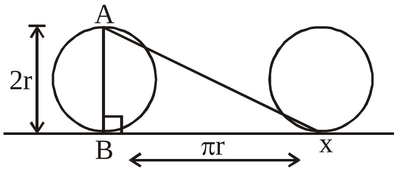
Hints & Solution
SECTION – II

Mathematics

1. (B)

Point A reaches point X, when the wheel makes half a rotation i.e. BX is equal to half the perimeter of the wheel $BX = \frac{1}{2}(2\pi r) = \pi r$



$Ax = \sqrt{4r^2 + (\pi r)^2} = r\sqrt{4+\pi^2}$

Also AB = 2r

Required ratio $\frac{Ax}{AB} = \frac{r\sqrt{4+\pi^2}}{2r} = \frac{1}{2}\sqrt{\pi^2+4}$.

2. (D)

Initial order 100, 99, 98, …,. 3, 2, 1

The cards to be arranged 1, 2, 3, 4, ….., 100

Minimum no. of switches needed between two adjacent cards at a time is

$99 + 98 + \ldots\ldots + 3 + 2 + 1 = \frac{99\times 100}{2} = 4950$

3. (A,C,D)

$f(x, y) = (x^{2010} + y^{2010})^{2010} - (x^{2009})^{(2010)^2} - (y^{2009})^{(2010)^2}$

$\Rightarrow (x^{2010} + y^{2010})^{2010} - (x^{2009\times 2010})^{2010} - (y^{2009\times 2010})^{2010}$

$\Rightarrow (x^{2010} + y^{2010})^{2010} - (x^{2010})^{2010} - (y^{2010})^{2010}$

[$\because (a+b)^n - a^n - b^n$ is always divisible by ab for all n $\in$ N]

$\therefore$ f(x) is divisible by $(xy)^{2010}$

4. (C)

Let $P(A) = 10\times A$

$P(X) = (X-1)(X-2)(X-3)(X-A) + 10\times X$

5. (A,B,C,D)

The point (5, a) will lie on the perpendicular bisector of the line joining (8, -2) and (2, -2). So it's true for every values of 'a'

6. (A,B,C).

Volume of water raised = Volume of the sphere

$\cancel{\pi r^2}\,(H) = \frac{4}{3}\cancel{\pi} r^{\cancel{3}}$

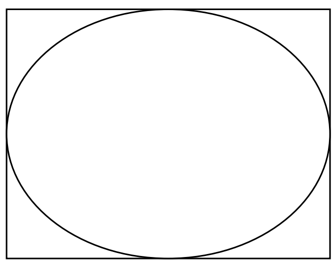
$H = \frac{4}{3} r$

$H = \frac{2}{3} h$

raised height $= \frac{2}{3} h$

water available = $\frac{1}{3} h$

The water will stand at top of the end.



2r = h (given)

7. (D)

$$(x+1)=\log_2\frac{(2^x+3)^2}{(1980-2^{-x})}$$

$$2^{(x+1)}=\frac{(2^x+3)^2}{1980-2^{-x}}$$

$$2\times 2^x=\frac{4^x+9+6\times 2^x}{1980-2^{-x}}$$

$$3960\times 2^x-2=(2^x)^2+6\times 2^x+9$$

Let $2^x = y$

$y^2 - 3954\, y + 11 = 0$

$$(2^x)^2-3954(2^x)+11=0$$

$\Rightarrow 2^{x_1}\times 2^{x_2}=11$

$\Rightarrow 2^{x_1+x_2}=11$

$\Rightarrow x_1 + x_2 = \log_2 11$

8. (A, D)

$P(a)=1+0+0=1$

$P(b)=0+1+0=1$

$P(c)=0+0+1=1$

As it is a quadratic polynomial and P(x) is coming constant at x=a,b,c. So, it means that it is a constant polynomial and its value is 1.

9. (B)

Diameter of sphere = diagonal of cube = $\sqrt{3}\,r=\sqrt{r^2+r^2+r^2}$

Radius of sphere = $\dfrac{\sqrt{3}r}{2}$

For right circular cone h = 2R

So $l=\sqrt{R^2+(2R)^2}=\sqrt{5}\,R$

then radius of sphere = $\dfrac{\Delta}{S}=\dfrac{\frac{1}{2}\times 2R\times 2R}{\frac{(2R+2\sqrt{5}R)}{2}}$

$$\frac{\sqrt{3}r}{2}=\frac{2}{(1+\sqrt{5})}R$$

$$\Rightarrow R=\frac{\sqrt{3}(1+\sqrt{5})r}{4}$$

So curved surface area of collinear is $2\pi R\times h$

$= 2\pi \times R \times 2R$

$= 4\pi R^2$

$$=4\times\pi\times\frac{(\sqrt{3}+\sqrt{15})^2}{16}\times r^2=\pi\left(\frac{\sqrt{3}+\sqrt{15}}{2}\right)^2\times r^2$$

10.(C)

Hypotenuse = $\sqrt{12^2+5^2}=13\text{ cm}$

Volume of resultant double cone = $\dfrac{\pi}{3h}l^2b^2=\dfrac{\pi}{3\times 13}\times 12^2\times 5^2=\dfrac{1200\pi}{13}\text{cm}^3$

Matrix match

1. (A – Q, B – P, C – S, D – R)

2. (A – S, B – PQ, C – PQR, D – S)

(A) (S)

$$x^{\log_3 x} = 9$$

$$\log_3 x \log_3 x = \log_3 3^2$$

$$(\log_3 x)^2 = 2$$

$$\log_3 x = \pm\sqrt{2} \Rightarrow x = 3^{\pm\sqrt{2}}$$

(B) (PQ)

$$\log(35 - x^3) = 3\log(5 - x)$$

$$35 - x^3 = (5 - x)^3$$

$$\Rightarrow 35 - x^3 = 125 - x^3 - 3\times 25x + 3x^2 \times 5$$

$= 15x^2 - 75x + 90 = 0$

$x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

$x(x - 3) - 2(x - 3) = 0$

$(x - 2)(x - 3) = 0$

$x = 2, x = 3$

(C) (PQR)

$$9^{\log_{1/3}(x+2)} = 5^{\log_{1/5}(2x^2+8)}$$

$$\Rightarrow 3^{2\log_{\left(\frac{1}{3}\right)}(x+2)} = 5^{\log_{\left(\frac{1}{5}\right)}(2x^2+8)}$$

$$(x+2)^{-2} = (2x^2+8)^{-1}$$

$$2x^2 + 8 = x^2 + 4x + 4$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$\therefore$ x = 2, 2

(D) (S)

$$2^{2(x^2+1)} - 9.2^{(x^2+1)} + 8 = 0$$

Let $2^{x^2+1} = y$

$$y^2 - 9y + 8 = 0$$

y = 8, y = 1

$$2^{x^2+1} = 8,\ 2^{x^2+1} = 2^3$$

$$x = \pm\sqrt{2}.$$

Integer

1. (5)

$$(x^2 - 7x + 11)^{x^2 - 11x + 30} = 1$$

The LHS can only be 1 if the base is 1 or –1 or the exponent is 0

Case – 1: $x^2 - 11x + 30 = 0$

X = 6, 5

Case – 2: $x^2 - 7x + 11 = 1$

x = 5, 2

Case – 3: $x^2 - 7x + 11 = -1$ and $x^2 - 11x + 30$ must be even

x = 3 or x = 4 (for x = 3, x = 4, power is even)

The possible solution is 2, 3, 4, 5, 6

No. of real solution are 5.

2. (2)

3. (8)

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots=\frac{\pi^2}{6}$$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots..\right)-\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\ldots.\right)$$

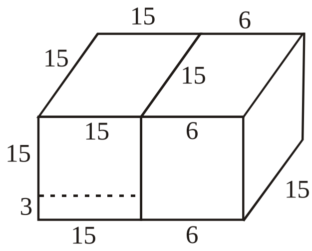
$$=\frac{\pi^2}{6}-\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots.\right)$$

$$=\frac{\pi^2}{6}-\frac{1}{4}\left(\frac{\pi^2}{6}\right)=\frac{4\pi^2-\pi^2}{24}=\frac{3\pi^2}{24}=\frac{\pi^2}{8}$$

4. (1)

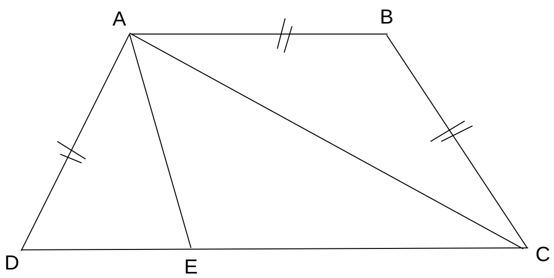
5. (4)

To have least no of cube one must cut largest cube of all with side 15
So left with 3 × 15 × 15 and 6 × 15 × 18



From $3\times15\times15=\frac{3\times15\times15}{3\times3\times3}=25$ cubes
From 6 × 15 × 18, 6 × 6 × 6 cube will be cut.
So 6 such cube can be cutoff.
Remaining size = 3 × 6 × 18
Cubes $\frac{3\times6\times18}{3\times3\times3}=12$
1 + 25 + 6 + 12 = 44
a = 4

6. (2)
Construct AE such that
DE = EC
Now, ΔADE becomes equilateral



$\Rightarrow \angle BAE=180^0-\angle D-\angle DAE=60^0$

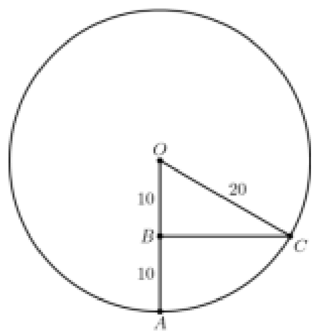
$\therefore \angle EAC=\frac{60^0}{2}=30^0$

$\Rightarrow \angle DAC=90^0$

$\Rightarrow \frac{AD}{AC}=\frac{1}{\sqrt{3}}$

$\Rightarrow k=2$

7. (5)
We can let this circle represent the ferris wheel with center O, and C represent the desired point 10 feet above the bottom. Draw a diagram like the one above. We find out ΔOBC is a $30-60-90$ triangle. That means $\angle BOC=60^0$ and the ferris wheel has made $\frac{60}{360}=\frac{1}{6}$ of a revolution. Therefore, the time it takes to travel that much of a distance is $\frac{1}{6}$th of a minute, or 10 seconds. The answer is 10 so, k= 5. Alternatively, we could also say that ΔABC is congruent to ΔOBC by SAS, so AC is 20, and ΔAOC is equilateral, and $\angle BOC=60^0$



8. (1)
Probability = $\frac{4\cdot3\cdot3+3\cdot4\cdot2}{7\cdot6\cdot5}=\frac{60}{210}=\frac{2}{7}=\frac{p}{q}$.

$4p - q = 4 \times 2 - 7 = 1.$