ **Ans. (A)**

53. The value of

$$\int_{-\pi/2}^{\pi/2} e^{-|\tan x|}\left(\cos(\tan x) + \sin(\tan x)\right)\sec^2 x\,dx \text{ .....}$$

Sol. (A) $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{-|\tan x|}(\cos\tan x)\sec^2 x dx + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{-|\tan x|}\sin(\tan x)\sec^2 x dx$

$I = 2\int_0^{\frac{\pi}{2}} e^{-\tan x}(\cos\tan x)\sec^2 x dx + 0$

tanx = t

$\sec^2 x dx = dt$

$I = 2\int_0^{\infty} e^{-t}\cos t\,dt$

$I = 2\left.\frac{e^{-t}(-\cos t + \sin t}{2}\right]_0^{\infty} = 0 - (1(-1 + 0)) = 1$

$I = 1$

Ans. A

54. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$ .....

Sol. (C) point on the L_1 $(2\lambda + 1, \lambda + 2, \lambda + 3)$
point on the L_2 $(\mu, -\mu, 3\mu + 5)$

$2\lambda + 1 = \mu$
$\lambda + 2 = -\mu = -2\lambda - 1$
$3\lambda = -3$
$\lambda = -1, \ \mu = -1$

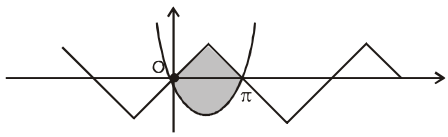
dr' of perpendicular to the L_1 and L_2 $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix}$

$= 4\hat{i} - 5\hat{j} - 3\hat{k}$

equation of line $\vec{r} = (-\hat{i} + \hat{j} + 2\hat{k}) + \lambda(4\hat{i} - 5\hat{j} - 3\hat{k})$

55. Area bounded by the curve .....

Sol. (D)



Area $= 2\int_0^{\pi/2}(x - x^2 + \pi x)dx$

$= 2\left[\frac{x^2}{2} + \frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^{\pi/2}$

$= 2\left[\frac{(1+\pi)}{2}.\left(\frac{\pi^2}{4}\right) - \frac{\pi^3}{24}\right] = \frac{\pi^2}{4}(1+\pi) - \frac{\pi^3}{12}$

$= \frac{\pi^2}{4} + \frac{\pi^3}{4} - \frac{\pi^3}{12} = \frac{\pi^2}{4} + \frac{2\pi^3}{12} = \frac{\pi^2}{4} + \frac{\pi^3}{6}$

56. Let $f(x) = \int_3^x \frac{dt}{\sqrt{t^4 + 3t^2 + 13}}$ .....

Sol. (B) $\frac{dy}{dx} = \frac{1}{\sqrt{x^4 + 3x^2 + 13}}$ when y = f (x)

$\therefore$ $g'(y) = \frac{1}{dy/dx} = \sqrt{x^4 + 3x^2 + 13}$

when y = 0 then x = 3

hence $g'(0) = \sqrt{3^4 + 27 + 13} = \sqrt{121} = 11$ Ans.

57. If the line ax + by + c = 0 .....

Sol. (BC) $xy = 1 \Rightarrow y = \frac{1}{x} \quad \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2}$

$\therefore$ slope of normal $= x^2 = -\frac{a}{b} > 0 \quad \Rightarrow \quad \frac{a}{b} < 0$

$\Rightarrow$ either $a < 0, b > 0$ or $a > 0, b < 0$.

58. Let the minimum value .....

Sol. (BC) $ax^2 - bx + \frac{1}{2a} = a\left(x^2 - \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \frac{1}{2a} - \frac{b^2}{4a} = a$

$\left(x - \frac{b}{2a}\right)^2 + \frac{2-b^2}{4a}$

$\therefore \quad k = \frac{b}{2a}$ and $y_0 = \frac{2-b^2}{4a}$

thus $\frac{b}{2a} = 2.\frac{2-b^2}{4a}$ i.e $b^2 + b - 2 = 0$ i.e $b = -2, 1$

59. Consider two lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ .....

Sol. (BCD) lines $L_1 = \frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and

$L_2 = \frac{x-2}{-4} = \frac{y+1}{1} = \frac{z-6}{1}$

Let shortest distance = d

$d = \left|\frac{(\vec{a}_1 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$\vec{a}_1 = -3\hat{i} + 6\hat{j}$; $\vec{a}_2 = 2\hat{i} - \hat{j} + 6\hat{k}$

$\vec{b}_1 = -4\hat{i} + 3\hat{j} + 2\hat{k}$; $\vec{b}_2 = -4\hat{i} + \hat{j} + \hat{k}$

$d = \frac{|(-5\hat{i} + 7\hat{j} - 6\hat{k}).(\hat{i} - 4\hat{j} + 8\hat{k})|}{|\hat{i} - 4\hat{j} + 8\hat{k}|} = 9$

60. If $3A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ x & -2 & y \end{bmatrix}^T$ .....

Sol. (ABD) $3A \,.\, 3A^T = \begin{bmatrix} -1 & 2 & x \\ -2 & 1 & -2 \\ -2 & -2 & y \end{bmatrix}\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ x & -2 & y \end{bmatrix}$

$= \begin{bmatrix} 1+4+x^2 & 2+2-2x & 2-4+xy \\ 2+2-2x & 4+1+4 & 4-2-2y \\ 2-4+xy & 4-2-2y & 4+4+y^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$\therefore$ $5 + x^2 = 9$, $4 - 2x = 0$, $4 - 2 - 2y = 0$ $xy = 2$

i.e. $x = 2, y = 1$

61. If $z_1 = 5 + 12i$ & $|z_2|$ .....

Sol. (AD) $z_1 = 5 + 12i$, $|z_2| = 4$

$|z_1 + iz_2| \le |z_1| + |z_2| = 13 + 4 = 17$

$\therefore$ $|z_1 + (1 + i)\, z_2| \ge ||z_1| - |1 + i|\, |z_2||$

$= 13 - 4\sqrt{2}$

$\therefore$ min $(|z_1 + (1 + i)\, z_2|) = 13 - 4\sqrt{2}$

$\left|z_2 + \frac{4}{z_2}\right| \le |z_2| + \frac{4}{|z_2|} = 4 + 1 = 5$

$\left|z_2 + \frac{4}{z_2}\right| \ge |z_2| - \frac{4}{|z_2|} = 4 - 1 = 3$

$\therefore$ max $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$ and min $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{5}$

62. If f(x) is a differentiable function .....

Sol. (BCD) $\frac{f(3) - f(1)}{3 - 1} = f'(c)$, for some c such that $1 < c < 3$

$f'(c) = \frac{f(3) - 2}{2} \ge 2$

$\therefore$ $f(3) \ge 6$

63. Let S denote the set .....

Sol. (ABC) The given system of linear equations will have a non-trivial solution if

$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

Expanding the determinant along C_1 we get

$\lambda(-\cos^2\alpha - \sin^2\alpha) - (-\sin\alpha\cos\alpha - \sin\alpha\cos\alpha) - (\sin^2\alpha - \cos^2\alpha) = 0$

$\Rightarrow$ $-\lambda + \sin 2\alpha + \cos 2\alpha = 0$

$\Rightarrow$ $\lambda = \sin 2\alpha + \cos 2\alpha = \sqrt{2}\sin(\pi/4 + 2\alpha)$

$\Rightarrow$ $-\sqrt{2} \le \lambda \le \sqrt{2}$

64. If $f(x) = \begin{cases} \frac{1-[x]}{1+x} & , x \ne 0 \\ 1 & , x = 0 \end{cases}$ .....

Sol. (ABC) $f(x) = \begin{cases} \frac{2}{1+x} & , -1 < x < 0 \\ 1 & , x = 0 \\ \frac{1}{1+x} & , 0 < x < 1 \\ 0 & , 1 \le x < 2 \end{cases}$

$\lim_{x\to 0^-} f(x) = 2$; $\lim_{x\to 0^+} f(x) = 1$; $\lim_{x\to 1^-} f(x) = \frac{1}{2}$; $\lim_{x\to 1^+} f(x) = 0$

65. **STATEMENT -1 :** If x, y, z are .....

Sol. (D) $x + y > z$ i.e. $1 - z > z$ i.e. $1 > 2z$

similarly $1 > 2x$ and $1 > 2y$

$\therefore$ $2x - 1 < 0$, $2y - 1 < 0$, $2z - 1 < 0$

$\therefore$ AM < GM

i.e. $\frac{(2x-1) + (2y-1) + (2z-1)}{3} < \{(2x-1)(2y-1)(2z-1)\}^{1/3}$

$\therefore$ statement-1 is false $\therefore$ statement-2 is true

66. Let y = sinx and y_r .....

Sol. (A) $y = \sin x$ $y_n = \sin\left(x + \frac{n\pi}{2}\right)$

$y_1 = \cos x = \sin\left(x + \frac{\pi}{2}\right)$

$y_2 = \cos\left(x + \frac{\pi}{2}\right) = \sin\left(x + \frac{2\pi}{2}\right)$

$\vdots$

$y_n = \sin\left(x + \frac{n\pi}{2}\right)$

$y_{4(n+1)+k} = \sin\left(x + (4(n+1) + k)\frac{\pi}{2}\right)$

$= \sin\left(x + (4n + k)\frac{\pi}{2} + 2\pi\right) = \sin\left(x + (4n + k)\frac{\pi}{2}\right) = y_{4n+k}$

$\therefore$ Statement-2 is true

Thus $y_{117} = y_{109+8} = y_{109}$

$y_{119} = y_{111+8} = y_{111}$

$y_{125} = y_{113+12} = y_{113}$

$\therefore$ two rows are identical $\therefore$ statement-1 is true

67. **STATEMENT-1 :** Coefficient of x^{51} .....

Sol. **(A)** Coefficient of $x^{\frac{n(n+1)}{2}-4}$ in the expansion of (x – 1) (x^2 – 2) ..... (x^n – n) is – 4 + (–1) (–3) = –1

68. If the equation of common tangent .....

Sol. **(4)** $\lambda^2 = 2 + 2\sqrt{2}$

69. If largest subset of (0, **p**).....

Sol. **(5)** $f'(x) = 12\cos^3 x(-\sin x) + 30\cos^2 x(-\sin x) + 12\cos x(-\sin x)$

$= -3\sin 2x(2\cos^2 x + 5\cos x + 2)$

$= -3\sin 2x(2\cos x + 1)(\cos x + 2)$

When, $f'(x) = 0 \Rightarrow \sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi$

or, $2\cos x + 1 = 0 \Rightarrow x = \frac{2\pi}{3}$

or, $\cos x + 2 = 0$ (not possible)

Sign scheme for f′(x) in [0, π] is as below.

0 – ve π/2 + ve 2π/3 – ve π

f(x) decreases on $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$

and increases on $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

70. The number of three digit numbers .....

Sol. **(2)** (i) $x = y > z \rightarrow {}^{10}C_2$

(ii) $x = y < z \rightarrow {}^{9}C_2$

(iii) $x < y = z \rightarrow {}^{9}C_2$

(iv) $x > y = z \rightarrow {}^{10}C_2$

Total numbers $= 2[{}^{10}C_2 + {}^{9}C_2] = 2.81$

$\Rightarrow a = 2$

71. Let tangent at a point P .....

Sol. **(4)** $x^{2m}\, y^{\frac{n}{2}} = a^{\frac{4m+n}{2}}$

$$2m\,\ell nx + \frac{n}{2}\,\ell ny = \frac{4m+n}{2}\,\ell na$$

$$\frac{(2m)}{x} + \left(\frac{n}{2}\right)\cdot\frac{1}{y}\frac{dy}{dx} = 0\,;\ \frac{dy}{dx} = \left(\frac{-2m}{x}\right)\frac{2y}{n}$$

Let P be the point (x_1, y_1).

So equation of tangent is

$$y - y_1 = \left(\frac{-4m}{n}\cdot\frac{y_1}{x_1}\right)(x - x_1) \quad \text{... (1)}$$

Let tangent meets the axes at A and B respectively

$$A \equiv \left(\frac{4m+n}{4m}x_1, 0\right) \quad B = \left(0, \frac{4m+n}{n}.\, y_1\right)$$

Let P devides AB in ratio μ : 1

$$\therefore\ x_1 = \frac{\mu.0 + 1\cdot\left(\frac{4m+n}{4m}\right)x_1}{\mu+1} \qquad \therefore\ 4m + n = 4m(\mu + 1)$$

Also $\mu(4m + n) = n(\mu + 1)$ $\therefore\ \mu = \frac{n}{4m}$ $\therefore\ \lambda = 4$

72. A ray of light moving parallel .....

Sol. **(2)** Focus of $(y - 2)^2 = 4(x + 1)$ is x + 1 = 1, y – 2 = 0

i.e. (0, 2)